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RISK AND RETURN ANALYSIS FOR ASSESSING PERFORMANCE OF SOME PRIVATE COMPANIES LISTED ON THE GHANA STOCK EXCHANGE THROUGH AN ASSET PRICING MODEL

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Abstract

In the presence of risk and uncertainty, investors still want good returns and maximal profits on their investments. For investors in stocks, to minimize loss, it is important to assess the risk and return inherent in the stocks, before investing in them. In this work, risk and return for some assets and portfolios of some companies quoted on the Ghana stock exchange (GSE) from January 2015 - February 2020 were assessed. The assets are representative of four major sectors of the economy. The systematic risk and performance of the portfolios were measured, by calculating alpha and Beta using the regression equation of the Capital asset pricing model. It was found that assets from the banking sector were more volatile, the asset of the food company was less risky, and that of the oil company seemed to mirror the market. Since private companies are investment drivers and economy boosters for growth; stability and performance analysis of some private sector companies namely CAL, GCB, MTN, GOIL, UNIL, FML,and TOTAL, was done. The price of the asset of one of the companies, were predicted for the next three months, using the model. The theoretical values were very close to the empirical values. Then, model performed well for risk and return assessment, and can be used to predict prices of assets on the GSE.

Keywords: Model, assets, risk and return, portfolio, Ghana stock exchange.
MSC2010: 26A18.

1 Introduction

Some degree of risk accompanies every investment. In the face of risk and uncertainty, investors still want optimal returns for their investments. When considering putting resources into a financial asset, the instrument must be analyzed for profitability; that is the concept of investment analysis. Recently in 2019, Ghanaian investors had a crisis of confidence concerning investment in some

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financial institutions. This further raises awareness of the fact that care must be taken in deciding where and how to invest. According to Fama and French [1] the attraction of the Capital asset pricing model, CAPM is that it offers powerful predictions, explains how to measure risk, and presents the relation between expected return and risk. Going by the model, investors aim to minimize the variance and maximize the expected return of their portfolios. The standard version of the model, was developed by Sharpe [2] and Lintner [3], and relates the expected rate of return of an individual security to a measure of its systematic risk. The systematic risk which is measured by beta, captures that aspect of investment risk which cannot be eliminated by diversification. One property of the model is that investors are compensated with a higher expected return only by accepting systematic risk. By the work of Elton et al [4] [16] an addition to this property, is that it suggests that securities with higher beta values are expected to give higher expected returns than securities with lower beta values.

In this paper, we use this model to assess some assets of companies quoted on the Ghana stock exchange. Thus the paper aims at using the Capital Asset Pricing model, to determine risk, return and performance of the companies studied by obtaining alpha and beta of their assets and portfolio. This analysis with respect to risk and return is to guide the investor and aid his investment decision. In portfolio management risk meets with return to determine a viable investment. Also the paper analyses stability and performance for some of the companies. The paper aims at obtaining useful information for the investor on assets on GSE, with respect to risk and return. This is to guide him, and aid in his investment decision. The model is used on both individual assets and portfolios consisting of five assets. The listings and activities of a country's stock exchange are an indication of the strength and performance of the companies whose stocks are traded herein. As a result, the Ghana stock exchange is the source of data used in the analysis. The period under study is 2015-2019. In Literature the CAPM has been used to assess assets on various stock exchanges, with varying results. Muhammad et al. [5] performed a study for ten companies listed on Karachi Stock Exchange (KSE)-in Pakistan. They used it to calculate their Beta Coefficient. In their work, the risk-free rate was the rate of the national savings certificate. They validated the results of Eatzaz and Attiya [6], and Hanif and Bhatti [7], thus they confirmed the inapplicability of the model to the KSE-Pakistan. Huseyn-Zada [8] in his work acknowledges that even though there are a lot of uncertainties surrounding the CAPM, it is still in use by many establishments and corporations because of its simplicity and effectiveness. The main model of his work, the Carhart four- factor model, is derived from the CAPM. Some work has been done on the GSE using this model. Some of the work was not done within the five years (2015 – 2020) on which we based our study. The five year time period was chosen because beta and alpha values are affected by time. i.e. the longer the period, the less reliable the alpha and beta values become. [9]. This period is also the most recent timing as the exchange has become more robust over time, now listing 42 equities (from 37 companies) and 2 corporate bonds. The GSE was incorporated in 1989 and started trading in 1990. For individual assets listed on GSE. Danquah [10] adopted and modified CAPM with an introduction of the crises effect (2008 Financial Crises) and the January effect. Abonongo et al [11], used the CAPM to measure the systematic risk of seven stocks on the GSE, and obtained their alpha and beta values. They determined defensive stocks (stocks with a beta less than 1) as CAL, FML and Tullow. They also obtained stocks with beta equal to 1 (meaning the expected return is the same as the market returns on those stocks) as PBC, CLYD, EGL and UNIL. Asamoah G. N [12], estimated beta risk of 32 companies listed on the Ghana stock exchange (2001-2006).He obtained the betas using various models and suggested that investors trade in infrequently traded stocks, whose beta risks are not constant over time. Menyah K. and Abor, J. [13] applied CAPM to individual securities on the GSE.(2000- 2009).They found positive linear relationship between equity risk premium and market beta. They suggest likely other risk factors other than beta. Basically, in this paper, we extend the assessment of risk and return beyond individual assets to sets of portfolios, this makes the work unique. The paper is structured as follows, section 1 presents the introduction, which includes a brief review of the literature, and section 2 presents the model and application of the model to assets of companies on the GSE. The results and analysis are presented in section 3, and the conclusion is drawn in section 4

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2 The Pricing model for Asset Pricing and Risk Assessment: Theoretical background

It is proposed that investors should look to either augment returns at a given risk level, or reduce risk given a targeted return value [14]. The main assertion of the CAPM is that the future price movement of an asset can be predicted by how well the asset has correlated to stock market movements in the past. This is similar to the standard modeling approach in determining what model best describes the behaviour of a price series, given information about past price trends. The beta and alpha factors in the CAPM introduce the advantage of refinement over the simple calculation provided by the Sharpe ratio [1]

2.1

2.1.1 Assumptions of the model

The assumptions of the model are given as follows: (i) Markets are perfect, (ideal)—no transaction fees, taxes, inflation, or short selling restrictions, all investors are averse to risk and all investors have equal access to all available information. (ii) All investors can borrow and lend unlimited amounts under a risk-free rate. (iii) Single period transaction horizon (iv) Investors hold diversified portfolios. The CAPM assumes a single-period investment with all investors having same time horizon. However, in reality, individual have multiple target dates for wealth accumulation which depends on their consumption pattern. Levy [15] analyzed the effect of investment horizon on performance measures in the CAMP framework. He found that reward to variability index is a function of time horizon assumed. However, in this work we assume a single period horizon. Further work is proposed to be done to factor investment pattern of individuals investing in these companies. This work can also be improved by using a stochastic CAPM model which would account for changing values of beta over time. A standardized holding period is assumed by the CAPM in order to make comparable the returns on different securities. A return over six months, for example, cannot be compared to a return over 12 months. A holding period of one year is usually used. The assumption of a single-period 4 transaction horizon appears reasonable from a real-world perspective, because even though many investors hold securities for much longer than one year, returns on securities are usually quoted on an annual basis. As a result, in this work, a holding period of one year is used. Despite its viability in risk, return, and performance assessment, CAPM has some short comings such as challenges in (i) Assigning values to CAPM variables, which can be difficult, hence proxies are sometimes used. The yield on short-term Government debt, which is used as a substitute for the risk-free rate of return, is not fixed but changes on a daily basis according to economic circumstances. (ii) Using the CAPM in investment appraisal. Problems can arise when using the CAPM to calculate a project-specific discount rate. For example, one common difficulty is finding suitable proxy betas, since companies very rarely undertake only one business activity. Not so much has been done in exploring risk and return especially for portfolio of assets for private sector companies in the GSE, as a result, despite the disadvantages of the model, we proceed to obtain preliminary work which can be improved subsequently. The systematic risk or market risk β, is captured by the covariance between the stock and the market, hence covariance is used as a risk measure for stock (fundamental underlying the CAPM). This role of β in the CAPM analysis is its advantage over deterministic models such as the Sharpe ratio. The CAPM can be derived from the general state price Beta model. The CAPM has a linear stochastic discount factor

2.1.2 CAPM and the Stochastic Discount Factor

We consider the state price model

$P_j = \sum_{s=1}^{S} q_s x_s^j$ where x^j is a security vector

$x^j = \begin{bmatrix} x_i^j \\ \cdots \\ x_s^j \end{bmatrix}, \in \{1, \cdots, J\}$, q is a state price vector.

The state price model can be written as $P_j = \sum_{s=1}^{S} \pi_s \frac{q_s}{\pi_s} x_s^j$ where π_s is the probability distribution of states.[16]. Let the random variable stochastic discount factor be defined as

$W_s = \frac{q_s}{\pi_s}$, then

$$P_j = \sum_{s=1}^{S} \pi_s w_s x_s^j = E\left[w x^j\right] \tag{1}$$

and

$$E\left[m^w x^j\right] = E\left[x^j\right].E\left[m^w\right] + cov\left[w, x^j\right] \tag{2}$$

For a risk-free bond, $x_s^b = 1$ for all S

Then we have $P_b = E\left[w\right] = \frac{1}{R^f}$

R^f is gross risk free return. Thus, for any asset j

$P_j = \frac{E\left[x^j\right]}{R^f}. + cov\left[w, x^j\right]$ usually $cov[w, x^j] < 0$

Defining the return vector for an asset j as $R^j = \frac{x^j}{P_j}$ then $E\left[w.R^j\right] = 1$

From equation (2) a risk-free bond R^f, is given as $R^f = \frac{1}{E(w)}$

Then we can write $E\left[w(R^j - R^f)\right] = E\left[(wR^j - R^f)\right] = 0$, consequently

$$E\left[w(R^j - R^f)\right] = E\left[w\right]\left(E[R^j] - R^f\right) + cov(w, R^j) = 0$$

Hence,

$$E[R^j] - R^f = \frac{-cov(w, R^j)}{E[w]} \tag{3}$$

$R^j - R^f$ is excess return. The Equation (3) states that excess return for an asset, is determined by the covariance with the stochastic discount factor. This also implies, that an investor is compensated with a higher return , for holding systematic risk, β.

2.1.3 A State-Price Beta Model

The CAPM can be derived from the general state-price Beta model. The stochastic factor was given in section 2.1.2 as $w_s = \frac{q_s}{\pi_s}$. We consider the stochastic discount factor

obtained from the pricing kernel $w^* = \begin{bmatrix} \frac{q_1^*}{\pi} \\ \cdots \\ \frac{q_s^*}{\pi} \end{bmatrix}$ and define its return as $R^* = \frac{w^*}{Pw^*} = \alpha w^*$ for $\alpha > 0$.

Replacing discount factor w, with its return R^*, we can write

$$E[R^j] - R^f = \frac{-cov(R^*, R^j)}{E[R^*]}$$

By defining $\beta_j = \dfrac{-cov(R^*, R^j)}{var[R^*]}$

For the asset j, we can write

$$E[R^j] - R^f = -\beta_j \frac{var[R^*]}{E[R^*]}$$

$$E[R^*] - R^f = -\frac{var[R^*]}{E[R^*]}$$

Therefore, for security j, we have

$$E[R^j] - R^f = \beta_j (E[R^*] - R^f) \tag{4}$$

If we assume a linear model for R^j and R^*equation (4) can be specified as

$$R^j_k - R^f = \beta_j (R^*_k - R^f) + \varepsilon_k$$

It can also be tested empirically with this form. This establishes notion of the CAPM equation as a Beta model. See equations 5 and 6. The fundamental theory of the model has been mathematically derived from the foregoing.

2.2 Model Equations for Application

In application, the model computes the expectation of an asset return. The model equation is given in equation 5.

$$E(R_i) = R_f + \beta_i * E(R_m) + R_f) \tag{5}$$

where $E(R_i)$is the expected return of an individual asset, R_f is the risk-free rate of return, β_i is the beta of the stock, which is the sensitivity coefficient between an asset's return and the stock market, and R_m is the market returns. The regression function is given

$$R_i - R_f = \alpha + \beta_t * (R_m - R_f) + \varepsilon_i \tag{6}$$

where α is the abnormal rate of return on security in excess of that predicted by the model, ε_i is the residual of the regression model, i.e. the error term which is a normal white noise with zero mean and variance σ^2,R_i,R_f, R_m and β_m are as defined above .

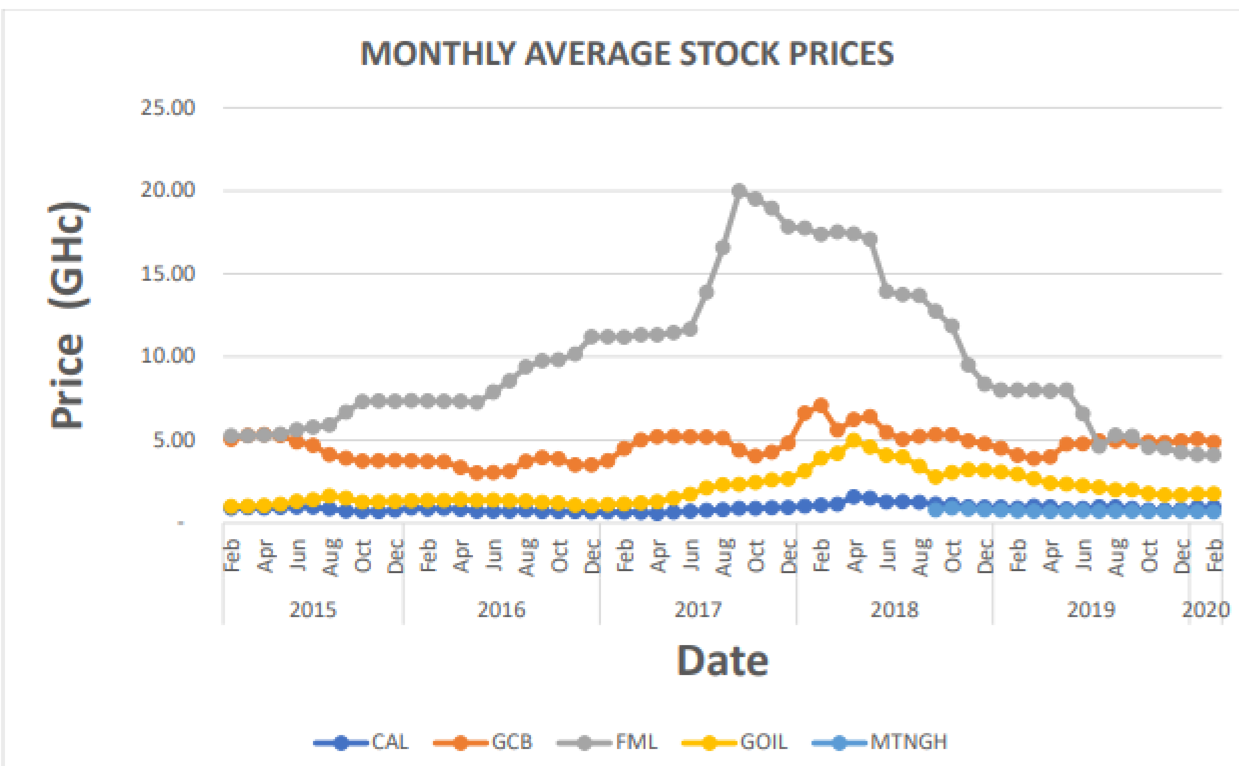
2.2.1 Application of Model to Assets of Companies Quoted on the Ghana Stock Exchange: Methodology

The 91-Day Treasury bill backed by the Bank of Ghana was used in this work, for the risk-free interest rate of return, and the Ghana Stock Exchange Composite Index, GSE composite index, was used as market returns. The paper aims at using the Capital Asset Pricing model, to determine risk, return and performance of the companies studied by obtaining alpha and beta of their individual assets and portfolios. To do this we first find the expected return of our assets and subtract our risk-free rate from it making $R_i - R_f$ our independent variable and $R_m - R_f$ our dependent variable. From there we regress $R_i - R_f$ against $R_m - R_f$ to obtain both our alpha and beta values. The regression model was implemented using the python programming language. The beta values are not published for stocks on the GSE, hence in this paper, to be able to predict the future movement of a security listed on the GSE, we had to obtain the beta values. This was quite challenging.

Proxy for beta was not used in this work, it was calculated. The parameters of the Capital Assets Pricing model are obtained for the Ghanaian Stock Market. Five- year data is used (2015 - 2020). This choice is also informed by study performed by Bartholdy and Peare [9]. There is a stochastic discount factor for the CAPM which is linear, the value of the coefficients Beta and Alpha are time varying.

2.3 Data

The Bank of Ghana and Ghana Stock Exchange maintain an extensive, regularly updated, and publicly available data library; therefore, Market trend data, including the Risk-free rate of return R_i and Market return R_m, were obtained from their websites. Data for individual stocks were obtained from the Ghana Stock Exchange website. Stock price data from December, 2015 through February, 2020 were collected for five, (5) companies namely; CAL Bank Ghana (CAL), Ghana Commercial Bank (GCB), Fan Milk Limited (FML), Ghana Oil (GOIL), and MTN Ghana (MTNGH). Monthly average stock prices and monthly average returns on stocks for the period under study (January, 2015 to February, 2020), were calculated, from the obtained daily historical prices. By the Ordinary least square OLS time regression analysis, the betas of the Capital Assets Pricing model, were estimated, using the PYTHON programming language. The Ghana Stock Exchange Composite Index, GSE composite index, was used as market returns. The GSE C.I is the major stock market index which tracks the performance of all companies traded on the Ghana Stock Exchange [17]. The 91-Day Treasury bill backed by the Bank of Ghana was used for the risk-free interest rate of return. To obtain and use this factor in its appropriate unit as proposed by the model, the risk-free rate was expressed on a monthly basis.. The $R_m - R_f$ factor, the market premium is an important factor in the Model. Graphs of the prices of the stocks of the five companies are displayed in Fig 1 below.



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Monthly Stock Prices of Selected Companies

3

3.1 Analysis and Results

3.1.1 Analysis for Individual Assets

As stated, the betas of the CAPM, for the assets were estimated using OLS time regression analysis with PYTHON programming language. The results of the regressions analysis for the five companies are displayed on Table 1 below. The adjusted R^2 values and the P-values for each CAPM alpha and Beta are also displayed.

Table 1: **Regression results for the five stocks using CAPM showing Intercepts, Beta coefficients and adjusted** R^2

STOCK	α	$P-Value(\alpha)$	β	$P-Value(\beta)$	***Adjusted***R^2	***No. of Observation***
CAL	0.8287	0.7604	1.0258	0.00	0.4600	60
GCB	0.9294	0.723	1.0422	0.00	0.486	60
FML	-8.0529	0.004	0.5464	0.000	0.191	60
GOIL	-3.9331	0.189	0.7530	0.000	0.272	60
MTNGH	4.4787	0.3651	0.2340	0.4356	-0.0229	60

From table 1 the range of adjusted R^2 values is between -0.0229 and 0.486, the average R^2 value is 0.277. This means that 27.7% of the variation in the output variable is explained by the input variable. All Companies' assets have positive R^2 value. However, MTN Ghana has negative adjusted R^2 value. The negative value essentially means that the regressions consist of exogenous variables that give no explanation for the variation in the endogenous variable [8]. R^2 is the coefficient of determination which identifies predictors that are not significant in a regression model. From Table 1 above, CAL Bank, GCB, GOIL and Fan Milk Ghana stocks have a significant Beta coefficient $(R_m - R_f)$ at a five per cent significance level. The two banks' stocks have beta values greater than 1. This means that the bank stocks are more volatile than the three non-Bank stocks for the period under review. Beta value, β, measures the volatility of an asset with respect to the stock market, it determines the riskiness of an asset. The benchmark value of β, is 1. If $\beta > 1$, we have high risk, more return. If $\beta < 1$, we have less risk, less return.

3.1.2 Analysis for Portfolios

Portfolios contain a group of securities that are selected to achieve the highest return for a given level of risk. Individual asset returns were replaced with portfolio returns in the calculation. The aim of portfolio formation is to eliminate or reduce to the minimum non-systematic risk that comes with investing in individual assets. Guided by results obtained for the individual assets, five portfolios were constructed as follows: Portfolio 1 was constructed by allocating an equal percentage of an investor's capital to each asset. In Portfolio 2, the two financial companies GCB and CAL bank, which had β coefficient greater than 1, were allocated 5% each. This is because as stated, they were more volatile, hence riskier. For portfolio 3, we reversed the idea in portfolio 2, and assigned larger capital to GCB and CAL banks. Since beta greater than 1 implies both higher risk and higher expected return, we assign 30% to both of them in this case. Being more volatile than the market implies additional returns on one's investment. Then 10%, 15% and 15% of the total capital was invested in FML, GOIL and MTNGH respectively. Portfolio 4 was generated by allocating an equal capital share of 24% each to the assets FML, GOIL, and MTN, and shared the remaining 18% equally among the more risky two. FML had a β coefficient of 0.54 which is neutral when compared to the benchmark of 1. This implies that its asset is not exposed to much risk and is not entitled to much return. As a result for Portfolio 5, the highest allocation of 40% is given to FML, 30% each to

International Journal of Mathematical Sciences and Optimization: Theory and Applications
Vol. 9, No. 1, pp. 1 - 14
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GOIL, and MTN. These stocks are considered to be less volatile than the market. CAL and GCB are ignored at this point due to high risk. This breakdown on portfolio allocation is displayed on Table 2 below. The results of the regressions analysis for the five portfolios are displayed on Table 3.

Table 2: **Percentages used in the construction of five portfolios**

	CAL	*GCB*	*FML*	*GOIL*	*MTNGH*	*TOTAL*
***Port*1**	20%	20%	20%	20%	20%	100%
Port 2	5%	5%	30%	30%	30%	100%
***Port*3**	30%	30%	10%	15%	15%	100%
Port 4	9%	9%	24%	24%	24%	100%
***Port*5**	-	-	40%	30%	30%	100%

Table 3: **Regression results for five portfolios using the Model.**

	α	$P-Value(\alpha)$	β	$P-Value(\beta)$	$AdjustedR^2$	*No. of Observation*
***Port*1**	-3.7755	0.000	0.7757	0.000	0.783	60
***Port*2**	-5.9316	0.000	0.6606	0.000	0.635	60
***Port*3**	-2.0791	0.058	0.8717	0.000	0.798	60
***Port*4**	-5.5235	0.000	0.6829	0.000	0.737	60
***Port*5**	-6.8250	0.000	0.6118	0.000	0.511	60

From Table 3, the model gives a range of adjusted R^2 between 0.511 and 0.798, the average adjusted R^2 value is 0.6928. This implies that 69.28% of the variations in the output variable is explained by the input variable. It is observed from results on Table 3, that all five portfolios have significant β values at five percent significance level and β values are less than 1. This means that the portfolios are less volatile than the stock market. α values for all the portfolios were less than zero, but only one portfolio, Port 3, had an insignificant α value. A beta value for a portfolio being close to 1, means that it mirrors the volatility of the benchmark index (GSE-CI). It is observed that the risk coefficients beta for the various portfolios responded directly, to combination of weight of risks, in the asset of companies, which made up the components of the portfolios.For example, portfolio comprising of assets with high β, values, i.e. also showed high β, values.

3.1.3 Measurement of company performance: Volatility and Performance

Companies rely on both financial and non-financial measures for performance evaluation. In this work we considered a performance of the assets of companies listed on the stock exchange as a possible indicator of company performance. Financial performance can affect movement of stock price, which has to do with volatility. A link exists between volatility and stability. The study by Murithi [18] investigated the effect of financial performance on the volatility of share prices for commercial banks in Kenya. The work stated that return on equity (Roe) has a big impact on share price volatility. Financial stability of a firm trading at the stock market could remedy the volatility of its share (lower beta value implies less volatility which in turn signifies more stability).Going by the work of Osu, Ogbogbo et al [19], a low volatility means a stable and consistent market, low volatility values for assets of a company could indicate stability and consistence in the market. The model was applied to assets of the five Companies. Graphs of the prices of the stocks of the five companies which basically describe the behavior of the stocks, are displayed on Figures 2 – 6 below

Fig 2: Stock Prices for Fan Milk Ghana

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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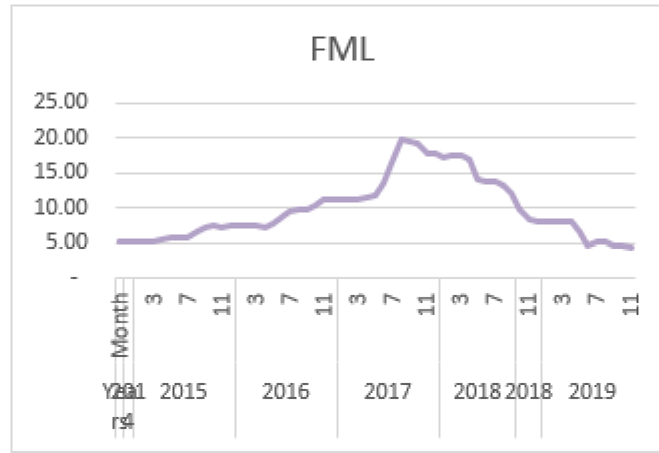

Fig 3: Stock Prices for Unilever Ghana Limited

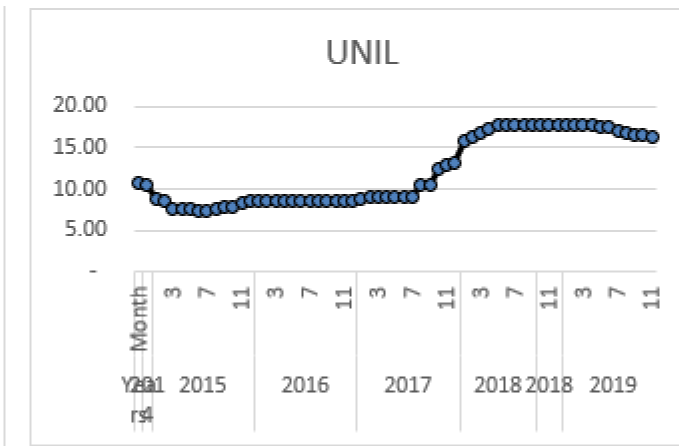

Fig 4: Stock Prices for Total Petroleum Ghana Limited

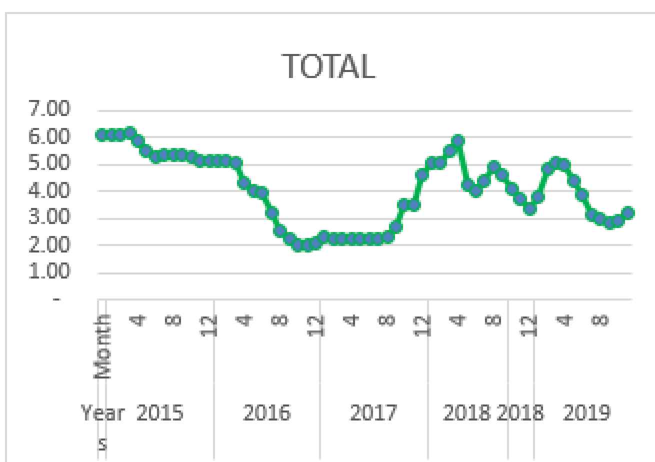

Fig 5: Stock Prices for Cal Bank Ghana

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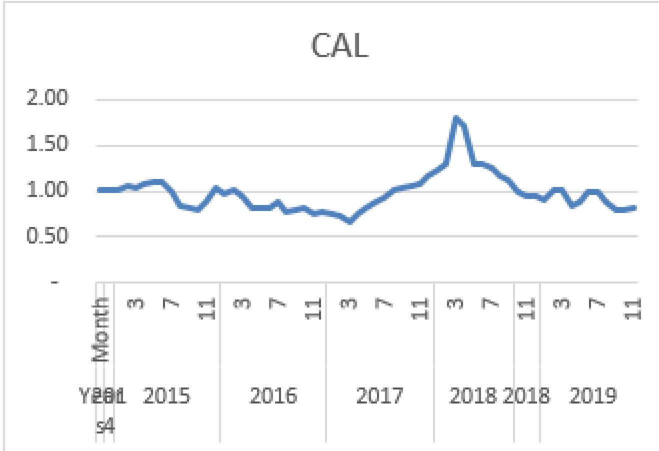
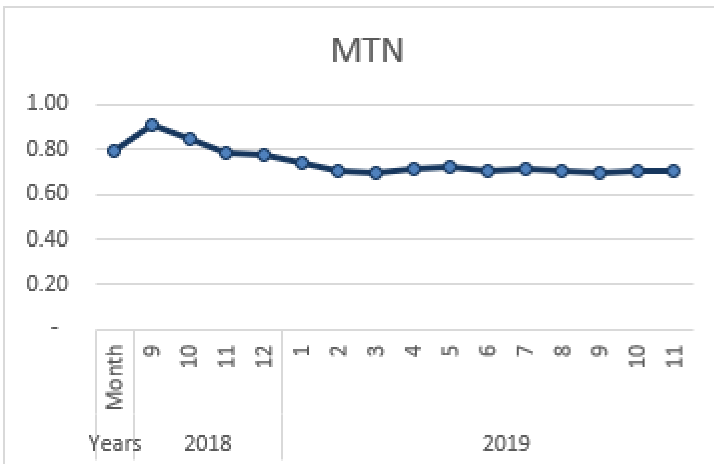


Fig 6: Stock prices for MTN Ghana



Since private companies are investment drivers and Economy boosters for growth, stability, and development, the companies used for Performance analysis par the model are private sector companies. As a result GCB and GOIL are are replaced by TOTAL and Unilever Ghana (UNIL). As stated, the betas of the model, for the assets were estimated using OLS time regression analysis with PYTHON programming language. The results of the regressions analysis for the five companies are displayed on Table 4 below. The adjusted R^2 values and the P-values for each alpha and Beta are also displayed.

Table 4: **-: Regression results for the five stocks. The table includes Stocks, Intercept, Beta coefficients and adjusted R^2**

	α	$P-Value(\alpha)$	β	$P-Value(\beta)$	$AdjustedR^2$	*Observation*
$\boldsymbol{CAL}$	0.8287	0.7604	1.0258	0.00	0.46	60
TOTAL	3.3451	0.3012	1.2157	0.00	0.4732	60
$\boldsymbol{FML}$	-8.0524	0.0037	0.5464	0.0003	0.19	60
UNIL	-2.1380	0.2131	0.8367	0.00	0.6045	
$\boldsymbol{MTNGH}$	4.4787	0.3651	0.2340	0.435	-0.02	60

Also from Table 4 above, CAL Bank, TOTAL, UNILEVER and FAN Milk Ghana stocks have a significant Beta coefficient $R_m - R_f$ at five percent significance level. Thus, MTN Ghana is the only stock with a non-significant Beta coefficient $Rm - Rf$ at five percent significance level. CAL and TOTAL stocks have beta values greater than 1. This means that their stocks are more volatile than the other three stocks for the period under review. CAL Bank, TOTAL and MTNGH had α values greater than zero, which indicates that they performed better than the bench mark index (GSE-CI in this case). However, Fan Milk limited and GOIL recorded α values less than zero which implies that the other three, performed better than them. Low α value may be an indication of decline in performance which could arise from a number of issues: low productivity, poor management, losing customers to competitors, or low patronage of their products and services (lack of confidence of consumers of the products). Fan Milk Limited recorded a significant α value at five percent significant level.

3.1.4 Prediction of Asset Returns

α and β values for FML were significant at 5% level of significance, therefore the model was used to predict its price for the months of March, April and May 2020. Results obtained are displayed on Table 5 below. Predicted prices are very close to actual prices, indicating that the model can be used to forecast returns.

Table 5: **Theoretical and Actual Stock Price for FML for March – May 2020 .**

Month	Actual Price	Theoretical Price	Variance	Standard Dev
March	4.07	4.44	0.03	0.1732
April	4.01	4.36	0.03	0.1732
May	3.71	4.21	0.06	0.2449

3.1.5 Possible Extension of the Model

A possible extension of the model is presented below:
Equation (6) can be written as

$$ER_i - R_f = \beta_i E(R_m - R_f) + E_i \tag{7}$$

The market model is defined as

$$R_t = \alpha + \beta m_t + V_t \tag{8}$$

where $t = 1, 2, ..., T$
Coefficients of (8) may be estimated using OLS. In application, the coefficients may not be time-variant. A model extension where α and β are time varying is necessary. This introduces the CAPM-VAR [19] model defined by

$$R_t = H_t^T \theta_t + V_t \tag{9}$$

Where $H_t^T = [1, m_t]$ and

$$(\theta_t - \theta_{med}) = F((\theta_{t-1} - \theta_{med}) + w_t \tag{10}$$

θ_t is the coefficient vector of CAPM i.e α,β and other factors of interest to the model. F is transition matrix of unknown elements. θ_{med} is the mean of θ_t, V_t is a normal white noise with zero mean and finite variance σ^2. The term w_t is a gaussian white noise vector with zero mean covariance matrix Q. The initial value θ_0 is a normal random variable with mean μ and covariance matrix $\sum$. Eqn (9) and (10) can then be easily rewritten in a state-space form:

$$\begin{bmatrix}\theta_t - \theta_{med} \\ \theta_{med}\end{bmatrix} = \begin{bmatrix}F & 0 \\ 0 & 1\end{bmatrix} \Bigg| \begin{bmatrix}\theta_t - \theta_{med} \\ \theta_{med}\end{bmatrix} + \begin{bmatrix}1 \\ 0\end{bmatrix} w_t$$

$$R_t = \begin{bmatrix}H_t & H_t\end{bmatrix} \begin{bmatrix}\theta_t - \theta_{med} \\ \theta_{med}\end{bmatrix} + V_t$$

EM algorithm can be used to estimate the hyperparameters, and the Kalman filter used to estimate the parameters.

4 Conclusion

The Model explains portfolio excess return better than it does for individual assets' excess returns. While beta, β, is an indication of riskiness or volatility of an asset, alpha, α values indicate the performance of an asset in comparison with GSE composite index. By these values, there is higher risk inherent in investing in financial institutions, considering their alpha values they also have the best performance out of the 5 assets studied. These five assets were chosen as representative of four major sectors of the Economy. Asset of the food company FML, was less volatile, while that of the oil company, GOIL, having a beta value close to 1, seem to mirror the market. Exogenous factors which may include customer reaction, copious advertisement of telecom companies, customer preferences, etc may have attributed to MTN's high alpha and low beta values, since it has negative R^2 value. Only Fan Milk Limited (FML) recorded significant α and β values at five percent significant level. As a result, prediction was made for the price of stock of FML for the months of March, April and May 2020 The theoretical values obtained were very close to the empirical values. Introduction of high-risk stocks in our five portfolios increased their beta values. Also, for alpha values, adding to a portfolio, stocks that underperformed close to twice the value of the best performing stocks accounts for the underperformance of our five portfolios, hence they all have negative alpha values.. The model did well in assessing risk, return and performance for the assets, of companies on the GSE. With a significant regression outcome, it can also be used to predict prices of assets on the GSE. Significance of α values should be taken into consideration when discussing the performance of individual assets because a non-significant alpha value cannot exactly be used to determine the performance of an asset. This motivates further analysis for assets, using a better model. Further work has been recommended as use of a stochastic extension of the model.

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