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On Full Fuzzy Parameterized Soft Set

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Abstract

Some Years back, researchers made attempts in trying to fuzzify set of parameters of soft set. In this paper, we consider a review of "fuzzy parameterized soft set", and improve it by introducing a way to fully fuzzify the set of parameters of soft set. We define the concept "full fuzzy parametrized soft set" and study some of its basic operations. We also illustrate the concept with example.

Keywords: Fuzzy set; Soft set; Fuzzy soft set; Fuzzy parametrized soft set; Full fuzzy parameterized soft set.
MSC2010: 03E72.

Abbreviations
S-set: Soft Set
F-set: Fuzzy Set
FS-set: Fuzzy Soft Set
FPS-set: Fuzzy Parameterized Soft Set
$FPFS$-set: Fuzzy Parameterized Fuzzy Soft Set
$FFPS$-set: Full Fuzzy Parameterized Soft Set
$FPSE$-set: Fuzzy Parameterized Soft Expert Set
$FPBFSE$-set: Fuzzy Parameterized Bipolar Fuzzy Soft Expert Set
$FPIFS$-set: Fuzzy Parameterized Intuitionistic Fuzzy Soft Set
$FPHFLTS$-set: Fuzzy Parameterized Hesitant Fuzzy Linguistic Term Soft Set.

1 Introduction

Many fields like Economics, Engineering and Environmental Sciences deal with uncertainty that may not be successfully modeled classically. Therefore, over the years, many non-classical set theories have been developed for modeling imprecision and uncertainty. F-Set theory proved to be more accurate due to its unique way of describing each element of a set by its membership degree, [1]. But it has a difficulty of inadequacy in parametrization tool associated with the theory, which was pointed out and handled by introducing S-set theory, [2]. S-set served as a mathematical tool in dealing with uncertainties in a parametric manner. The author also outlined several areas of application of the theory such as: Riemann integration, game theory, operations research, probability theory, etc.

With the rapid increase in works relating to S-set theory, Maji *et al* [3] defined some operations on S-set and used the theory to solve some decision making problems ([4]). In [5], Chen *et al* presented a new definition of S-set as an improvement of [4]. Also, scholars like Cagman & Enginoglu [6], introduced and investigated soft matrix theory and applied it to a decision making problem. In sequel, many researchers in various directions also contributed, like Ali *et al* [7], defined some new operations such as: restricted union, extended intersection, restricted intersection, etc. Furthermore, Researchers in a bid to solve problems which are more complicated have introduced F-set into the study of S-set theory, leading to the notion of FS-set. In this direction, Maji *et al* in [8] are the first contributors to defined FS-set. Following this definition, many researchers have come up with interesting applications of the theory. In [9], Roy & Maji investigated some application of FS-set. Yang *et al* [10,11] made some improvement on this concept. In [13], Cagman *et al* defined fuzzy soft set theory and its related properties, and fuzzy soft aggregation operator that allows for more efficient way in dealing with decision making problems. Again, in [14,15], Cagman *et al* introduced the concepts of FPS-set and $FPFS$-set together along with their associated properties. In a related development, Alkhazaleh *et al* [16] introduced the concept of fuzzy parameterized interval-valued FS-set and gave its application in decision making. In [17], the authors introduced the concept of multi Q-fuzzy parameterized soft set.

In addition, many authors consider the application parts of S-set and FS-set theories in different areas of decision making. In [18], Rodriguez *et al* proposed the comparison score based approach to solving FS-set based decision making problems. Also, in [19], Nasef *et al* present another application of S-set in a decision making problem for real estate marketing with the help of rough mathematics, and provide an algorithm to select the optimal choice of an object. Again in [20], the authors applied the notion of FS-set in Sanchez's method [21,22] of decision making. Furthermore, in [23], the authors developed a way to solve intertemporal choice problems like: savings, investments, spendings, etc., for FS-sets. Also, in [24], the authors combined hesitant F-set and multi fuzzy soft set to develop hesitant multi fuzzy soft set and presented an algorithm and a novel approach to hesitant multi fuzzy soft set based decision making problems.

In recent developments, the idea of the concept of FPS-set theory has been a great influence to numerous researchers towards solving more realistic decision making problems. For instance; the work of Rodzi and Ahmad [25] on $FPHFLTS$-set in multi-criteria decision making, which came up by studying the work on hesitant fuzzy linguistic term soft set [26] in a fuzzy parameterized environment. The authors also described some related concepts and consider the fundamental operations of $FPHFLTS$-set, and they were able to develop three different algorithm for solving group decision making. Other contributions include: [27–33].

Up to the present era, all these works, starting from the example given in [2], are choice based on some certain factors; the influence of choice are more to be considered as fuzzy than crisp. This is because, for instance; the word “expensive" is not well defined in a classical sense, and of course, cannot be precisely measured, given that the price of a particular house cannot be said to be just “expensive" or “not expensive". Also, the other parameters are considered not to be well defined as cannot be precisely measured. Just like in the case of FPS-set, [14], with reference to the example given in [2], the parameters were just given fuzzy values. For example; a certain parameter “expensive" is given a fuzzy value, say 0.6, then the fuzzy value of the other parameters with respect to the subset are automatically always 0, i.e to say; $f(0.6/e_1) = \{h_1\}$, Thus, we consider this as a type of generalization of soft set. This is why, in this present work, we consider a more encompassing way in fuzzifying the set of parameters. In this case, the description of each element of the power set of the universe set ($P(U)$) is entirely based on the whole set of parameters, i.e., instead of this function $f\big(\mu(e_1)/e_1\big) = \{h_1\}$ as presented in [14], we use $f\big(\mu(e_1)/e_1, \mu(e_2)/e_2, ..., \mu(e_n)/e_n\big) = \{h_1\}$, which FULLY describe the set $\{h_1\}$ in terms of all the parameters. One reason for our generalization is that in real life, decision making is most time based on several factors (parameters) and not just one. Clearly, the degree to which each parameter contributes to the final decision varies. Thus, our work is a kind of generalization of the work presented in [14], since in our work, the function $f\big(0.6/e_1, 0/e_2, 0/e_3, 0/e_4\big) = \{h_1\}$ means that the subset $\{h_1\}$ is considered expensive (e_1) to the degree 0.6 and to the degree 0 for the other parameters, and is considered to be equivalent to the

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function $f(0.6/e_1) = \{h_1\}$ as presented in [14]. Therefore, the decision on the selection of any element in $P(U)$ is based on the contribution of each of the parameters. This motivated us to introduce the new concept "$FFPS$-set".

The remaining part of this paper is organized as follows: Section 2 contains some basic relevant definitions. Section 3 introduces the new concept ($FFPS$-set). Section 4 studies some basic operations, namely: $A-empty$ and $A-Universal$ $FFPS$-set, $FFPS$-subset, Complement, Intersection, Set difference and union of $FFPS$-set. While section 5 draws conclusion and suggestion for further research.

2 Preliminaries

In this section, we provide some basic definitions following from: [1], [2], [8], [14]. [1] (F-**set**): A pair (f, U) is called a F-set, where U is a universe set and f is a mapping from U into the unit interval $[0, 1]$, (i.e, $f : U \to [0, 1]$) and for every $x \in f$, $\mu_f(x)$ is called the grade of membership of $x \in f$.

[2] (S-**set**): Let U be a universe set, and E be the set of parameters. A pair (F, E) is called a S-set over U if and only if F is a mapping from E into the set of all subsets of the universe set U, i.e., $F : E \to P(U)$, where $P(U)$ is the power set of U.
In other words, S-set over U is a parameterized family of subsets of U.
Every set $F(e)$, for every $e \in E$, from this family may be considered as the set of e-elements of the S-set (F, E) or considered as the set of e-approximate elements of the soft set. Accordingly, we can view a soft set (F, E) as a collection of approximations: $(F, E) = \{F(e) : e \in E\}$.

[8] (FS-**set**): Let I^U be the set of all fuzzy sets of U. Then a pair (f, A) is called a FS-set over U, where A is a subset of the set of parameters E, and f is a mapping from A into I^U. That is, $f : A \to I^U$, and for each $a \in A$, $f(a) = f_a : U \to I$, is a F-set on U.

[14] (FPS-**set**): Let U be an initial universe, E be the set of parameters and A be the fuzzy set over E. A FPS-set $\digamma_A$ on the universe U is defined by the set of ordered pairs

$$\digamma_A = \big\{\big(\mu_A(x)/x, \gamma_A(x)\big) : x \in E, \gamma_A(x) \in P(U), \mu_A(x) \in [0, 1]\big\},$$

Where $P(U)$ is the power set of U and the function

$$\gamma_A : E \to P(U)$$

is called approximate function such that $\gamma_A(x) = \varnothing$ if $\mu_A(x) = 0$

Characterized by the membership function

$$\mu_A : E \to [0, 1]$$

The value $\mu_A(x)$ of an element x of the parameters represents its degree of importance. And it is solely based on the desirability of the decision maker.

Hence, this means that the approximate function is defined from fuzzy subset of E to the crisp subset of the Universe set U.

Note that from now on, the sets of all FPS-sets over U will be denoted by $FPS(U)$.

[14] (A-**empty** FPS-**set**): Let $\digamma_A \in FPS(U)$. If $\gamma_A(x) = \varnothing$ for all $x \in E$, then $\digamma_A$ is called an A-empty FPS-set, denoted by $\digamma_{\varnothing_A}$.

If $A = \varnothing$, then $\digamma_A$ is called an empty FPS-set, denoted by $\digamma_{\varnothing}$.

[14] (A-**universal** FPS-**set**): Let $\digamma_A \in FPS(U)$. If A is a crisp subsets of E and $\gamma_A(x) = U$ for all $x \in A$, then $\digamma_A$ is called A-universal FPS-set, denoted by $\digamma_{\tilde{A}}$.

If $A \in E$, then the A-universal FPS-set is called universal FPS-set, denoted by $\digamma_{\tilde{E}}$

[14] (**FPS-subset**): Let $\digamma_A, \digamma_B \in FPS(U)$. Then, $\digamma_A$ is an FPS-subset of $\digamma_B$, denoted by $\digamma_A \tilde{\subseteq} \digamma_B$, If $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \subseteq \gamma_B(x)$ for all $x \in E$.

[14] (**FPS-equal sets**): Let $\digamma_A, \digamma_B \in FPS(U)$. Then, $\digamma_A$ and $\digamma_B$ are FPS-equal, written as $\digamma_A = \digamma_B$, if and only if $\mu_A(x) = \mu_A(x)$ and $\gamma_A(x) = \gamma_B(x)$ for all $x \in E$.

[14] (**Complement of FPS-set**): Let $\digamma_A \in FPS(U)$. Then, complement $\digamma_A$, denoted by $\digamma_A^{\tilde{c}}$, is an FPS-set defined by the approximate and membership functions as

$$\mu_{A^{\tilde{c}}}(x) = 1 - \mu_A(x) \text{ and } \gamma_{A^{\tilde{c}}}(x) = U \backslash \gamma_A(x)$$

[14] (**Union of FPS-set**): Let $\digamma_A, \digamma_B \in FPS(U)$. Then, union $\digamma_A$ and $\digamma_B$, denoted by $\digamma_A \tilde{\cup} \digamma_B$, is defined by

$$\mu_{\digamma_A \tilde{\cup} \digamma_B}(x) = \max\{\mu_A(x), \mu_B(x)\} \text{ and } \gamma_{\digamma_A \tilde{\cup} \digamma_B}(x) = \gamma_A(x) \cup \gamma_B(x),$$
$$\text{for all } x \in E.$$

[14] (**Intersection of FPS-set**): Let $\digamma_A, \digamma_B \in FPS(U)$. Then, intersection of $\digamma_A$ and $\digamma_B$, denoted by $\digamma_A \tilde{\cap} \digamma_B$, is an FPS-sets defined by the approximate and membership functions

$$\mu_{\digamma_A \tilde{\cap} \digamma_B}(x) = \min\{\mu_A(x), \mu_B(x)\} \text{ and } \gamma_{\digamma_A \tilde{\cap} \digamma_B}(x) = \gamma_A(x) \cap \gamma_B(x),$$
$$\text{for all } x \in E.$$

3 The Concept of Full Fuzzy Parameterized Soft Set

In this section, we introduce the notion of $FFPS$-**set**.

Let E be the set of all parameters and let U be an initial universe with $P(U)$ the power set of U. Here, we fuzzify E to $\tilde{E}$. In this case, $\tilde{E}$ is considered to be the set of all possible fuzzy set over E. Therefore, a fuzzy set $\hat{y}$ over E is as:

$$\hat{y} = \Big(\mu_{\hat{y}}(x_1)/x_1,\ \mu_{\hat{y}}(x_2)/x_2, ..., \mu_{\hat{y}}(x_n)/x_n\Big),$$

characterized by the membership function

$$\mu_{\hat{y}} : E \to [0, 1],\ E = \{x_1, x_2, ..., x_n\}.$$

This is so, since every fuzzy set is completely and uniquely defined.
Hence, $\tilde{E}$ is said to contain the elements y_i, for $i = 1, 2, 3, \ldots$
Therefore, each $\hat{y} \in \tilde{E}$ is considered as an indicator of the degree to which the parameters in E are considered intra-dependently.
Meanwhile, $\mu_{\hat{y}}(x)$ is the degree to which the parameter $x \in E$ is considered.

[14] Let $\tilde{A} \subset \tilde{E}$. An $FFPS$-set $F_{\tilde{A}}$ on the universe U is given as:

$$F_{\tilde{A}} = \Big\{\Big(\hat{y},\ f_{\tilde{A}}(\hat{y})\Big) : \hat{y} \in \tilde{A},\ f_{\tilde{A}}(\hat{y}) \in P(U),\ \mu_{\hat{y}}(x) \in [0, 1],\ x \in E\Big\}$$

Where $f_{\tilde{A}} : \tilde{E} \to P(U)$ represents the approximation function of $F_{\tilde{A}}$ such that
$f_{\tilde{A}}(\hat{y}) = \emptyset$ whenever $\mu_{\hat{y}}(x) = 0\ \forall x \in E$ and $\hat{y} \in \tilde{A}$.

Therefore, throughout this work, the set of all $FFPS$-set will be denoted by $FFPSS(U, \tilde{E})$.

As an illustration, we use the following example, presented in [2] for more detail discussion.

Given the following initial universe, U = the set of houses under consideration for sales, E be the set of parameters.
Suppose: $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{x_1, x_2, x_3, x_4\}$
Where we have six houses in the defined universe, and $x_i \in E$ for $i = 1, 2, 3, 4$, stands for the parameters: x_1 = expensive, x_2 = beautiful, x_3 = wooden, x_4 = in green surrounding.

Assume:

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$$\tilde{A} = \Big\{\hat{y}_1 = \Big(\frac{0.1}{x_1}, \frac{0.2}{x_2}, \frac{0.3}{x_3}, \frac{0.4}{x_4}\Big), \hat{y}_2 = \Big(\frac{0.3}{x_1}, \frac{0.5}{x_2}, \frac{0.1}{x_3}, \frac{0.2}{x_4}\Big), \hat{y}_3 = \Big(\frac{0.4}{x_1}, \frac{0.2}{x_2}, \frac{0.2}{x_3}, \frac{0.3}{x_4}\Big)\Big\}$$

$$\tilde{B} = \Big\{\hat{z}_1 = \Big(\frac{0.2}{x_1}, \frac{0.2}{x_2}, \frac{0.3}{x_3}, \frac{0.1}{x_4}\Big), \hat{z}_2 = \Big(\frac{0.5}{x_1}, \frac{0.3}{x_2}, \frac{0.5}{x_3}, \frac{0.4}{x_4}\Big), \hat{z}_3 = \Big(\frac{0.5}{x_1}, \frac{0.5}{x_2}, \frac{0.8}{x_3}, \frac{0.4}{x_4}\Big)\Big\}$$

Suppose that:
$f_{\tilde{A}}(\hat{y}_1) = \{h_1\}$
$f_{\tilde{A}}(\hat{y}_2) = \{h_2, h_4\}$
$f_{\tilde{A}}(\hat{y}_3) = \{h_3, h_4, h_5\}$
$f_{\tilde{B}}(\hat{z}_1) = \{h_1, h_2, h_3\}$
$f_{\tilde{B}}(\hat{z}_2) = \{h_4\}$
$f_{\tilde{B}}(\hat{z}_3) = \{\}$

Where $f_{\tilde{A}}(\hat{y}_i)$ is a subset of U whose elements match the fuzzy set $\hat{y}_i$ over E.
Therefore, $f_{\tilde{A}}(\hat{y}_2)$ represents houses which are considered
expensive to the degree 0.3
beautiful to the degree 0.5
wooden to the degree 0.1
in green surrounding 0.2
The functional value of $f_{\tilde{A}}(\hat{y}_2)$ is $\{h_2, h_4\}$. Hence, this means that $\hat{y}_2$ indicates the degree to which the parameters in E intra-dependently give a description of the houses h_2 and h_4 (i.e., describes the degree of attractiveness of the houses h_2 and h_4). In this case, both houses are considered expensive to the degree 0.3, considered beautiful to the degree 0.5, considered wooden to the degree 0.1 and considered in a green surrounding to the degree 0.2.

Thus, considering $\tilde{E}$ as a set of customers for the purchase of the houses, then the customer $\hat{y}_2$ values the houses h_2 and h_4 as above. Considering the example above, it is clear in general, that crisp parameterized things contains high degree of uncertainty. Therefore, dealing with this, we will need to fuzzify the set of parameters in use.

Now, considering example 2.1 in [3], parameter e_1, is the predicate "expensive house" having the approximate value set {house h_2; house h_4}. In this example, the question of uncertainty arises. Therefore, the question is "does it mean houses h_2 and h_4 can only be expensive or not?". Assuming this is so, then another question arises "what condition(s) or situation(s) necessitated this decision?". Before looking into the questions above, we first consider the work as presented by Cagman *et al* in [14], the authors considered dealing with the notion of "not absoluteness" of valuation (i.e., not using the valuation 0 or 1 only) by partially fuzzifying the set of parameters E. i.e, attaching an independent degree to each of parameters in E. We consider this attempt by the authors in [14] not sufficient to answer the questions. This is because all they did was to individually attach numeric values within the unit interval to the elements in E. This still have the same kind of valuation as in the classical soft set, in which case, parameters are still mapped individually. Therefore in this research, we attempt to proffer a solution to the questions above. Using example 3, we illustrate our proposed solution to the above questions.

Thus, by example 3, our predicate part for each approximation is fuzzy. In our case, unlike [14], all parameters in E are attached with choice numeric values and all together assigned against possible approximate value set. Therefore we have that parameters intra-dependently relate, i.e, putting into consideration the impact parameters have on one another.

With this, we consider that to a good extent, imprecision and uncertainty in crisp valuation are dealt with.

So, $f_{\tilde{A}}(\hat{y})$ with $\hat{y} = \big(\mu_{\hat{y}}(x_1)/x_1,\ \mu_{\hat{y}}(x_2)/x_2,...,\mu_{\hat{y}}(x_n)/x_n\big)$ is considered to be $\hat{y}$-approximate elements of $FFPSS(U, \tilde{E})$. Thus $FFPSS(U, \tilde{E})$ gives a description of the "attractiveness of the house" under consideration base on the valuation of the parameters $x_1, x_2, \ldots, x_n$.

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Therefore in this work, we will also be dealing with multiple entries.
From the example given above, we consider:
$f_{\tilde{A}}(\hat{y}_2) = \{h_2, h_4\}$
$f_{\tilde{A}}(\hat{y}_3) = \{h_3, h_4, h_5\}$

Thus, the element (house h_4) will have the valuations $(0.3/x_1,\ 0.5/x_2,\ 0.1/x_3,\ 0.2/x_4)$ and $(0.4/x_1,\ 0.2/x_2,\ 0.2/x_3,\ 0.3/x_4)$ respectively.

In this case, to find the $\hat{y}-approximate$ of h_4, we consider the arithmetic mean of $\hat{y}_2$ and $\hat{y}_3$ componentwise.

So h_4 approximation is:

$$\left(\tfrac{0.3+0.4}{2}/x_1,\ \tfrac{0.5+0.2}{2}/x_2,\ \tfrac{0.1+0.2}{2}/x_3,\ \tfrac{0.2+0.3}{2}/x_4\right)$$

$$= \left(0.35/x_1,\ 0.35/x_2,\ 0.15/x_3,\ 0.25/x_4\right).$$

$$\text{Thus, } f_{\tilde{A}}(\hat{y}) = (0.35/x_1,\ 0.35/x_2,\ 0.15/x_3,\ 0.25/x_4) = \{h_4\}$$

In a tabular form, we can represent the $FFPS$-set as follows:

Table 1: $FFPS$-set Table

U/E	x_1	x_2	x_3	x_4
h_1	$\frac{0.1+0.2}{2} = 0.15$	$\frac{0.2+0.2}{2} = 0.2$	$\frac{0.3+0.3}{2} = 0.3$	$\frac{0.4+0.3}{2} = 0.35$
h_2	$\frac{0.3+0.2}{2} = 0.25$	$\frac{0.5+0.2}{2} = 0.35$	$\frac{0.1+0.3}{2} = 0.2$	$\frac{0.2+0.3}{2} = 0.25$
h_3	$\frac{0.4+0.2}{2} = 0.3$	$\frac{0.2+0.2}{2} = 0.2$	$\frac{0.2+0.3}{2} = 0.25$	$\frac{0.3+0.3}{2} = 0.3$
h_4	$\frac{0.3+0.4+0.2}{3} = 0.3$	$\frac{0.5+0.2+0.2}{3} = 0.3$	$\frac{0.1+0.2+0.3}{3} = 0.2$	$\frac{0.2+0.3+0.3}{3} = 0.27$
h_5	0.4	0.2	0.2	0.3

Interpretation (for house h_1):
h_1 is expensive to the degree 0.15, beautiful to the degree 0.2, wooden to the degree 0.3 and in green surrounding to the degree 0.35.

Clearly, this tabular representation of *FFPS-Set* is a form of generalization of table 1(Tabular representation of a soft set) given by Maji et al in [3].

In which case:
$f(e_1) = f_{\tilde{A}}(\hat{y}_1) = \{h_2, h_4\}$

$\hat{y}_1 = (1/e_1,\ 0/e_2,\ 0/e_3,\ 0/e_4,\ 0/e_5)$

$f(e_2) = f_{\tilde{A}}(\hat{y}_2) = \{h_1, h_3\}$

$\hat{y}_2 = (0/e_1,\ 1/e_2,\ 0/e_3,\ 0/e_4,\ 0/e_5)$

$f(e_3) = f_{\tilde{A}}(\hat{y}_3) = \{h_3, h_4, h_5\}$

$\hat{y}_3 = (0/e_1,\ 0/e_2,\ 1/e_3,\ 0/e_4,\ 0/e_5)$

$f(e_4) = f_{\tilde{A}}(\hat{y}_4) = \{h_1, h_3, h_5\}$

$\hat{y}_4 = (0/e_1,\ 0/e_2,\ 0/e_3,\ 1/e_4,\ 0/e_5)$

$f(e_5) = f_{\tilde{A}}(\hat{y}_5) = \{h_1\}$

$\hat{y}_5 = (0/e_1,\ 0/e_2,\ 0/e_3,\ 0/e_4,\ 1/e_5)$

Table 2: S-set Table

U/E	e_1	e_2	e_3	e_4	e_5
h_1	0	1	0	1	1
h_2	1	0	0	0	0
h_3	0	1	1	1	0
h_4	1	0	1	0	0
h_5	0	0	1	1	0

Interpretation (for house,h_1):
h_1 is not expensive, not wooden, but beautiful, cheap and in green surrounding. Thus it is clear that the question of "to what extent...?" is not put into consideration. Therefore for example the phrase "not expensive" is absolute!

Also, in the same way we represent in tabular form, the FPS-set presented by Cagman et al in [14]:

$f_X(x_2) = f_{\tilde{A}}(\hat{y}) = \{u_2, u_5\}$

$\hat{y_1} = (0/x_1,\ 0.8/x_2,\ 0/x_3,\ 0/x_4,\ 0/x_5)$

$f_X(x_3) = f_{\tilde{A}}(\hat{z}) = \{u_1, u_2, u_3, u_4\}$

$\hat{y_2} = (0/x_1,\ 0/x_2,\ 0.3/x_3,\ 0/x_4,\ 0/x_5)$

$f_X(x_4) = f_{\tilde{A}}(\hat{t}) = \{u_1, u_2, u_4, u_5\}$

$\hat{y_3} = (0/x_1,\ 0/x_2,\ 0/x_3,\ 0.5/x_4,\ 0/x_5)$

$f_X(x_5) = f_{\tilde{A}}(\hat{y}) = \{u_1, u_3\}$

$\hat{y_4} = (0/x_1,\ 0/x_2,\ 0/x_3,\ 0/x_4,\ 0.6/x_5)$

Interpretation (for object u_2):
Not x_1, it is x_2 to the degree 0.8, it is x_3 to the degree 0.3, it is x_4 to the degree 0.5 and not x_5.
Viewing table 3 above, we still have the presence of crispness in the representation even though with the presence of other degree such as 0.2. But is not always the case that other parameters do not have some degree of influence on the value set.
Thus, we consider the work by Cagman et al in [14] as a partial fuzzy parameterized soft set ($PFPS$-set). This gives rise to our name; $FFPS$-set. So, our work seek for full graded membership of set of parameters.

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Table 3: FPS-set Table

U/E	x_1	x_2	x_3	x_4	x_5
u_1	0	0	0.3	0.5	0.6
u_2	0	0.8	0.3	0.5	0
u_3	0	0	0.3	0	0.6
u_4	0	0	0.3	0.5	0
u_5	0	0.8	0	0.5	0

4 Defining Operations on FFP-soft set

[14] Let $F_{\tilde{A}} \in FFPSS(U, \tilde{E})$, then;

1. $F_{\tilde{A}}$ is called the $\tilde{A} - empty$ $FFPS$-set if $F_{\tilde{A}}(\hat{y}) = \emptyset$, $\forall \hat{y} \in \tilde{A}$ and denoted by $F_{\emptyset_{\tilde{A}}}$. If $\tilde{A} = \emptyset$, then $\tilde{A} - empty$ $FFPS$-set is called the empty $FFPS$-set, denoted by $F_{\emptyset}$.
2. $F_{\tilde{A}}$ is called $\tilde{A} - Universal$ $FFPS$-set if $F_{\tilde{A}}(\hat{y}) = U$ $\forall \hat{y} \in \tilde{A}$, denoted by $F_{\tilde{A}}$. If $\tilde{A} = \tilde{E}$, then $\tilde{A} - Universal$ $FFPS$-set is called the universal $FFPS$-set, denoted by $F_{\tilde{E}}$.

Let $\tilde{A}$, $\tilde{B} \subseteq \tilde{E}$ and $F_{\tilde{A}}$, $F_{\tilde{B}} \in FFPSS(F, \tilde{E})$, then $F_{\tilde{A}}$ is Full Fuzzy Parameterized Soft Subset ($FFPS$-subset) of $F_{\tilde{B}}$, denoted by $F_{\tilde{A}} \subseteq F_{\tilde{B}}$ if the following condition holds: $\forall$ $y_i \in \tilde{A}$ $\exists$ $z_j \in \tilde{B}$ such that $y_i \leq z_j$ and $f_{\tilde{A}}(y_i) \subseteq f_{\tilde{B}}(z_j)$.

Let $F_{\tilde{A}}$, $F_{\tilde{B}} \in FFPSS(F, \tilde{E})$, then $F_{\tilde{A}} \subseteq F_{\tilde{B}}$ does not imply that every element in $F_{\tilde{A}}$ are in $F_{\tilde{B}}$ compare to the case of crisp set.
Also, the element z_j in $\tilde{B}$ is not unique.
(This Remark was highlighted from [14])

Let E=$\{x_1, x_2, x_3, x_4\}$ be set of parameters and U=$\{h_1, h_2, h_3, h_4\}$ be the initial universe. Assume $\tilde{A}$, $\tilde{B} \subset \tilde{E}$, such that
$\tilde{A} = \big\{\hat{y}_1 = (0.1/x_1,\ 0.5/x_2,\ 0.6/x_3,\ 0.4/x_4),\ \hat{y}_2 = (0.3/x_1,\ 0.4/x_2,\ 0.3/x_3,\ 0.5/x_4)\big\}$
$\tilde{B} = \big\{\hat{z}_1 = (0.4/x_1,\ 0.5/x_2,\ 0.4/x_3,\ 0.6/x_4),\ \hat{z}_2 = (0.3/x_1,\ 0.3/x_2,\ 0.4/x_3,\ 0.7/x_4),\ \hat{z}_3 = (0.7/x_1,\ 0.6/x_2,\ 0.6/x_3,\ 0.5/x_4)\big\}$,
where: $f_{\tilde{A}}(\hat{y}_1) = \{h_2, h_3\}$, $f_{\tilde{A}}(\hat{y}_2) = \{h_1, h_4\}$, $f_{\tilde{B}}(\hat{z}_1) = \{h_1, h_2, h_3\}$, $f_{\tilde{B}}(\hat{z}_2) = \{h_2\}$, $f_{\tilde{B}}(\hat{z}_3) = \{h_2, h_3\}$.

Since $\mu_{\hat{y}_1} \leq \mu_{\hat{z}_3}$ $\forall$ $x \in \tilde{E}$, Then $\hat{y}_1 \leq \hat{z}_3$ and $f_{\tilde{A}}(\hat{y}_1) \subseteq f_{\tilde{B}}(\hat{z}_2)$
Likewise, since $\mu_{\hat{y}_2} \leq \mu_{\hat{z}_1}$ $\forall$ $x \in \tilde{E}$, Then $\hat{y}_2 \leq \hat{z}_1$ and $f_{\tilde{A}}(\hat{y}_2) \subseteq f_{\tilde{B}}(\hat{z}_1)$

An immediate consequence of the above definition of subsetness is the following.

Let $F_{\tilde{A}} \subseteq F_{\tilde{B}}$. Then if $F_{\tilde{A}}$ and $F_{\tilde{B}}$ are $\tilde{A}$-universal, then $\tilde{F}_{\tilde{A}} \tilde{\subseteq} \tilde{F}_{\tilde{B}}$

Proof. Assuming that $F_{\tilde{A}} \subseteq F_{\tilde{B}}$, thus follows that $\forall$ $\hat{y}_i \in \tilde{A}$ $\exists$ $\hat{z}_j \in \tilde{B}$ such that $\hat{y}_i \leq \hat{z}_j$ and $f_{\tilde{A}}(\hat{y}_i) \subseteq f_{\tilde{B}}(\hat{z}_j)$
Now, let $\tilde{F}_{\tilde{A}}$ and $\tilde{F}_{\tilde{B}}$ be $\tilde{A}$-universal and $\tilde{B}$-universal FFP-soft set, it follows that $\tilde{f}_{\tilde{A}}(\hat{y}) = U$ $\forall$ $\hat{y} \in \tilde{A}$ and $\tilde{f}_{\tilde{B}}(\hat{z}) = U$ $\forall$ $\hat{z} \in \tilde{B}$. Then $\tilde{f}_{\tilde{A}} = U$, $\forall \hat{y} \in \tilde{A}$ and $\forall \hat{z} \in \tilde{B}$.
Thus, we have that
$\forall \hat{y}_i \in \tilde{A}$ $\exists$ $\hat{z}_j$ such that $\hat{y}_i \leq \hat{z}_j \wedge \tilde{f}_{\tilde{A}}(\hat{y}_i) \subseteq \tilde{f}_{\tilde{B}}(\hat{z}_j)$
Hence, $\tilde{F}_{\tilde{A}} \tilde{\subseteq} \tilde{F}_{\tilde{B}}$. □

Let $F_{\tilde{A}}$, $F_{\tilde{B}} \in$ *FFPSS(U,E)*. If $\tilde{A} \subseteq \tilde{B}$ and $f_{\tilde{A}}(\hat{y}) \subseteq f_{\tilde{B}}(\hat{y})$, then $F_{\tilde{A}} \tilde{\subseteq} F_{\tilde{B}}$

Proof. The prove follows easily. □

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The converse of proposition 4 does not hold. This can easily be seen from example 4

Let $F_{\tilde{A}} \in$ *FFPSS(U,$\tilde{E}$)*. Then the complement of $F_{\tilde{A}}$ denoted by $F^c_{\tilde{A}^c}$ is given as

$$F^c_{\tilde{A}^c} = \{(\hat{y}^c, f_{\tilde{A}^c}(\hat{y}^c)) : \hat{y}^c \in \tilde{E} \backslash \tilde{A},\ f_{\tilde{A}^c}(\hat{y}^c) \in P(U) \backslash f_{\tilde{A}}(\hat{y})\}$$

Where $\hat{y}^c$ is associated with the membership function $\mu_{\hat{y}^c} : E \to [0, 1]$ defined by $\mu_{\hat{y}^c}(x) := 1 - \mu_{\hat{y}}(x)$, $\forall x \in E$ and $f_{\tilde{A}^c}(\hat{y}^c) := U \backslash f_{\tilde{A}}(\hat{y})\ \forall \hat{y} \in \tilde{A}$

The cardinality of $\tilde{A}$ and $\tilde{A}^c$ are equal (i.e, $|\tilde{A}| = |\tilde{A}^c|$) and $|F^c_{\tilde{A}^c}| = |F_{\tilde{A}}|$.

Assume that $U = \{h_1, h_2, h_3\}$ and $E = \{x_1, x_2, x_3\}$ are the initial universe and the set of parameters respectively.

$$\text{Suppose } \tilde{A} = \{\hat{y}_1 = (0.1/x_1, 0.2/x_2, 0.5/x_3), \hat{y}_2 = (0.3/x_1, 0.1/x_2, 0.5/x_3), \hat{y}_3 = (0.2/x_1, 0.3/x_2, 0.1/x_3)\}$$

and $f_{\tilde{A}}(\hat{y}_1) = \{h_1\}$, $f_{\tilde{A}}(\hat{y}_2) = \{h_1, h_3\}$, $f_{\tilde{A}}(\hat{y}_3) = \{h_3\}$.

$$\text{Then } \tilde{A}^c = \{\hat{y}^c_1 = (0.9/x_1, 0.8/x_2, 0.5/x_3), \hat{y}^c_2 = (0.7/x_1, 0.9/x_2, 0.5/x_3), \hat{y}^c_3 = (0.8/x_1, 0.7/x_2, 0.9/x_3)\}.$$

and $f_{\tilde{A}^c}(\hat{y}_1)^c = \{h_2, h_3\}$, $f_{\tilde{A}^c}(\hat{y}_2)^c = \{h_2\}$, $f_{\tilde{A}^c}(\hat{y}_3)^c = \{h_1, h_2\}$.

Let $F_{\tilde{A}}$, $F_{\tilde{B}} \in$ *FFPSS(U,$\tilde{E}$)*, then the intersection of $F_{\tilde{A}}$ and $F_{\tilde{B}}$ written as $F_{\tilde{A}} \cap F_{\tilde{B}}$ is given as

$$F_{\tilde{A}} \cap F_{\tilde{B}} = \{(\hat{t}, f_{\hat{C}}(\hat{t})) : \hat{t} \in \hat{C} = \tilde{A} \cap \tilde{B},\ f_{\hat{C}}(\hat{t}) = f_{\tilde{A}}(\hat{y}) \cap f_{\tilde{B}}(\hat{z}) \in P(U)\}$$

where $\mu_{\hat{t}}(x) : E \to [0, 1]$ is the membership function associated with $\hat{t} \in \tilde{C}$ and defined as:

$$\mu_{\hat{t}}(x) = \mu_{\hat{y}}(x) \wedge \mu_{\hat{z}}(x) = min\{\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\},\ \forall x \in E,\ \hat{y} \in \tilde{A},\ \hat{z} \in \tilde{B}$$

We note that the choice of $\hat{y}$ and $\hat{z}$ are arbitrary.

Let F_C be the intersection of F_A and F_B. Clearly, as in the classical set theory, $F_C \subseteq F_A$ and F_B. From our definition of subsetness, obviously from the example below, $F_C \subseteq F_A$ and $F_C \subseteq F_B$. Obviously if $|A| \leq |B|$, then $|C| = |A|$

Let $U = \{h_1, h_2, h_3, h_4\}$ and $E = \{x_1, x_2, x_3\}$ be the initial universe and the set of parameters respectively.

Suppose:

$$F_{\tilde{A}} = \{(\hat{y}_1 = (0.1/x_1, 0.2/x_2, 0.5/x_3), \{h_2, h_3\}), (\hat{y}_2 = (0.3/x_1, 0.1/x_2, 0.5/x_3), \{h_1, h_4\}), (\hat{y}_3 = (0.2/x_1, 0.3/x_2, 0.1/x_3), \{h_1, h_2, h_3\})\}.$$

$$F_{\tilde{B}} = \{(\hat{z}_1 = (0.2/x_1, 0.2/x_2, 0.3/x_3), \{h_2, h_4\}), (\hat{z}_2 = (0.5/x_1, 0.2/x_2, 0.4/x_3), \{h_1, h_3\})\}$$

$$\text{Thus, } F_{\tilde{A}} \cap F_{\tilde{B}} = \Big\{\Big((0.1, 0.2, 0.3), \{h_2\}), ((0.2, 0.2, 0.1), \{h_1, h_3\}\Big)\Big\}.$$

In notion of extended intersection of soft sets introduced in [7], considered in terms of FFP-soft set fails, since the condition of identical approximate element is not considered in our definition of subsetness.

Let $F_{\tilde{A}}$, $F_{\tilde{B}} \in$ *FFPSS(U,$\tilde{E}$)*. Then, the set difference of $F_{\tilde{A}}$ and $F_{\tilde{B}}$, written as $F_{\tilde{A}} - F_{\tilde{B}}$ is given as

$$F_{\tilde{A}} - F_{\tilde{B}} = \{(\hat{t}, f_{\tilde{C}}(\hat{t})) : \hat{t} \in \tilde{A} - \tilde{B},\ f_{\tilde{C}}(\hat{t}) = f_{\tilde{A}}(\hat{y}) - f_{\tilde{B}}(\hat{z}) \in P(U)\}$$

Where $\mu_{\hat{t}} : E \to [0, 1]$ is the membership function associated with $\hat{t} \in \tilde{C}$ and defined as

$\mu_{\hat{t}}(x) = \mu_{\hat{y}}(x) \wedge \big(1 - \mu_{\hat{z}}(x)\big), \forall x \in E$

Where $\mu_{\hat{y}}(x) \wedge \big(1 - \mu_{\hat{z}}(x)\big) = min\{\mu_{\hat{y}}(x), 1 - \mu_{\hat{z}}(x)\} \ \forall \ \hat{y} \in \tilde{A}$ and $\hat{z} \in \tilde{B}$

and $f_{\tilde{A}}(\hat{y})$ - $f_{\tilde{B}}(\hat{z}) = f_{\tilde{A}}(\hat{y}) \cap f_{\tilde{B}^c}(\hat{z}^c)$
From 4 above, we have that;
$F_{\tilde{A}} - F_{\tilde{B}} = \Big\{ \Big((0.1, 0.2, 0.5), \{h_2\} \Big), \big((0.2, 0.3, 0.1), \{h_2\} \Big) \Big\}$

Note: We used $y_1 - z_2$ and $y_3 - z_2$
Let $F_{\tilde{A}}$, $F_{\tilde{B}} \in$ *FFPSS(U,$\tilde{E}$)*, the union of $F_{\tilde{A}}$ and $F_{\tilde{B}}$ written as $F_{\tilde{A}} \cup F_{\tilde{B}}$ is given as;

$F_{\tilde{A}} \cup F_{\tilde{B}} = \big\{(\hat{t}, f_{\hat{C}}(\hat{t})) : \hat{t} \in \hat{C} = \tilde{A} \cup \tilde{B},\ f_{\hat{C}}(\hat{t}) = f_{\tilde{A}}(\hat{y}) \cup f_{\tilde{B}}(\hat{z}) \in P(U)\big\}$

Where $\tilde{A} \cup \tilde{B} = \{\hat{y} \vee \hat{z} : \hat{y} \in \tilde{A}, \hat{z} \in \tilde{B}\}$

and $\hat{y} \vee \hat{z} = max\{\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\}, \forall x \in E$

Therefore, for $\hat{t} = \hat{y} \vee \hat{z} \in \tilde{C}$ with the membership function $\mu_{\hat{t}} : E \to [0, 1]$, define as $\mu_{\hat{t}}(x) = max\{\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\} \ \forall x \in E$.
Also, from 4 above:

$$F_{\tilde{A}} \cup F_{\tilde{B}} = \Big\{ \Big((0.2/x_1, 0.2/x_2, 0.5/x_3), \{h_2, h_3, h_4\} \Big), \Big((0.5/x_1, 0.3/x_2, 0.4/x_3), \{h_1, h_2, h_3\} \Big), \Big((0.3/x_1, 0.1/x_2, 0.5/x_3), \{h_1, h_4\} \Big) \Big\}$$

Interpretation:
For $\big((0.2/x_1, 0.2/x_2, 0.5/x_3), \{h_2, h_3, h_4\}\big) \in F_{\tilde{A}} \cup F_{\tilde{B}}$, we can say that the buyer $\hat{t}_1$ considers the houses h_2, h_3, h_4 to be of 0.2 degree x_1, 0.2 degree x_2 and 0.5 degree x_3, what so ever, we consider parameter x_1, x_2, x_3 to be. Also, buyer $\hat{t}_2$, say considered h_1, h_2, h_3 to be 0.5 degree x_1, 0.3 degree x_2 and 0.4 degree x_3.

For example, combining these two buyers valuations could help the seller possibly decide on the approximate value for the houses, since $f_{\tilde{C}}(t_1)$ and $f_{\tilde{C}}(t_2)$ have intersection, so the seller is assumed to take the arithmetic mean of those such cases for his decision.

5 Conclusion

It is clear that in [14], the authors proposed a graduation of membership on the set of parameters as a kind of fuzzification, and so, a form of a deviation from the crisp nature inherent in the soft set. Thus, in this work, we proposed a new way of graduation of membership on the set of parameters as a more general way to fuzzify the set of parameters. In this regard, a numerical example is given to substantiate our argument in comparison with the examples given in [3] and [14], to show that the work generalizes these other works to a good extent. Also, the definitions of some of the basic set operations are studied in this context. We do hope our work as presented in this paper will facilitate further research in this direction of soft set theory and its relationship with fuzzy set. Presently, the authors are working on algebraic properties of the defined set operations of the $FFPS$-set, and developing new algorithm to solve both individual and group decision making problems. For future directions, the concept introduced in this article can be studied together with other concepts like: Soft Expert Set, Intuitionistic Fuzzy Set, Phythagorean Fuzzy Set, Fermatean Fuzzy Set, Bipolar Fuzzy Soft Expert Set, Hesitant Fuzzy Linguistic Term Soft Set, etc. It can also, be applied to solve uncertainties in Economic, Engineering and Environmental sciences, as well as in group decision making.

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Competing Financial Interests

The authors declare no competing financial interests.

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