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Coefficient Bounds for Certain Subclasses of Analytic Functions associated with Generalized Distribution Defined by Polylogarithm

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Abstract

In the present work, we use Subordination principle to obtain the bounds for the first few coefficients of the classes $SY_\lambda^m(b)$ and $RY_\lambda^m(b)$ in the unit disk.In addition, we established relevant connections of our results to Fekete-Szego theorem.

Keywords: Analytic function, Univalent function, Subordination, Spirallike function, Bounded turning function.
MSC2010: 30C45.

1 INTRODUCTION AND DEFINITIONS

Suppose that H denote the class of functions

$$f(z) = z + a_2z^2 + a_3z^3 + ... \tag{1.1}$$

which are analytic and univalent in the open unit disc $U = \{z : z \in C, |z| < 1\}$, and let $S \in H$ consist of univalent functions in U normalized with $f(0) = f'(0) - 1 = 0$.

The analytic function $f(z)$ is said to be Starlike, Convex, Spiralike, Close-to-Convex and Bounded turning provided the following geometric conditions are respectively satisfied: $Re\left\{\frac{zf'(z)}{f(z)}\right\} > 0$, $Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0$, $Re\left\{e^{i\theta}\frac{zf'(z)}{f(z)}\right\} > 0$, $Re\left\{\frac{zf'(z)}{g(z)}\right\} > 0$ and $Re\{f'(z)\} > 0$.

Let f and g be analytic in U, then f is said to be subordinate to g written as $f(z) \prec g(z)$, if there exists a function $\omega(z)$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ such that $f(z) = g(\omega(z))$ $(z \in U)$.

The generalized distribution with univalent functions were examined by Porwal [1] and interesting properties of the said distribution were established.

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Let Y denote the sum of the convergent series of the form

$$Y = \sum_{n=0}^{\infty} a_n,$$

where $a_n \geq 0$ for all $n \in N$. Also let $p(j)$ denote the probability mass function given as

$$p(j) = \frac{b_j}{Y}, j \geq 0$$

It is important to note that $p(j)$ is the probability mass function because $p(j) \geq 0$ and $\sum_j p(j) = 1$.

In addition, suppose

$$\wp(x) = a_0 + a_1 x + a_2 x^2 + ...,$$

then from $Y = \sum_{n=0}^{\infty} a_n$ series $\wp$ converges for both $|x| < 1$ and $x = 1$.

Now if X is a discrete random variable that takes values $x_1, x_2, ...$ associated with probabilities $p_1, p_2, ..$ then the expected X denoted by $E(X)$ is defined as

$$E(X) = \sum_{j=1}^{\infty} p(j) x^j.$$

The moment of a discrete probability distribution (r^{th}) about $x = 0$ is defined by

$$\mu_r^{'} = E(X^r)$$

where $\mu_1^{'}$ is the mean of the distribution and the variance is given as

$$\mu_2^{'} - (\mu_1^{'})^2.$$

Moment about the origin is given as

$$Mean = \mu_1^{'} = \frac{\psi^{'}}{Y},$$

$$Variance = \mu_2^{'} - (\mu_1^{'})^2 = \frac{1}{S}[\psi^{''}(1) + \psi^{'}(1) - \frac{(\psi^{'}(1))^2}{Y}].$$

The moment generating function of a random variable X is denoted by $M_X(t)$ and defined by

$$M_X(t) = E(e^{Xt})$$

and the moment generating function of generalized discrete probability is given as

$$M_X(t) = \frac{\psi(e^t)}{Y}.$$

Special values of a_n for various well known discrete probability distributions such as Yule-Simon distribution, Logarithmic distribution, Poisson distribution, Binomial distribution, Beta-Binomial distribution, Zeta distribution, Geometric distribution and Bernoulli distribution can be obtained (see [1] for details).

Here, we consider the polynomial whose coefficients are probabilities of the generalized distribution given by:

$$T(z) = z + \sum_{n=2}^{\infty} \frac{a_{n-1}}{Y} z^n, \tag{1.2}$$

where $Y = \sum_{n=0}^{\infty} a_n$. (see also [2]).

The convolution of f and u denoted by $(f * u)(z)$ is given by:

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$$(f*u)(z)=z+\sum_{j=2}^{\infty}a_jb_jz^j=(u*f)(z) \tag{1.3}$$

where

$$u(z)=z+\sum_{j=2}^{\infty}b_jz^j.$$

In 1696, Bernoulli and Leibniz [3] defined the polylogarithm function by the absolutely convergent series $Li_p(z)=\sum_{j=1}^{\infty}\frac{z^j}{j^p}$, provided $|z|<1$ and $p\geq 2$.

Several authors have investigated polylogarithm function in different perspective and their results have appeared in prints (see [4]).

However, Al-Shaqsi and Darus [5] generalized Ruscheweyh and Salagean derivative operators as follows:

Let f belongs to H,the generalized polylogarithms $D_\lambda^m f(z):H\longrightarrow H$

$$D_\lambda^m f(z)=z+\sum_{n=2}^{\infty}\frac{n^m(n+\lambda-1)!}{\lambda!(n-1)!}z^n \tag{1.4}$$

where $m,\lambda\in N_0=0,1,2,...,z\in U$ and H as earlier defined (see [6–8] among others).

If $\lambda=0$, the operator (4) reduces to Salagean differential operator and if $m=0$ the operator reduces to Ruscheweyh operator, (see [4] for more properties of polylogarithm).

Motivated by the earlier works of [1,2,5,9–11], we investigate bounds for a new class of analytic function defined with complex order in the open unit disk U.

Using (2), (3) and (4), We can write that

$$D_\lambda^m*T(z)=z+\sum_{n=2}^{\infty}\frac{n^m(n+\lambda-1)!}{\lambda!(n-1)!}\frac{a_{n-1}}{Y}z^n \tag{1.5}$$

where $m,\lambda\in\{0,1,2,...\}$ and $Y=\sum_{n=0}^{\infty}a_n$.

Definition 1.1 Let $f(z)\in SY_\lambda^m(b)$, then for $k\in[0,\infty)$, $\alpha\in[0,1)$, $m,\lambda\in N_0, b\neq 0$ and $|\theta|<\frac{\pi}{2}$

$$1+\frac{1}{b}\left\{e^{i\theta}\frac{z(D_\lambda^mT(z))'}{D_\lambda^mT(z)}-1\right\}\prec p_k(z)$$

Definition 1.2: Let $f(z)\in RY_\lambda^m(b)$, then for $k\in[0,\infty)$, $\alpha\in[0,1)$, $m,\lambda\in N_0, b\neq 0$ and $|\theta|<\frac{\pi}{2}$

$$1+\frac{1}{b}\left\{(D_\lambda^mT(z))'-1\right\}\prec p_k(z)$$

2 PRELIMINARY LEMMA

Let P denote the class of functions with positive real parts (Caratheodogy functions) such that $p\in P$ with $\{Rep(z)\}>0$ and $p(z)=1+\alpha_1z+...$ in U, then $|\alpha_k|\leq 2$, $k=1,2,3,...$

Lemma 2.1 : Let $\omega(z)=\omega_1z+\omega_2z^2+...\in\Omega$ be so that $|\omega(z)|<1$ in U. If ρ is a complex number, then

$$|\omega_2+t\omega_1^2|<max\{1,|\rho|\}$$

The sharp inequality for the functions $\omega(z)=z$ or $\omega(z)=z^2$, (see also [12–15].

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3 MAIN RESULTS

The first few coefficient bounds for the classes defined in the Definitions above are considered in the following results .

Theorem 3.1: Suppose $f(z) \in SY_{\lambda}^{m}(b)$, then for $k \in [0,\infty)$, $\alpha \in [0,1)$, $m, \lambda \in N_0, b \neq 0$ and $|\theta| < \frac{\pi}{2}$

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{2^m(\lambda+1)(2e^{i\theta}-1)},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{4|b|\alpha_1}{3^m(3e^{i\theta}-1)(\lambda+2)(\lambda+1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{b\alpha_1}{(2e^{i\theta}-1)}\right|\right\}$$

and

$$\left|\frac{a^2}{Y} - \mu\frac{a_1^2}{Y^2}\right| \leq$$

$$\frac{2|b|\alpha_1}{3^m(3e^{i\theta}-1)(\lambda+2)(\lambda+1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{2^{m+1}(\lambda+1)(e^{i\theta}-1) - 3^m\mu(3e^{i\theta}-1)(\lambda+2)}{2^m(\lambda+1)(2e^{i\theta}-1)}b\alpha_1\right|\right\}.$$

where λ is as earlier defined.

Proof

If $f \in SY_q^b(\alpha_k)$ and $\omega \in \Omega$ then by the definition of subordination, we have that

$$1 + \frac{1}{b}\left\{e^{i\theta}\frac{z(D_\lambda^m K(z))'}{D_\lambda^m T(z)} - 1\right\} = p_k(\omega(z)). \tag{3.1}$$

The first few terms of (6) gives

$$ze^{i\theta} + 2^{m+1}(\lambda+1)\frac{a_1}{Y}z^2e^{i\theta} + 3^{m+1}\frac{(\lambda+2)(\lambda+1)}{2}\frac{a_2}{Y}z^3e^{i\theta} - z - 2^m(\lambda+1)\frac{a_1}{Y}z^2 - 3^m\frac{(\lambda+2)(\lambda+1)}{2}\frac{a_2}{Y}z^3 - \ldots$$

$$= b\alpha_1\omega_1 z^2 + b\alpha_1\omega_1^2 z^3 + 2^m(\lambda+1)\frac{a_1}{Y}z^3 b\alpha^1\omega_1 + 3^m\frac{(\lambda+2)(\lambda+1)}{2}\frac{a_2}{Y}z^4 b\alpha_1\omega_1. \tag{3.2}$$

From (6) and (7), we have

$$\frac{a_1}{Y} = \frac{b\alpha_1\omega_1}{2^m(2e^{i\theta}-1)(\lambda+1)} \tag{3.3}$$

and

$$\frac{a_2}{Y} = \frac{2b\alpha_1}{3^m(3e^{i\theta}-1)(\lambda+2)(\lambda+1)}\left\{\omega_2 + \left(\frac{1}{2e^{i\theta}-1} - \left[\frac{\alpha_2}{\alpha_1} + b\alpha_1\right]\omega_1^2\right)\right\}. \tag{3.4}$$

Solving (8) and (9), we obtain

$$\frac{a_2}{Y} - \mu\frac{a_1^2}{Y^2} = \frac{2b\alpha_1}{3^m(3e^{i\theta}-1)(\lambda+2)(\lambda+1)}\left\{\omega_2 + \rho\omega_1^2\right\}$$

where

$$\rho = \frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{2^{m+1}(\lambda+1)(2e^{i\theta}-1) - 3^m\mu(3e^{i\theta}-1)(\lambda+2)}{2^{2m+1}(\lambda+1)(2e^{i\theta}-1)}b\alpha_1$$

The desired result is obtained by applying the Lemma 2.1.

corollary 3.2

For $k \in [0,\infty), \alpha \in [0,1), m = 0, b \neq 0$ in theorem 3.1 and if $f \in SY_\lambda^0(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{(\lambda+1)(2e^{i\theta}-1)},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{2|b|\alpha_1}{(3e^{i\theta}-1)(\lambda+2)(\lambda+1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{b\alpha_1}{(2e^{i\theta}-1)}\right|\right\}$$

and

$$\left|\frac{a^2}{Y} - \mu\frac{a_1^2}{Y^2}\right| \leq$$

$$\frac{2|b|\alpha_1}{(3e^{i\theta}-1)(\lambda+2)(\lambda+1)} \max\left\{1, \left\{\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{2(\lambda+1)(e^{i\theta}-1)-\mu(3e^{i\theta}-1)(\lambda+2)}{(\lambda+1)(2e^{i\theta}-1)}b\alpha_1\right|\right\}.$$

Corollary 3.3

For $k \in [0,\infty), \alpha \in [0,1), \lambda = 0, b \neq 0$ in theorem 3.1 and if $f \in SY_0^m(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{2^m(2e^{i\theta}-1)},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{2|b|\alpha_1}{3^m(3e^{i\theta}-1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{b\alpha_1}{(2e^{i\theta}-1)}\right|\right\}$$

and

$$\left|\frac{a^2}{Y} - \mu\frac{a_1^2}{Y^2}\right| \leq \frac{|b|\alpha_1}{3^m(3e^{i\theta}-1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{2^{m+1}(e^{i\theta}-1)-3^m\mu(6e^{i\theta}-2)}{2^m(2e^{i\theta}-1)}b\alpha_1\right|\right\}$$

Corollary 3.4

Let $k \in [0,\infty), \alpha \in [0,1), m = 0, \lambda = 0, b \neq 0$ and if $f \in SY_0^0(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{(2e^{i\theta}-1)},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{2|b|\alpha_1}{(3e^{i\theta}-1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{b\alpha_1}{(2e^{i\theta}-1)}\right|\right\}$$

and

$$\left|\frac{a^2}{Y} - \mu\frac{a_1^2}{Y^2}\right| \leq \frac{|b|\alpha_1}{(3e^{i\theta}-1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{2(e^{i\theta}-1)-\mu(6e^{i\theta}-2)}{(2e^{i\theta}-1)}b\alpha_1\right|\right\}.$$

Corollary 3.5

Let $k \in [0,\infty), \alpha \in [0,1), m = 1, \lambda = 1, b \neq 0$ and if $f \in SY_1^1(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{4(2e^{i\theta}-1)},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{2|b|\alpha_1}{3^2(3e^{i\theta}-1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{b\alpha_1}{(2e^{i\theta}-1)}\right|\right\}$$

and

$$\left|\frac{a^2}{Y} - \mu\frac{a_1^2}{Y^2}\right| \leq \frac{|b|\alpha_1}{3^2(3e^{i\theta}-1)} \max\left\{1, \left|\frac{\alpha_2}{\alpha_1(2e^{i\theta}-1)} + \frac{2^3(e^{i\theta}-1)-3^2\mu(3e^{i\theta}-1)}{2^3(2e^{i\theta}-1)}b\alpha_1\right|\right\}.$$

Corollary 3.6

Let $k \in [0,\infty), \alpha \in [0,1), m = 1, \lambda = 1, b \neq 0, \theta = 0, \alpha_1 = \alpha_2 = 2$ and if $f \in SY_1^1(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|}{2},$$

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$$\left|\frac{a_2}{Y}\right| \leq \frac{2|b|}{9}\max\{1,|1+2b|\}$$

and

$$\left|\frac{a^2}{Y}-\mu\frac{a_1^2}{Y^2}\right| \leq \frac{|b|}{9}\max\left\{1,\left|1-\frac{9\mu}{2}b\right|\right\}.$$

Theorem 3.7: Suppose $f(z) \in RY_\lambda^m(b,\alpha_k)$, then for $k \in [0,\infty)$, $\alpha \in [0,1)$, $m,\lambda \in N_0, b \neq 0$ and $|\theta| < \frac{\pi}{2}$

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{2^{m+1}(\lambda+1)},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{2|b|\alpha_1}{3^{m+1}(\lambda+2)(\lambda+1)}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}\right|\right\}$$

and

$$\left|\frac{a^2}{Y}-\mu\frac{a_1^2}{Y^2}\right| \leq \frac{2|b|\alpha_1}{3^{m+1}(\lambda+2)(\lambda+1)}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}-\frac{3^{m+1}(\lambda+2)}{2^{2m+3}(\lambda+1)}\mu b\alpha_1\right|\right\}.$$

Proof

The method of proof is similar to that of Theorem 3.1 .

Corollary 3.8

Let $k \in [0,\infty), \alpha \in [0,1), m,\lambda \in N_0, b \neq 0, m=0$. If $f(z)$ belong to $RY_\lambda^m(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{2(\lambda+1)},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{2|b|\alpha_1}{3(\lambda+2)(\lambda+1)}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}\right|\right\}$$

and

$$\left|\frac{a^2}{Y}-\mu\frac{a_1^2}{Y^2}\right| \leq \frac{2|b|\alpha_1}{3(\lambda+2)(\lambda+1)}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}-\frac{3(\lambda+2)}{8(\lambda+1)}\mu b\alpha_1\right|\right\}.$$

Corollary 3.9

Let $k \in [0,\infty), \alpha \in [0,1), m,\lambda \in N_0, b \neq 0, \lambda=0$. If $f(z)$ belong to $RY_\lambda^m(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{2^{m+1}},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{|b|\alpha_1}{3^{m+1}(\lambda+1)}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}\right|\right\}$$

and

$$\left|\frac{a^2}{Y}-\mu\frac{a_1^2}{Y^2}\right| \leq \frac{|b|\alpha_1}{3^{m+1}}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}-\frac{2.3^{m+1}}{2^{2m+3}}\mu b\alpha_1\right|\right\}.$$

Corollary 3.10

Let $k \in [0,\infty), \alpha \in [0,1), m,\lambda \in N_0, b \neq 0, m=0, \lambda=0$. If $f(z)$ belong to $RY_\lambda^m(b,\alpha_k)$ then

$$\left|\frac{a_1}{Y}\right| \leq \frac{|b|\alpha_1}{2},$$

$$\left|\frac{a_2}{Y}\right| \leq \frac{|b|\alpha_1}{3}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}\right|\right\}$$

and

$$\left|\frac{a^2}{Y}-\mu\frac{a_1^2}{Y^2}\right| \leq \frac{|b|\alpha_1}{3}\max\left\{1,\left|\frac{\alpha_2}{\alpha_1}-\frac{3}{4}\mu b\alpha_1\right|\right\}.$$

4 Conclusion

The principal significance of the sharp bounds for the coefficients is the information about geometric properties of the functions. For instance, the sharp bounds of the second coefficient of normalized univalent functions readily yields the growth and distortion bounds (see [17]. Also, sharp bounds of the coefficient functional $|a_3 - \mu a_2^2|$ helps in the investigation of univalence of analysis function. Also, apart from $n-th$ coefficients, bounds were used to determine the extreme points of the class of analysis functions;(see also [16])

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