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Combinatorial Model of 3 Dimensional Nildempotency Star-Like Classes $N_c\omega_n^*$ Partial One to One Semigroups

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Abstract

Let $X_n = \{1, 2, 3, ..., n\}$ be a distinct non negative integer. The 3 Dimensional Nildempotency star-like classes $N_c\omega_n^*$ partial one-one was studied using the Star-like operator $|\alpha w_i - \alpha w_{i+1}| + |\alpha^* w_i - w_{i+1}| \leq |w_i - w_{i+1}| + |\alpha^* w_{i+1} - w_i|$. The geometric model of 3 Dimensional star-like Nildempotency transformation semigroup was also generated by using a standard rectangular A4 paper.We show that for any star-like polygon with n star-like vertices V^*, star-like edges E^*, star-like faces F^* and n star-like angles α^* in $N_c\omega_n^*$ then $V^* - E^* + F^* = 2$.

Keywords: Cobinatorial composite function, Geometry, NIldempotency, Semigroup, Star-like.
MSC2010: 11T06, 06F05.

1 Introduction

Many researchers have studied semigroup theory, but few have worked on star-like transformation semigroup been newly established by Akinwunmi in [1]. A star-like semigroup S^* with empty mapping is said to be nildempotent provided that there exist $t, k \in V : S^t = \emptyset$, that is $x_1, x_2, x_3, \ldots, x_n = \emptyset$ for all $x_1, x_2, x_3, \ldots \in S$ implies $\alpha \in S$ where $e^k \in D(\alpha) : e^k = e$. Akinwunmi [2].

If S^* is nildempotent then the minimal element $t, k \in V : S^t = \emptyset \implies e^k = e$ is called the nildempotency degree of S^* and is denoted by ND(S^*). We observe that the nildempotent elements form a sub-semigroup class of their own then the combinatorial nature of the sequence of numbers and their triangular arrangement arise naturally thus making it essential to find the general relation which in turn highlight it's application to Mathematics and Science as a whole.

The Partial One to One transformation semigroup denoted as I_n given by $X_n = \{1, 2, 3, \ldots, n\}$ such that α:Dom $\alpha = X_n$, commonly known as inverse or partial one to one transformation semigroup with set S^* and operation $*$ Transformation semigroup are associative then:(α, β, μ):$(\alpha * \beta) * \mu$=$\alpha * (\beta * \mu)$.

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It is also known as the inverse symmetric semigroup or monoid with composition of mappings as the semigroup operator. The star-like partial one-one transformation semigroup denoted as $I\alpha\omega_n^*$ in I_n is also a semigroup in one-one transformation semigroup.

Transformation is used instead of mapping, the latter serves as another name for the former. More information on semigroup of mappings are obtainable from the works of Bhattacharya et al. [3], Howie [4].

The domain and image set of any given transformations $\alpha_i^* \in \alpha\omega_n^*$ was denoted by $D(\alpha^*)$ and $I(\alpha^*)$ respectively as used by Malik [5] and also see [?, ?] for the study and definition of partial transformation semigroup.

A Star-like transformation semigroup is said to satisfy collapse function if $c^+(\alpha^*) = |\bigcup t\alpha^{-1} : t \in T\alpha\omega_n^*|$ while Relapse function is denoted as $r^+(\alpha) =| n - c^+(\alpha^*) |$ where $n \in N$.

2 Preliminary Notes

2.1 Generalization of 3 Dimensional Star-like Sequence through Some Combinatorial Composite Functions

Let $X = \{w_1, w_2, w_3, \dots w_n\}$ be a distinct finite n-elements set where $N_c\omega_n^*$ is a 3 Dimensional star-like partial one to one classical semigroup such that

$$|\alpha w_i - \alpha w_{i+1}| + |\alpha^* w_i - w_{i+1}| \leq |w_i - w_{i+1}| + |\alpha^* w_{i+1} - w_i| \tag{2.1}$$

for all $w_i, w_{i+1} \in D(\alpha^*)$ and $\alpha^* w_i, \alpha^* w_{i+1} \in I(\alpha^*)$ where $i \geq n \geq 1$; $n \in \mathbb{N}$, then we give a transformation of X in the form:

$$\alpha_n^* = \begin{pmatrix} w_1 & w_2 & w_3 & \dots & w_n \\ \alpha^\star w_1 & \alpha^\star w_2 & \alpha^\star w_3 & \dots & \alpha^* w_n \end{pmatrix} \tag{2.2}$$

The element $\alpha^\star w_i$ is call the value of points in the image of the transformation of $\alpha_n^\star$ and w_i is call the value of points in the domain of the transformation of α_n^*. The fact that $N_c\omega_n^*$ is a 3 Dimensional star-like transformation implies that X can be written as $\alpha^* : X \longrightarrow X$. Then, the transformation of X is a function $\alpha^* : B \longrightarrow X$, where $B = \{f_1, f_2, f_3, \dots f_n\}$ is a subset of X then we write the element of α^* such that:

$$\alpha_n^* = \begin{pmatrix} f_1 & f_2 & f_3 & \dots & f_n \\ \alpha^* f_1 & \alpha^* f_2 & \alpha^* f_3 & \dots & \alpha^* f_n \end{pmatrix} \tag{2.3}$$

Then the set of all 3 dimensional star-like one to one (inverse) transformation of X is denoted by $N_c\omega_n^*$. We can rewrite the element of α^* in equation (1) in the form:

$$\alpha_n^* = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ k_1 & k_2 & k_3 & \dots & k_n \end{pmatrix} \tag{2.4}$$

here $k_i = \alpha^* \omega_i \in I(\alpha^*)$ and $n \in \mathbb{N}$. By adopting the methodology given by Akinwunmi et al. [2] on Nildempotency of partial one to one contraction Cl_n transformation semigroups, we obtained the 3 dimensional Nildempotency star-like classes $N_c\omega_n^*$ partial one to one transformation semigroups. The star-like nildempotency $N_c\omega_2^*$ degree of order 2 were generated from Equation (1): when $n = 1$

$$N_c\omega_1^* = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$$

when $n = 2$

$$N_c\omega_2^* = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \right\}$$

when $n = 3$

$$C\omega_n^* = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \end{pmatrix} \right\}$$

2.2 Geometric Model of 3 Dimensional Nildempotency star like classes $N_c\omega_n^*$ partial one to one transformation semigroups.

Geometric model of 3 Dimensional Nildempotency star like transformation semigroups was generated from star-like folding principle by using a standard rectangular $A4$ paper from the upper right corner down to produce figure 1:

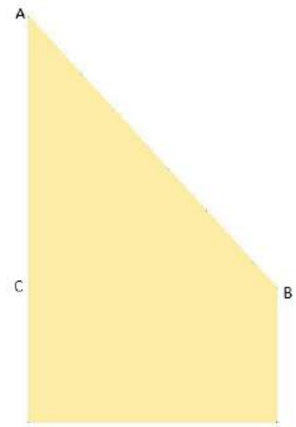

Figure 1: 3 Dimensional Nildempotency Star-like Trapezium

Then, Cutting out the remaining sheet below vertices C and B in Figure 1: we obtained Figure 2:

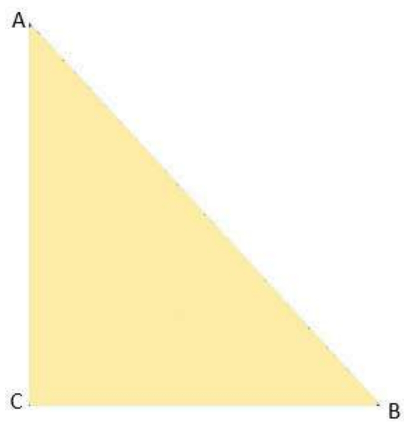

Figure 2: 3 Dimensional Nildempotency Star-like Right Angle Triangle

From the star-like folding principle, holding vertex of A and joining it to the vertex of B in Figure 2: we obtain Figure 3: below

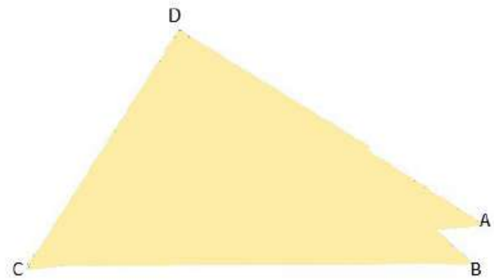

Figure 3: 3 Dimensional Nildempotency Star-like Equilateral Triangle

Now, unfolding the Figure 1:, Figure 2: and Figure 3: above, we reverse the structure to a standard plain sheet with some star-like paths retained, we generated the star-like nildempotency classes of semigroup as shown in Figure 4:

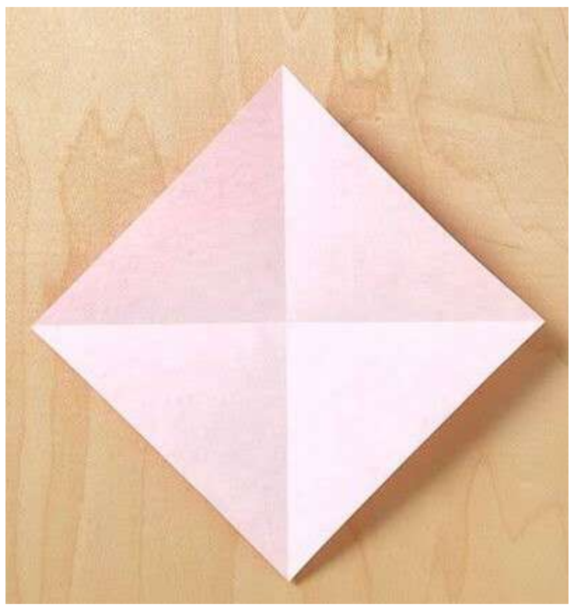

Figure 4: 3 Dimensional Nildempotency Star-like Square Pyramid

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Holding the vertices of A and B of the Figure 4: in such a way that the vertices of C and D will facing 'you' directly there by joining the vertices of A and B together allowing the middle of vertices A and B to be folded inward to obtain Figure 5:

Figure 5: 3 Dimensional Nildempotency Star-like Polygon

Repeating the procedure in Figure 5: of vertices C and D, we obtained Geometric model of 3 Dimensional Nildempotency Star-like Classes $N_c\omega_n^*$ partial one-one transformation semigroups represented in Figure 6:

Figure 6: Bottom View of 3 Dimensional Nildempotency Star-like Pyramid

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The star-like 3 Dimensional model in Figure7: represent the star-like Nildempotency rectangular prism with the composition of:

i Star-like Faces F^*

ii Star-like Edges E^*

iii Star-like Vertices V^*

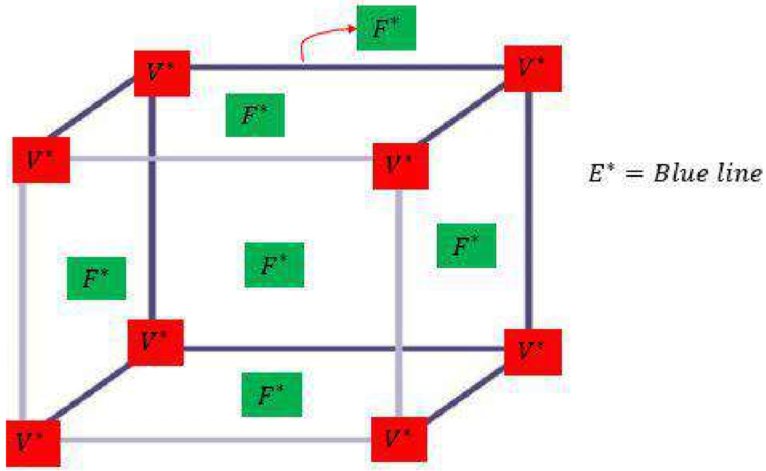

Figure 7: Star-like Nildempotency Square Prism

By the star-like folding principle structure we unfold the Fig 7: to obtained the general 3 Dimensional star-like Nildempotency equation.

$$F^* + V^* = E^* + 2 \tag{2.5}$$

which is a relation to the unfolded 3 Dimensional star-like Nildempotency rectangular prism. Therefore to obtained the volume of a 3 Dimensional star-like Nildempotency triangular prism, we must begin to construct a star-like triangular path with a 3 Dimensional star-like Nildempotency array of a control star-like Nildempotency disk point which form an n sided star-like Nildempotency 3 Dimensional depths. From equation 1 combining with 3 Dimensional general star-like Nildempotency, we obtained

$$V^* = \frac{1}{2} b \times h \times l \tag{2.6}$$

Equivalent to

$$\frac{1}{2} V^* = |\alpha\omega_i - \omega_{i+1}| \leq |\alpha\omega_i - \omega_i| \tag{2.7}$$

where $\omega_{i+1} \in D(\alpha^*)$ and $\alpha\omega_i \in I(\alpha^*)$.

Some notable notation used in this paper

$N_c\omega_n^*$	3 Dimensional Nildempotency star-like transformation
ϕ^*	Star-like nildempotency disk constant point
ϕ_n^*	Set of all star-like disk constant relation
C^*	Star-like convex polygon
R_n^*	Star-like polygon
$M(\alpha^*)$	Star-like Maximum element
$\phi^* = 2$	Inner star-like nildempotency disk constant

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3 Main Results

These are the main results of the paper.

Given any star-like transformation α^* in partial one-one star-like semigroup $N_c\omega_n^*$, then the following statement are equivalent:

i Any $\alpha \in N_c\omega_n^*$ has a maximum element $m(\alpha^*)$

ii $\mid N_c\omega_n^* \mid = \sum_{q=2}^{n} \binom{n+2q^*}{3q^*}$

iii $F(n, c^*, m^*) = \left(2^{(n-c^*)+1} - 1\right) : n, m^* \geq q^*$

Proof. $(i) \longrightarrow (ii)$
Suppose $F(n, c^*, m^*)$ is a Nildempotency star-like composition combinatorial function, let $N_i = \{i, i+1, i+2, ...\}, i = \{0, 1, 2, ...\}$ be a distinct non negative integer such that $N_c\omega_n^*$ contain a star-like transformation $\alpha^* \in N_c\omega_n^*$. then $i \in N_i$ of $Dam(\alpha^*)$ can be chosen from

$$N_i \text{ in } \binom{q^*}{n} \text{ ways.}$$

If α^* is a bijective map and $N_c\omega_n^*$ is reducible. $Im(\alpha^*) = 0$
if $\mid q^*(\alpha^*) \mid = 1$ but if

$$n = q^*; \mid q^*(\alpha^*) \mid = -1$$

for each value of $\alpha \in N_c\omega_n^*$ such that

$$\mid N_c\omega_n^* \mid = \binom{n+2q^*}{3q^*}$$

$(ii) \longrightarrow (iii)$ Since $\alpha \in N_c\omega_n^*$ is star-like Nildempotency , there exist finitely many element such that $Dm(\alpha^*) = Im(\alpha^*) where\ \lambda_n^* \bigcap r_n$ form a total of

$$\sum_{n=1}^{k} \binom{2^{(n-q)+1}}{2^{(n-m)+1} - 1} \binom{n+2q^*}{3q^*} = F(n, q, m)$$

$(iii) \longrightarrow (i)$ Suppose $m(\alpha^*)$. denote maximum element in $Im(\alpha^*)$ and
$N_c\omega_n^* \subseteq \alpha\omega_n^*$, then consider a mapping

$$r^* = \begin{pmatrix} u_1 & u_2 & u_0 \\ m_1 & m_2 & m_0 \end{pmatrix} \in m(\alpha)$$

such that there exist another element $\lambda^* \in m(\alpha$ with $r^* \leq \lambda^* \text{and} I_0 < \lambda^*$. since r and λ_0 are bijective we have

$$(rI\omega_n^*) \bigcap (\lambda_0 I\omega_n^*) = m(\alpha^*)$$

□

Let $N_c\omega_n^*$ be a set of star-like Nildempotency classes, where $\alpha \in N_c\omega_n^*$ such that $D(\alpha^*) \subseteq I(\alpha^*)$, then $V(N_c\omega_n^*)$ is

$$\frac{1}{2}V^* = \frac{E^* - F^*}{2} + \phi; E^* \in D(N_c\omega_n^*), F^* \in I(N_c\omega_i)$$

Proof. ,
Suppose $N_c\omega_n^*$ is a set of star-like Nildempotency finite transformation semigroup with a star-like composite relation,

$$a_i^* + b_j^* + c_k^* = N_c\omega_n^* \tag{3.1}$$

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Such that for any Star-like nildempotency semigroup, there exist a star-like operator $\phi^* = 2$, then by Star-like general 3 Dimensional Nildempotency equation and operator

$$F^* + V^* = E^* + 2 \tag{3.2}$$

$$\mid N_c\omega_n^* \mid = \mid \alpha_{i+1} - \alpha_i \mid \leq \mid \alpha_i - \omega_i \mid \tag{3.3}$$

we have that

$$\frac{1}{2}V^* = \mid \alpha\omega_i - \omega_{i+1} \mid \leq \mid \alpha_i\omega_i - \omega_i \mid \tag{3.4}$$

Since $N_c\omega_n^*$ satisfy (7), it shows that

$$\frac{1}{2}V^* = \frac{E^* - F^*}{2} + \phi^* \tag{3.5}$$

Thus; $\frac{1}{2}V^*(N_c\omega_n^*)$ implies $V^* = E^* - F^* + \phi^*$ which is the required Star-like Nildempotency vertices of any $\alpha_n^* \in N_c\omega_n^*$ transformation semigroup with a star-like Nildempotency disk constant point ϕ^*.

□

Example 1 Given $\alpha^* = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & \emptyset \end{pmatrix}$; such that $\alpha^* \in N_c\omega_n^*$ with star-like Nildempotency constant point $\phi^* \in \alpha^*$. Then the folded $\alpha^* \in N_c\omega_n^*$ is illustrated in figure 8

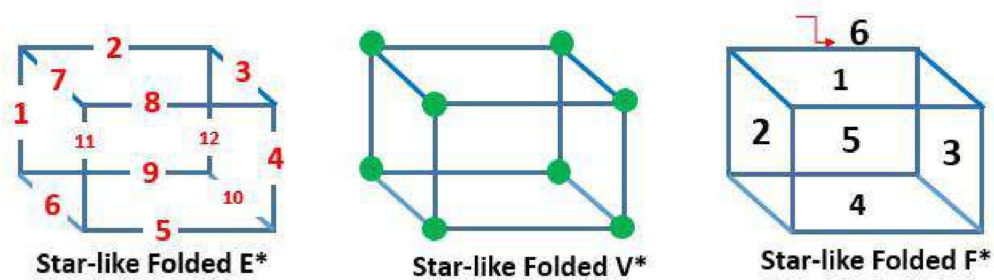

Figure 8: Star-like Nildempotency cube

obtain the star-like structure general Nildempotency structure of $\alpha^* \in N_c\omega_n^*$;
solution:
From equation (9) and (12) we can obtain $F^* + V^* = E^* + 2$ with a star-like Nildempotency constant $\phi \in \alpha^*$, Then by star-like folding principle, unfolding the figure(Star- like Nildempotency cubes) we have that $F^* = 6; V^* = 8$ implies $6 + 8 = E + 2$ implies $E^* = 12$

Given a star-like nildempotency transformation $N_c\omega_n^*$, Let R_n^* be a star-like polygon with n star-like vertices and n sides with star-like interior angles, $\alpha_i^*, ...\alpha_n^*$. Then

$$Area(R_n^*) = (\sum_{n=1}^{\infty} \alpha_n) - (n-2)\pi$$

Proof. Let C_n^* be a star-like convex polygon in R_n^*, this can be done rigorously by arranging C_n^* so that the origin lies in the interior of C_n^* and projecting the boundary of C_n^* on S^2 using the function

$$f(a,b,c) = \frac{(a,b,c)}{\sqrt{a^2+b^2+c^2}}$$

It's easy to check that vertices of ϕ_n^* go to part on S^2, edges go to part of great circles (ϕ_n^*) and faces go to polygon.

Let V^*, E^* and F^* denote the number of star-like vertices, edges, and faces of C_n^* respectively. Let $R_1^* + \cdots R_n^*$ be the polygon on S^2. Since their union is S^2

$$Area(R_1^*) + Area(R_2^*) + \cdots + Area(R_n^*) = Area(S^2)$$

Let n_i be the number of edges of R_i and $\alpha_{ij} for j = 1, \cdots n_i$ be the interior angles. Then by theorem 3.2, we have

$$\frac{1}{2}V^* = \frac{E^* - F^*}{2} + \phi^* \; where \; \phi^* = 2$$

implies

$$V^* = E^* - F^* + 4$$

since $n_i \in R_i$ and α_{ij} for $j = 1, \cdots n_i$ then

$$\sum_{i=1}^{n}(\sum_{j=1}^{n_i} \alpha_{ij} - n_i\pi + 2\pi) = 4\pi$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i} \alpha_{ij} - \sum_{i=1}^{n} n_i\pi + \sum_{i=1}^{n} 2\pi = 4\pi$$

Since every edges is shared by two star-like polygon

$$\sum_{i=1}^{n} n_i\pi = 2\pi E^*$$

Also, since the sum of angles at every vertex is 2π

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i} \alpha_{ij} = 2\pi V^*$$

Hence

$$2\pi V^* - 2\pi E^* + 2\pi F^* = 4\pi$$

which is equivalent to $V^* - E^* + F^* = 2$ □

4 Discussion and Conclusion

Given a star-like nildempotency transformation semigroup $N_c\omega_n^*$, Let R_n^* be a star-like polygon such that $R_n^* \in N_c\omega_n^*$ which satisfies the following:

i Any two star-like vertices of R_n^* can be connected by chain of star-like edges.

ii Any star-like loop on R_n^* which made up of star-like straight line segment then $V^* - E^* + F^* = 2$ for all $R_n^* \in N_c\omega_n^*$

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