International Journal of Mathematical Sciences and Optimization: Theory and Applications
Vol. 8, No. 1, pp. 109 - 116
https://doi.org/10.6084/m9.figshare.20710417

Forecasting Nigeria annual yam production using Grey System Model

Lawal Adamu[1*], Saidu Daudu Yakubu[2], Didigwu Ndidiamaka Edith[3], Abdullahi Abubakar[4], Isaac Adaji[5], Mohammed Yahaya[6]

1,6 Department of Mathematics, Federal University of Technology, Minna, Nigeria
2 Department of Mathematical Science, Ibrahim Badamasi Babagida University, Lapai, Nigeria
3 Department of Mathematics, Enugu State University of Science and Technology,Nigeria
4 Department of Mathematics, Faculty of Science, Air Force Institute of Technology, Kaduna, Nigeria
5 Department of Mathematics and Statistics, Confluence University of Science and Technology, Osara, Kogi State. Nigeria
* Corresponding author: lawal.adamu@futminna.edu.ng, saiduyaku2014@gmail.com, edith.diddigwu@esut.edu.ng , aawachin@gmail.com, isaacadaji4real@gamail.com, yahayamohammed1986@gmail.com

Article Info
Received: 25 March 2022 Revised: 26 April 2022
Accepted: 08 June 2022 Available online: 29 August 2022

Abstract

Demand for reliable and accurate annual crop production forecast has been on the increase in recent time, because it avails the famers and the government opportunity to plan ahead of time, with regard to import or export in the event of shortfall or surplus to ensure food security of the respective nation. A GM(1,1) system model that forecast the Nigeria annual yam production has been developed and tested on known data. The data used in the research were collected from the archive of Central Bank of Nigeria for a period of Nine years (2010-2018). The fitted result from the model showed 98.97% accuracy, this indicate that, the model is reliable and dependable. Therefore, results from the model could serve as a guide to the yam farmers and the government to plan strategies for high yam production in the country.

Keywords: Yam, Production, Grey-System, Forecasting, Nigeria.
MSC2010: 60G25

1 Introduction

Timely and accurate crop production estimation before growing season begins is an important information for the government in decision making and policies formulation with regard to import or export in the event of shortfall or surplus. At the present time, one of the most important sectors of Nigeria economy is agriculture. It is also the major means of livelihood of many homes in the country. With population of about Two hundred million people, more than 70% of the population

International Journal of Mathematical Sciences and Optimization: Theory and Applications
Vol. 8, No. 1, pp. 109 - 116
https://doi.org/10.6084/m9.figshare.20710417

is engaged in agricultural activities. To feed the increasing population of Nigeria, there is a need to incorporate the latest technology and tools in the agricultural sector. It is on this note, that this study is adopting GM(1,1) forecasting model to determine the long time trend for Nigeria yam production with a view to helping the government with information to plan strategy for high yam production in the country and also to maintain its first position in Global ranking in term of yam production.Nigeria produces about 70% of the world's yam accounting for about 39.9 million tons [1]. GM(1,1) forecasting model is being adopted in this study because of its ability to use small sample size and make long time forecasting with minimal error ([1–3]) .The prediction of crop production is one of the most desirable yet challenging tasks for every nation. Nowadays, due to the unpredictable climatic changes, farmers are struggling to obtain a good amount of yield from the crops [4]. Accurate and reliable seasonal forecasts of crop yields are among the most valuable pieces of information that stakeholders such as farmers, commodity traders, and government officials can have at their disposal to make strategic decisions in their respective roles [5].Successful prediction of crop production has also been presented in the following, researches ([6–9]). This paper is aim at using small sample size available to determine long time trend of Nigeria annual Yam production with minimal error, and with a view to providing the authority with reliable information that could help in decision making regarding import or export in the event of shortfall or surplus.

2 Materials and Methods

The Grey-System Model GM(1,1)

The grey GM(1,1) model make use of the discrete data series to establish a equation of grey continuous differential equation by adding these data from the first in Accumulating Generation Operator (AGO), and the equation can then be solved to perform forecast [10].
Let the raw data series be represented by $x^{(0)}(k), k = 1, 2, 3, ..., n, x^{(0)}(k) \geq 0$ which can also be represented as:

$$X^{(0)}(k) = \left(x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n)\right) \tag{2.1}$$

Let the accumulated generating sequence be represented as:

$$X^{(1)}(k) = \left(x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), ..., x^{(1)}(n)\right) \tag{2.2}$$

Where

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, ..., n \tag{2.3}$$

$X^{(1)}(k)$ is called accumulated generating operation of $X^{(0)}(k)$ denoted as 1-AGO. By differentiating $X^{(1)}_{(k)}$, a whitened differential equation is obtained. See ([11–14]). Which is grey differential equation.

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{2.4}$$

Where a and $b$are parameters to be identified. a is called developing coefficient and b is grey input. The difference form is given as:

$$x^{(0)}(k) + ax^{(1)}(k) = b \tag{2.5}$$

Equation (5) represents the original form of the GM(1,1) model. The symbol GM(1,1) stands for first order Grey Model in one variable.
Equation (6) is the solution of equation (4)

$$\hat{x}^{(1)}(k+1) = \left(x^{(1)}(0) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \tag{2.6}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
Vol. 8, No. 1, pp. 109 - 116
https://doi.org/10.6084/m9.figshare.20710417

Equation (6) is the time response function while parameters a and b are estimated using Least Square Method as follows:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left[B^T B\right]^{-1} B^T Y \tag{2.7}$$

Where,

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{2.8}$$

$$z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}\ , (k = 2, 3, 4...., n) \tag{2.9}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), ..., x^{(0)}(n)\right]^T \tag{2.10}$$

The reduction value of equation (6) is given in equation (11) below:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1 - e^a)\left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} \tag{2.11}$$

Prediction Accuracy Test

To determine the accuracy of our forecast, we shall adopt mean absolute percentage error (MAPE).This tool is often used for determining prediction accuracy showing the same characteristics i.e the smaller the value, the higher the prediction accuracy [15]

MAPE

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \widehat{y}_i}{y_i}\right| \times 100\% \tag{2.12}$$

Where;
$\hat{y}_i$ is the Grey Model predicted value.
y_i is the Grey-Model actual value.
n is the number of prediction samples [15]
[16] divided the prediction accuracy of models into four grades and the division of prediction accuracy grades is shown in the table below:

Table 1: Division of Prediction Accuracy Grades

MAPE	Prediction Accuracy
$< 10\%$	High
$10\% - 20\%$	Good
$20\% - 50\%$	Feasible
$> 50\%$	Low

3 Results and Discussion

Application of Grey System Model for Forecasting Nigeria Annual Yam Production

The data used in this research were collected from the archive of Central Bank 0f Nigeria for the period of Six years (2010-2015). It is the original data sequence of annual yam production in Nigeria. The summary of the data is presented in table 2 below:

Table 2: Nigeria Annual Yam Production from 2010 to 2015

S/N	Year of Production	Yam Production('000 Tonnes)
1	2010	37653.44
2	2011	39693.44
3	2012	41599.35
4	2013	42998.35
5	2014	43038.00
6	2015	44660.50

Source [17]: Central Bank of Nigeria Annual Report 2015

We begin the application by substituting the raw data in table 2 into equation (1), to obtain equation (13) below

$$X^{(0)} = (37635.44,\ 3969.44,\ 41599.35,\ 42998.35,\ 43038,\ 44660.5) \tag{3.1}$$

Using equation (2) we obtain the accumulated generating sequence from equation (14) as given below:

$$X^{(1)} = (37653.44,\ 77346.88,\ 118946.23,\ 161944.58,\ 204982.58,\ 349643.08) \tag{3.2}$$

Using equation (9), we obtain equation (15) below

$$Z^{(1)} = (57500.16,\quad 98146.55,\quad 140445.405,\quad 183463.58,\quad 277312.79) \tag{3.3}$$

using equation (10), we obtained equation (16)

$$Y = \begin{bmatrix} 39693.44 \\ 41599.35 \\ 42998.35 \\ 43038.00 \\ 44660.50 \end{bmatrix} \tag{3.4}$$

And using equation (8), we obtained equation (17)

$$B = \begin{bmatrix} -57500.16 & 1 \\ -98146.55 & 1 \\ -140445.40 & 1 \\ -183463.58 & 1 \\ -277312.79 & 1 \end{bmatrix} \tag{3.5}$$

equation (18) is obtained using equation (7) by the help of Maple 17 software

$$\hat{a} = \begin{bmatrix} -0.0208 \\ 39253.14 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \tag{3.6}$$

Where $a = -0.0208$, $b = 39253.14$
Substituting for a and b in equation (6), we obtained equation (19) below:

$$\hat{x}\,(k+1) = 1924823.632e^{0.0208k} - 1887170.192 \tag{3.7}$$

IJMSO

INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES AND OPTIMIZATION: THEORY AND APPLICATIONS
VOL. 8, NO. 1, PP. 109 - 116
HTTPS://DOI.ORG/10.6084/M9.FIGSHARE.20710417

Evaluating equation (19) for $k = 0, 1, ..., 5$ we obtained equation (20) below:

$$\hat{X}^{(1)} = (37653.44,\ 78109.05,\ 119414.951,\ 161589.013,\ 2046499.481,\ 248614.986\,) \tag{3.8}$$

We compute the simulated value of the using equation (21) below:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \tag{3.9}$$

The simulated values are presented in equation (22) below

$$\hat{X}^{(0)} = (37653.44,\ 40455.612,\ 41305.899,\ 42174.062,\ 43060.468,\ 43965.505) \tag{3.10}$$

Equation (22) is the simulated values from 2010-2015 and is presented in Table 3 below.

Table 3: Comparison of actualand Grey simulated value for Nigeria Yam production from year 2010-2015.

S/N	Year Of Production	Actual Yam Production ('000 Tonnes)'	Grey Simulated Yam Production('000 Tonnes)	Residual Error	Relative Error(%)
1	2010	37653.44	37653.44	0	0
2	2011	39693.44	40455.612	-762.172	-1.92
3	2012	41599.35	41305.899	293.451	0.705
4	2013	42998.35	42174.062	824.288	1.92
5	2014	43038.00	43060.468	-22.468	-0.052
6	2015	44660.05	43965.505	694.545	1.56

Using equation (12), we observed from Table 3 that:
$MAPE = 1.026\%$ which is the error of the simulated value and it is described as high accuracy [16].
Hence the accuracy is calculated as:
$ACCURACY = 100\% - 1.026\% = 98.974\%$ this shows that the simulated accuracy is high.

Figure 1: Comparison between simulated values and real values of Nigeria yam production from 2010-2015

Grey Forecasting for Nigeria Annual Yam Production From 2016 -2022

From Table 3 and Figure 1 as it can be observed, the grey forecasting model has shown high level of accuracy, this shows that the model is reliable and dependable, hence we make forecast for the next Seven years. To make forecast from 2016-2022, we evaluate equation (19) for$k = 6, 7, 8, 9,$ 10, 11, 12. Thus, we have

$$\hat{X}^{(1)} = (293505.55,\ 339337.596,\ 386133.953,\ 433913.869,\ 482698.014,\ 532507.495,\ 583363.62\,) \tag{3.11}$$

We then compute the forecast values using equation (21) that is:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1)$$

$$\hat{X}^{(0)} = (44889.564,\ 45833.046,\ 46796.357,\ 47779.916,\ 48784.145,\ 49809.481,\ 50856, 125) \tag{3.12}$$

The forecasted values of equation (24) is presented in table 4 below

Table 4: Grey forecasted values for Nigeria Annual Yam production from year 2016-2023

Year of Production	Grey forecast Yam Production ('000 Tonnes)
2016	44889.564
2017	45833.046
2018	46796.357
2019	47779.916
2020	48784.145
2021	49809.481
2022	50856.125

Table 5: Nigeria Annual Yam Production from 2016 to 2018

S/N	Year of Production	Actual Yam Production ('000 Tonnes)'
1	2016	45409.8
2	2017	46912.7
3	2018	48291.6

Source [18]: Central Bank of Nigeria Annual Report 2018

Table 6: Comparison of Actual and Forecasted values for Nigeria Yam Production from year 2016-2018

S/N	Year Of Production	Actual Yam Production ('000 Tonnes)'	Grey Forecasted Yam Production('000 Tonnes)	Residual Error	Relative Error(%)
1	2016	45409.8	44889.56	520.236	1.15
2	2017	46912.7	45833.046	1079.66	2.30
3	2018	48291.6	46796.357	1495.24	3.10

Using equation (12), we observed from Table 6 that:
$MAPE = 2.183\%$ which is the error of the forecasted value and it is described as high accuracy [16] . Hence the accuracy is calculated as:
$ACCURACY = 100\% - 2.183\% = 97.82\%$ this shows that the forecasting accuracy is high.

Figure 2: Comparison of Actual and Forecasted values of Nigeria Yam Production for years 2016-2018

Grey system model has been successfully applied to forecast Nigeria annual Yam production from 2016 to 2022. Table 3 is the comparison of Actual values of the production and simulated values of the production using the fitted model which recorded 98.974% accuracy. Table 5 is the comparison of the actual values of the Nigeria annual Yam production with forecasted values from years 2016 to 2018 and this has recorded 97.82% accuracy. In both cases, the model showed good performance.

4 Conclusion

Forecasting trend of agriculture production in Nigeria using empirical data has always been a difficult task, this is as a result of small sample size available and its strong connectivity with climatic elements that exhibit randomicity and fluctuation. However, with the mathematical prowess of grey forecasting model, that not only requires minimal data but also has the capability of long-time forecasting with minimal error. The study was able to make forecast of Nigeria annual yam production with high level of accuracy.

5 Competing Interests

The authors of this paper declared that there are no competing interests as regards this article.

6 Acknowlegement

The authors wish to sincerely appreciate the inputs of the reviewers which have added value to the paper.

References

[1] Waziri, A., Tsado, E.K., Likita .T , Gana, A.S.(2014).Socio-economic factors influencing adoption of yam Minisett Technology in Niger State of Nigeria*Journal of Biology, Agriculture and Healthcare* 4(5), 98-105.

[2] Jian-Yi L., Ying Z. (2014). Application of Grey Markov SCGM $(1,1)_c$ Model to prediction of Accidents Death in Coal Mining.Hindawi Publishing Corporation International Scholarly Research Notices Vol.2014, Article ID 632804, 7 Pages http://dxidoi.org /10.1155/2014/632804.

[3] Wei, Z., Jian-Min,H.,(2013).Generalized GM(1,1) Model and its Application in Forecasting of Fuel Production. *Applied Mathematical Modelling.* 37: 6234-6243.

[4] Yong, S., Yang, Z., Changshun, Y., (2016). Gray Prediction of Air Pollutants in Beijing Based onImproved Residual GM(1,1) Model. 4^{th} International Conference on Machinery, Materials and Information Technology Applications. Atlantis Press, Advance in Computer Science Research, Volume 71.

[5] . Khosla, E., Dharavath, R. & Priya, R. Crop yield prediction using aggregated rainfall-based modular artificial neural networks and support vector regression. *Environ Dev Sustain22,* 5687–5708(2020). https://doi.org/10.1007/s10668-019-00445-x.

[6] Saeed, K., and Lizhi, W. (2019) Crop Yield Prediction Using Deep Neural Networks. *frontiers in plant science.*

[7] Syngenta (2018). Syngenta Crop Challenge In Analytics. Available online at: https://www.ideaconnection.com/syngenta-crop- challenge/challenge.php/.

[8] You, J., Li, X., Low, M., Lobell, D., and Ermon, S. (2017). "Deep gaussian process for crop yield prediction based on remote sensing data," in Thirty-First AAAIConference on Artificial Intelligence (San Francisco, CA), 4559–4566.

[9] Zhang, S., Tao, F., and Zhang, Z. (2017). Spatial and temporal changes in vaporpressure deficit and theirimpacts on crop yields in China during 1980–2008. J.Meteorol. Res., 31 , 800–808. doi: 10.1007/s13351-017-6137-z.

[10] Li Q., Hu Q., Zangh P., (2007). Application of Grey-Markov Model in Predicting Traffic Volume. Proceeding of 2007 IEEE International Conference on GreySystem and Intelligent Services, November 18-2-, 2007, Nianjin, China.

[11] Deng, J.(1989). Introduction to Grey System Theory. Journal of Grey System Vol(1): 1-24.

[12] Deng, J. L. (1990). Grey system theory tutorial, Huazhong University of Science and Technology Press, Wuhan, China.

[13] Mao Z., Sun J., (2011). Application of Grey-Markov Model in Forecasting Fire Accidents. TheFifth Conference on Performance-based Fire Protection Engineering, ELSEVIER.

[14] Saeed, O. B., Abubakar, U., Lawal, A., & Usman, A. (2021).Prediction Of Vehicular Accidents Along Lokoja-Abuja-Kaduna Express Way Nigeria Using Grey-System Model GM(1,1). Journal of Science, Technology, Mathematics and Education (JOSTMED), 17(3),pp87-94.

[15] Xin, Z., Wen, Long, Y., Chengjiang, L.,(2018). Road Traffic Accident Prediction Using an SCGM(1,1)c-Markov Model. Asian Transport Studies: Volume 5(1): 191-205.

[16] Lewis, C.D.(1982). Industrial and Business Forecasting Methods: A Practical guide to exponential smoothing and curve fitting, Butterworth-Heinemann, London.

[17] Central Bank of Nigeria Annual Report 2015: retrieved at https://www.cbn.gov.ng [12/06/2021].

[18] Central Bank of Nigeria Annual Report 2018: retrieved at https://www.cbn.gov.ng [23/04/2022].