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Workdone by m-Topological Transformation Semigroup Regular Spaces (M_{ψ_n})

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Abstract

This paper introduces a new class of topological Semigroup called the m-Topological Transformation Semigroups; this uses the notion of topological spaces to solve semigroup problem with primary focus on Transformation Semigroups. m-Topological Transformation Semigroup Regular space moves an element x of its domain to a distance of $|x - \alpha x|$ units. The workdone $w(\psi)$ by ψ is defined as the sum of all of the distances. In this paper, we derive formula for total work done, average work done and the power by transformations on m-Topological transformation Semigroup Regular space of degree n.

Keywords: Workdone, Transformations, Regular space, Power, Elements.
MSC2010: 26A18.

1 Introduction and Preliminaries

Topology, a branch of mathematics that finds practical applications in various fields, has emerged as a vital tool in theoretical computer science [1], [2], [3], [4], [22], [6]. Herlihy and Shavit [7], explores the applications of algebraic topology, including finite topological spaces and their connections to computer science. The challenges presented in computational settings have provided new and stimulating opportunities for topology, fostering increased interaction between topology and related mathematical areas such as order theory and algebraic topology. Algebraic topology is one approach to studying algebraic structures in mathematics, specifically, the relationship between topological spaces and group theory. It utilizes algebraic tools to investigate topological spaces, aiming to transform topological problems into more amenable algebraic forms, such as groups. Notable works by Dylan [8] and Hatcher [9] exemplify, this approach. Conversely, it is also possible to translate algebraic concepts into the realm of topology. Building upon this idea, our research introduces a novel concept called m-topological transformation semigroups, which leverages topology to address problems in semigroup theory. Our primary focus is on transformation semigroups, and by employing topological principles, which is aimed at providing solutions within this domain. It is

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important to note that; transformation semigroups are mainly functions from a given set to itself and one of the important transformation in semigroup theory is the finite partial transformations semigroups [10]. m-Topological Transformation Semigroup spaces are Transformation semigroups that admits the properties of topological spaces. Some researchers have published works on the transformation semigroup theory, researchers like Laradji and Umar [11], Laradji and Umar [12] have put together alot of useful contents on different concepts of the transformation semigroup. In fact, this work is an advancement of the research work put together by Umar [13], James and Peter [14] and Kehinde [15], in their work on workdone, avarage workdone and power. They investigated; Finite order-perserving injective partial transformation semigroup, order-perserving full transformation semigroup and order-perserving partial transformation semigroup. To derive our findings, we explore certain methodologies employed in their work. The displacement of point x, denoted as $d(x, \alpha x) = |x - \alpha x|$ units, represents the distance it has been moved for a given belonging to the domain of n. Other works that relate to this work includes [16], [17], [18] and [19]. Francis and Adeniji [20], work on number of element in open and clopen topological space and gave numerical values for $n \leq 4$. Francis [21], considered the number of equivalent and non-equivalent homeomorphic topological spaces for $n \leq 4$. Edeghagba and Muhammad [22], investigated the notion of Ω-subgroup of an Ω-group with one binary operation, they further resented and investigated some particular notions as in the case of classical group theory: Ω-centralizer, Ω-center and Ω-normalizer in an Ω-group. In this paper, the total work done on m_ψ will be the sum of these distances as ψ varies over the domain of n. Throughout this work, the following shall be denoted as; m-topological transformation semigroup regular m_ψ, total work done by m-topological transformation semigroup regular spaces $w(m_\psi)$, element of m-topological transformation semigroup regular ψ. We shall consider the total work $w(m_\psi)$, average work denoted as $\bar{w}(m_\psi)$, and power of m_ψ denoted by $P_t(m_\psi)$. Let $\alpha, \beta \in \delta$ be two elements of M_δ the intersection, union and complement of α and β are given as follows $\alpha \cap \beta = min\{\alpha x, \beta y\}$, $\alpha \cup \beta = max\{\alpha x, \beta y\}$ and $\alpha^c = |n - \alpha x|$ where $n = max(X)$ for $x \in Dom(\alpha)$ and $\alpha x \in Im(\alpha)$. If $\alpha \in M_\delta$ is open, then $\alpha^c \in M_\delta$ is closed

Definition 1([14])
Let $\psi \in w(m_\psi)$ and $x \in n$. for $n \in \{1, 2, 3, ...\}$
We define the work done by α to move x to αx as :

$$w(\psi) = \begin{cases} (|x - \alpha x|) & \text{if } x \in Dom(\alpha) \\ 0 & \text{otherwise} \end{cases}$$

and we define the (total) work done by ψ to be

$$w(m_\psi) = \sum_{\alpha \in n} \sum_{x \in n} w_x(M_\psi) = \sum_{x \in n} w(M_\psi)$$

we define the (average) work done by ψ as

$$\bar{w}(M_\psi) = \sum_{x \in n} \frac{w(M_\psi)}{|(M_\psi)|}.$$

We also define the power of the transformations as

$$(M_\psi) = \sum_{x \in n} \frac{w_t(M_\psi)}{t}, \forall t \geq 1$$

Let $\Delta_{x\alpha x}(M_\psi) = \alpha \in M_\psi$ For $x, \alpha x \in n$ be the set of all elements of M_ψ which move x to αx, and such that

$$\Delta_{x\alpha x}(M_\psi) = |\Delta_{x\alpha x}(M_\psi)|$$

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and

$$w_x(\alpha) = |x - \alpha x|, \forall \alpha \in \Delta_{x\alpha x} M_\psi$$

So

$$(\psi) = \sum_{\alpha x} |x - \alpha x| \Delta_{x\alpha x}(M_\psi)$$

Definition 2
Let δ be the chart on $X_n = \{1, 2, 3, ...\}$. The map $\alpha : Dom(\alpha) \subseteq X_n \to Im(\alpha) \subseteq X_n$ is said to be a full transformation semigroup; denoted by T_n, if $Dom(\alpha) = X_n$, and partial transformation if $Dom(\alpha) \subseteq X_n$; denoted by P_n.

Definition 3
A set of transformations in δ, is said to be m-topological transformation semigroup (shorten as M_δ) if it satisfies the following properties:
(i) α and $\emptyset$ are in M_δ
(ii) if α is closed under arbitrary union in M_δ
(iii) if α is closed under finite intersection in M_δ.

Definition 4
Let the triple $(X, \delta, M_{\delta_n})$ be a m-topological transformation semigroup. the triple (X, ψ, M_{ψ_n}) is said to be m-topological transformation semigroup Regular Spaces denoted as M_ψ, if υ is closed subset of X, and $\mu \in X$. $\mu \notin \upsilon$ then, there exist $\alpha, \beta \in M_\psi$ such that $\upsilon \in \alpha$, $\mu \in \beta$ and $\alpha \cap \beta = \emptyset$. Hence, the triple (X, ψ, M_ψ) is a m-topological transformation semigroup Regular Space.
We select m-Topological transformation semigroups that fulfill the regularity condition (T_3) from a collection of m-topological partial transformation semigroups. In this context, we denote an element of the Regular space for a given value of n as $\psi(n)$, and the Regular space on the m-topological transformation semigroup as $m_{\psi(n)}$. To ensure clarity and organization, we begin by presenting a list of the m-topological transformation semigroups $m_{\psi(n)}$.

When $n = 1$

$$M_{(\psi_1)} = \left\{ \begin{pmatrix} 1 \\ \emptyset \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\} : 1$$

For $n = 2$

$$M_{(\psi_2)} = \left\{ \begin{array}{r} \upsilon_1 = \left\{ \begin{pmatrix} 1 & 2 \\ \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \right\} \\ \upsilon_2 = \left\{ \begin{pmatrix} 1 & 2 \\ \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ \emptyset & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \right\} \end{array} \right\} : 2 \quad (1.1)$$

For $n = 3$

$$m_{(\psi_3)} = \left\{ \begin{array}{r} \upsilon_1 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 3 & 3 \end{pmatrix} \right\} \\ \upsilon_2 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix} \right\} \\ \upsilon_3 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 3 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & \emptyset & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix} \right\} \\ \upsilon_4 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix} \right\} \end{array} \right\} : 4 \quad (1.2)$$

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For $n = 4$

$$m_{(\psi_4)} = \left\{ \begin{array}{r} \upsilon_1 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \\ \upsilon_2 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \\ \upsilon_3 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & \emptyset & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & 4 & \emptyset & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \\ \upsilon_4 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & \emptyset & \emptyset & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & 4 & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \\ \upsilon_5 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & 4 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \\ \upsilon_6 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & 4 & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & \emptyset & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \\ \upsilon_7 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \\ \upsilon_8 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\} \end{array} \right\} : 8 \tag{1.3}$$

Example 1 Consider an arbitrary M_{ψ_4} on $X = \{1, 2, 3, 4\}$

$$M_{\psi_4} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\}$$

We establish that m_{ψ_n} satisfies the condition of m-topological transformation semigroup space Axiom 1

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & \emptyset \end{pmatrix} \cap \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & 4 \end{pmatrix} \in M_{\psi_4}$$

Axiom 2

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & \emptyset \end{pmatrix} \cup \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & 4 \end{pmatrix} \in M_{\psi_4}$$

Axiom 3

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \in M_{\psi_4}$$

the axioms are satisfied.

We proceed to show that it is a regular space

$$M_{\psi_4} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix} \right\}$$

$$M^c_{\psi_4} = \upsilon = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix} \right\}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \notin \begin{pmatrix} 1 & 2 & 4 \\ 4 & 4 & 4 \end{pmatrix}$$

then

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$$\alpha = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 4 & 4 \end{pmatrix}$$

$$\alpha \cap \beta = \emptyset$$

$$\alpha = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \in \begin{pmatrix} 1 & 2 & 3 & 4 \\ \emptyset & \emptyset & 4 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 4 \\ 4 & 4 & 4 \end{pmatrix} \subset \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & \emptyset & 4 \end{pmatrix}$$

Lemma 1.1. *In m-topological transformation semigroup, if $M_{\delta_4} = \upsilon$, then (X, δ, M_δ) is regular*

Proof. Let υ be closed transformations of M_δ. this implies that $\alpha x \notin \alpha$ and $\alpha x \in \beta$ $\alpha \cap \beta = \emptyset$ or this also implies that $\upsilon \cap \upsilon^c = \emptyset$ □

2 Element of m-Topological Transformations

Theorem 2.1. *Let $\psi \in M_\psi$. Then $|\psi| = 2^n$*

Proof. $\psi \in M_\psi$. The number of elements in a regular space are of binomial theorem form in a special class that is

$$\begin{pmatrix} n \\ 0 \end{pmatrix} + \begin{pmatrix} n \\ 1 \end{pmatrix} + \begin{pmatrix} n \\ 2 \end{pmatrix} + ... + \begin{pmatrix} n \\ n \end{pmatrix}. \tag{2.1}$$

Implies that

$$\sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} a^k b^{n-k}. \tag{2.2}$$

If $a = 1$ and $b = 1$ implies

$$\sum_{k=0}^{n} \binom{n}{k} 1^k 1^{n-k} = \sum_{k=0}^{n} \binom{n}{k} = (1+1)^n = 2^n. \tag{2.3}$$

□

Theorem 2.2. *Let $\psi \in M_\psi$. Then $|M_\psi| = 2^{n-1}$*

Proof. $\psi \in M_\psi$. The number of elements in a regular space has n points to be partition into exactly 2 points this applies to the formula of Stirling numbers of the second kind that is $S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^j (k-j)^n$. with n having maximum chose of 2 points at a time. However, this can best remembered in terms of Pascal's triangle. Therefore, if we consider the row sum and alternating row sum of Pascal's triangle we have Equation (2.4) and (2.5) respectively.

$$2^n = \sum_{k=0}^{n} \binom{n}{k} = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2k} + \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n}{2k+1}, \tag{2.4}$$

and

$$0 = \sum_{k=0}^{n} (-1)^n \binom{n}{k} = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2k} - \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n}{2k+1}. \tag{2.5}$$

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From equation (2.5) we can deduce that

$$\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2k} = \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n}{2k+1}. \tag{2.6}$$

Substituting equation (2.6) in (2.4) for $n \geq 1$ yields

$$\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2k} = \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{2n}{2k+1} = \sum_{k=0}^{n} \binom{n}{2k+1} = 2^{n-1}. \tag{2.7}$$

However, a more concise statement can be made:

$$S(n,2) + S(n,0) = \frac{1}{2!}\sum_{j=0}^{2}(-1)^j(2-j)^n + \frac{1}{0!}\sum_{j=0}^{0}(-1)^j(0-j)^n, \tag{2.8}$$

which is simplified as

$$2^{n-1} - 1 + 1 = 2^{n-1}.$$

As mentioned □

Table 1 : Values obtained for m-Topological Transformation Semigroup Regular Spaces and their number of Elements

n	$\|m_\psi\| = 2^{n-1}$	$\|\psi\| = 2^n$
1	1	2
2	2	4
3	4	8
4	8	16
5	16	32
6	32	64
7	64	128
6	128	256
8	256	512
9	512	1024
10	1024	2048

3 Workdone of m-Topological Transformation

Theorem 3.1. *Let $x, \alpha x \in n$. Then for $\psi \in M_\psi$*

$$w(m_{\psi_n}) = 2^{(n-1)}n^2$$

Proof. Let $a_0, a_1, a_2, \therefore, a_n$ be set of points in $\sum_{x\alpha x\in n}|x - \alpha x|\Delta_{x\alpha x}(m_\psi)$. There exist points $a_0 = 0$ and $a_n = 2^{n-1} \neq 0$. There exist other points called $a_1, a_2, a_3, \therefore, a_{n-1}$ where each distinct point gives 2^n by taking the sum of the points we have

$$n2^{n-1} + 2^n\Big\{0, 1, 2, \therefore, \Big\}.$$

This implies

$$n2^{n-1} + 2^n\frac{n(n+1)}{2}$$

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$$\implies n2^n2^{-1} + 2^n\left\{\frac{n(n+1)}{2}\right\}$$

$$\implies 2^n\left\{n2^{-1} + \left\{\frac{n(n+1)}{2}\right\}\right\}$$

$$\implies 2^n\left\{\frac{n}{2} + \left\{\sum_{k=0}^{n-1} k\right\}\right\} = 2^{(n-1)}n^2$$

□

Theorem 3.2. *Let $x, \alpha x \in n$. Then for $\psi \in M_\psi$*

$$\bar{w}(m_{\psi_n}) = \frac{n^2}{2}$$

, and

$$P_t w(M_{\psi_n}) = \frac{2^{(n-1)}n^2}{t}$$

Proof. From Theorem (3.1)

$$w(m_{\psi_n}) = 2^{(n-1)}n^2 \tag{3.1}$$

have be established .

$$P_t(m_\psi) = \sum_{i\in n} \frac{w_t(m_\psi)}{t}, \tag{3.2}$$

and (average) work done by ψ is

$$\bar{w}(m_\psi) = \sum_{x\in n} \frac{w(m_\psi)}{|(m_\psi)|}. \tag{3.3}$$

That is

$$\bar{w}(m_\psi) = \sum_{x\in n} \frac{w(m_\psi)}{|(m_\psi)|} = \frac{2^{(n-1)}n^2}{2^n}, \tag{3.4}$$

implies

$$\bar{w}(m_\psi) = 2^{(n-1)} \cdot n^2 \cdot 2^{-n}. \tag{3.5}$$

Hence, we have

$$2^{-1} \cdot n^2 = \frac{n^2}{2}. \tag{3.6}$$

Similarly, since power is define as

$$P_t(m_\psi) = \sum_{i\in n} \frac{w_t(m_\psi)}{t}, \tag{3.7}$$

this implies that

$$P_t(m_\psi) = \frac{2^{(n-1)}n^2}{t}. \tag{3.8}$$

As required □

Remark
The triangle of numbers and sequence obtained, as at the time of submitting this paper not in Sloane [23].

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Table 2. Triangular array of numbers in M_{ψ_n}

$n\backslash k$	0	1	2	3	4	5	6	7	8	9	$\sum_{k=0}^{n-1} k$
1	0										00
2	0	1									01
3	0	1	2								03
4	0	1	2	3							06
5	0	1	2	3	4						10
6	0	1	2	3	4	5					15
7	0	1	2	3	4	5	6				21
8	0	1	2	3	4	5	6	7			28
9	0	1	2	3	4	5	6	7	8		36
10	0	1	2	3	4	5	6	7	8	9	45

Table 3. Values obtained for the total work done, Average Work done and Power of m-Topological Transformation Semigroup Regular Spaces elements in M_{ψ_n}

n	$\|w(m_{\psi_n})\|$	$\bar{w}(m_{\psi_n})$	$\|P_t w(m_{\psi_n})$
1	1	$\frac{1}{2}$	$\frac{1}{5}$
2	8	2	1.6
3	36	$4\frac{1}{2}$	$7\frac{1}{2}$
4	128	8	25.6
5	400	$12\frac{1}{2}$	80
6	1152	18	$230\frac{2}{5}$
7	3136	$24\frac{1}{2}$	$627\frac{1}{5}$
8	8192	32	$1628\frac{2}{5}$
9	20736	$40\frac{1}{2}$	$4147\frac{1}{5}$
10	51200	50	10240

Table 4. Formulas for Results Obtained on m-Topological Transformation Semigroup Regular Spaces

m_{δ_n}	Formula
M_ψ	2^{n-1}
ψ	2^n
$w(M_{\psi_n})$	$2^{(n-1)}n^2$
$\bar{w}(M_{\psi_n})$	$\frac{n^2}{2}$
$P_t w(M_{\psi_n})$	$\frac{2^{(n-1)}n^2}{t}$

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5 CONCLUSION

We have introduced a novel class of regular spaces called m-Topological Transformation semigroup regular spaces. In our research, we have developed both explicit and implicit formulas to calculate the work performed by these m-Topological Transformation semigroup regular spaces.

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