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OPTIMIZATION OF EXPONENTIALLY WEIGHTED MOVING AVERAGE STATISTICS USING EMPIRICAL BAYESIAN WEIGHTING FACTOR

Adewara, J. Ademola[1] and Mbata, U. Ahamefula.[2]

[1] Distance Learning Institute, University of Lagos, Akoka, Lagos, Nigeria
[2] Departments of Mathematics, University of Lagos, Akoka, Lagos, Nigeria

* Corresponding author: jadewara@unilag.edu.ng, adewaraja@gmail.com

Abstract

Quality control managers are faced with the challenge of detecting out-of-control during production which may be assignable or common causes. To monitor and get quality products there is need for the use of Statistical Process Control (SPC). The study uses empirical Bayesian (EB) models for estimating Exponentially Weighted Moving Average (EWMA) statistic weighting factor. This is applied to data collected from a tyre producing company on the weight of radial car tyre of sizes 185, rim 14 Elite. A random sample of size 30 containing five subgroups was taken. The simulation was done using Markov chain Monte Carlo (MCMC) of 10000 samples. The study further obtained the values for λ (weighting factor) in the two EB models as $0.493 \leq \lambda \leq 0.506$ for beta-Bernoulli model while uniform-Bernoulli model is $0.494 \leq \lambda \leq 0.506$, which are useful for EWMA quality control charts. The results show that the uniform-Bernoulli model and the beta-Bernoulli model give almost identical results, which are reliable in plotting EWMA quality control charts as the classical approach.

1. Introduction

Quality control engineers are faced with the challenge of the detection of the shift during the production process. This shift is mostly due to assignable or common causes. To monitor and get quality products there is need for the use of Statistical Process Control (SPC). SPC is an applied statistical method that improves the quality of characteristics by monitoring the process under consideration, continuously, in order to detect assignable causes such that a required action is taken as quickly as possible [6]. A control chart consists of three horizontal lines: Upper Control Limit (UCL), Centre Line (CL) and Lower Control Limit (LCL). The centre line in a control chart denotes the average value of the quality characteristic under study. If a point lies between UCL and LCL, then the process is deemed to be under control. Otherwise, a point outside the control limits can be regarded as evidence that the process is out of control and, hence preventive or corrective actions are necessary in order to find and eliminate the assignable cause or causes [5]. The CUSUM chart helps in detecting such small permanent shifts that may go undetected when using the X-bar chart [14]. Therefore, it is important to detect the shift as soon as it occurs and to provide corrective actions in order to eliminate or minimize future occurrences of similar shifts. Various statistical techniques have been applied to detect shifts in the production process, such as, Shewhart charts for detecting moderate to large process shifts, while the Cumulative Sum (CUSUM) and the Exponentially Weighted Moving-Average (EWMA) charts are commonly used for small shift detection [1]. EWMA control charts were introduced by [2] followed by [8], and [4]. Since the weights decline geometrically when connected by the smooth curve, the EWMA is sometimes called the geometric moving average (GMA). The EWMA is used extensively in time series modeling and in forecasting [3]. Since the EWMA can be viewed as a weighted average of all past and current observations, it is very insensitive to the normality assumption. The choice of weighting factor λ, in the EWMA control procedure determines its sensitivity to a small or gradual drift in the process. Several authors have tried to estimate a value for the weighting factor λ or fixing the desired in-control Average Run Length (ARL). [12] Suggested that the value of λ will work well in the interval $0.05 \leq \lambda \leq 0.25$. On the other hand, [9] suggested that the weighting factor (λ) value should be between 0.03 and 1.0. However, despite advanced scholarly works in this area, the use of empirical Bayesian methods has not been employed. This paper focuses on estimating the weighting factor λ using beta-Bernoulli and uniform-Bernoulli empirical Bayesian (EB) models, the EWMA statistic and EWMA quality control chart.

2.0 Material and Method

2.1 Optimization of EWMA with Empirical Bayesian Weighting Factor

The exponentially weighted moving average (EWMA) is a statistic for monitoring the process that averages the data and gives it a less as weight of λ such that the smaller the value of λ, the further it is removed in time. The EWMA statistic as established by [15] is:

$$EWMA_t = \lambda Y_t + (1-\lambda)EWMA_0 \qquad for\ \ t = 1, 2, \ldots, n \tag{1}$$

while the estimated variance of the EWMA statistic is approximately:

$$\sigma^2 = \left(\tilde{\lambda} \big/ (2-\tilde{\lambda}) \right) \sigma^2 \tag{2}$$

where

- $EWMA_0$ is the mean of historical data
- σ^2 is the variance of historical data
- Y_t is the observation at time t
- n is the number of observations to be monitored including $EWMA_0$

- $0 < \lambda \leq t$ is a constant that determines the depth of memory

The parameter λ is the weighting factor (0<λ<1) which determines the rate at which "older" data enters into the estimation of the EWMA statistic. An empirical Bayesian (EB) procedure was introduced to estimate λ using prior information about the weighting factor λ. Two different estimates for λ were achieved through the use of beta-Bernoulli and uniform-Bernoulli models. In the first model, since the smoothing constant takes values between 0 and 1 we assume a Bernoulli data likelihood distribution where values between 0 and 0.49 are taken as 0, and values between 0.5 and 1 are taken as 1. This is taken in this manner because Bernoulli is a discrete distribution. Then a beta conjugate prior is assumed for the probability of success in Bernoulli data likelihood distribution. In the second model, Bernoulli data likelihood is again assumed, and a conjugate uniform prior distribution is assigned for the probability of success in the Bernoulli distribution. The product of the likelihood and the prior distribution gives the most important distribution; the posterior distribution. The posterior distributions are sampled to give a large amount of information about the parameter of interest (weighting factor λ). The commonest method for this is called Gibbs sampling. Thus, the Bayes theorem is given as:

$$P(\lambda|Y) = \frac{P(Y|\lambda)\,P(\lambda)}{\int_{\theta} P(Y|\lambda)\,P(\lambda)} \propto P(Y|\lambda)\,P(\lambda) \qquad (3)$$

2.2 Beta-Bernoulli Model

According to [7], a beta prior distribution can be assigned to the probability of success (which is the parameter of interest) of a Bernoulli random variable. The conjugacy is beta-Bernourlli model. This is adopted in this paper to make inference about the weighting factor λ for estimation of EWMA statistic. Therefore, let $P(Y_t|\lambda_t)$ be the probability of success that older data enters into the distribution for estimation of EWMA statistic, so;

$$(Y_t|\lambda_t) \sim Bern(\lambda_t)$$

$$(\lambda_t|\alpha,\beta) \sim Beta(\alpha,\beta)$$

Where Y_t is the observed data at time $t, for \;\; t = 1, 2, \dots, n$

The Bernoulli distribution is described as

$P(Y_t|\lambda_t) = \lambda^y(1-\lambda)^{1-y}, \quad 0 < \lambda_t < 1$

while the data likelihood function, which is the joint probability density function, is given as

$$l(Y_t|\lambda_t) = \prod_{i=1}^{n} \lambda^{y_i}(1-\lambda)^{1-y_i} = \lambda^{\sum_{i=1}^{n} y_i}(1-\lambda)^{\sum_{i=1}^{n}(1-y_i)} = \lambda^{Y_t}(1-\lambda)^{n-Y_t}$$

where $\sum y_i = Y_t$.

The prior distribution is beta distribution defined as

$$P(\lambda_t|\alpha,\beta) = \frac{1}{\mathrm{B}(\alpha,\beta)} \lambda^{\alpha-1}(1-\lambda)^{\beta-1} \qquad 0 \le \lambda \le 1$$

where α is the shape parameter and β is the inverse scale parameter.

The expectation and variance of the prior distribution are given as

$$E(\lambda) = \frac{\alpha}{\alpha+\beta} \ and\ V(\lambda) = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

Hence, the posterior distribution using Bayes theorem is derived as

$$P(\tilde{\lambda}_t|Y_t;\ \alpha,\beta) = \frac{l(\lambda_t|Y_t)P(\lambda_t|\alpha,\beta)}{\int l(\lambda_t|Y_t)P(\lambda_t|\alpha,\beta)} \propto l(\lambda_t|Y_t)P(\lambda_t|\alpha,\beta)$$

$$= (\lambda^{Y_t}(1-\lambda)^{n-Y_t})\left(\frac{1}{\mathrm{B}(\alpha,\beta)}\lambda^{\alpha-1}(1-\lambda)^{\beta-1}\right)$$

$$= \frac{1}{\mathrm{B}(\alpha,\beta)}\lambda^{\alpha+Y_t-1}(1-\lambda)^{\beta-Y_t+n-1}$$

where, $\tilde{\lambda}_t$ is the posterior estimates of the weighting factor.

This yields the beta-Bernoulli model. Since the posterior and the prior distributions are of the same functional form, we deduce that $\alpha^{/} - 1 = \alpha + Y_t - 1 \Rightarrow \alpha^{/} = \alpha + Y_t$ $\beta^{/} - 1 = \beta - Y_t + n - 1 \Rightarrow \beta^{/} = \beta - Y_t + n$

2.3 Estimation of the posterior mean and variance of beta–Bernoulli model:

$$E(\tilde{\lambda}_t|Y_t;\ \alpha,\beta) = \frac{\alpha^{/}}{\alpha^{/}+\beta^{/}} = \tilde{\lambda} = \frac{\alpha+Y_t}{\alpha+\beta+n}$$

$$V(\tilde{\lambda}_t|Y_t;\ \alpha,\beta) = \frac{\alpha^{/}\beta^{/}}{(\alpha^{/}+\beta^{/}+1)(\alpha^{/}+\beta^{/})^2} = \frac{(\alpha+Y_t)(\beta+n-Y_t)}{(\alpha+\beta+n+1)(\alpha+\beta+n)^2}$$

2.4 Uniform-Bernoulli Model

The use of Uniform-Bernoulli model in statistical application is not common in the literature. However, in this paper, the model is developed and used in the estimation of the weighting factor λ for the estimation of EWMA statistic. Therefore, let $P(Y_t|\lambda_t)$ be the probability of success that an older data enter into the estimation of EWMA statistic, so;

$$(Y_t|\lambda_t) \sim Bern(\lambda_i)$$

$$(\lambda_t|\alpha,\beta) \sim Unif(\alpha,\beta)$$

Where Y_t is the observed data at time , $for\ \ t - 1,2,\ldots,n$.

The Bernoulli distribution is described as

$P(Y_t|\lambda_t) = \lambda^y(1-\lambda)^{1-y}, \quad 0 < \lambda_i < 1$

While the data likelihood function is given as

$$l(Y_t|\lambda_t) = \prod_{i=1}^{n} \lambda^{y_i}(1-\lambda)^{1-y_i} = \lambda^{\sum_{i=1}^{n} y_i}(1-\lambda)^{\sum_{i=1}^{n}(1-y_i)} = \lambda^{Y_t}(1-\lambda)^{n-Y_t}$$

where $\sum y_i = Y_t$.

The prior distribution is uniform distribution defined as

$$P(\lambda_t|\alpha,\beta) = \frac{1}{\beta-\alpha}, \qquad \alpha < \lambda < \beta$$

The expectation and variance of the prior distribution is given as $E(\lambda) = \frac{\alpha+\beta}{2}$ $and\ V(\lambda) = \frac{(\beta-\alpha)^2}{12}$

where α and β are the lower and upper limits. Hence, the posterior distribution using Baye's theorem is derived as

$$P(\tilde{\lambda}_i|Y_t;\ \alpha,\beta) = \frac{l(\lambda_i|Y_t)P(\lambda_i|\alpha,\beta)}{\int L(\lambda_i|Y_t)P(\lambda_i|\alpha,\beta)} \propto l(\lambda_i|Y_t)P(\lambda_i|\alpha,\beta)$$

$$= (\lambda^{Y_t}(1-\lambda)^{n-Y_t})\left(\frac{1}{\beta-\alpha}\right)$$

$$= \frac{1}{\beta \quad \alpha}\lambda^{Y_t}(1-\lambda)^{n-Y_t}$$

This yields the beta-Bernoulli model.

Estimation of the posterior mean and variance of uniform–Bernoulli model using the k'th moment:

$$E(\tilde{\lambda})^k = \frac{1}{\beta-\alpha}\int_0^1 \lambda^{Y_t+k}(1-\lambda)^{n-Y_t}d\lambda$$

$$= \frac{1}{\beta-\alpha}\int_0^1 \lambda^{Y_t+k}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\lambda^j\,d\lambda$$

$$= \frac{1}{\beta-\alpha}\int_0^1 \sum_{j=0}^{\infty}\binom{n-Y_t}{j}\lambda^{Y_t+k+j}\,d\lambda$$

$$= \frac{1}{\beta-\alpha}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\int_0^1 \lambda^{Y_t+k+j}d\lambda$$

$$= \frac{1}{\beta-\alpha}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\left[\frac{\lambda^{Y_t+k+j+1}}{Y_t+k+j+1}\right]_0^1$$

$$= \frac{1}{\beta-\alpha}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\frac{1}{Y_t+k+j+1}$$

Therefore, **Mean**:

$$E(\tilde{\lambda}) = \frac{1}{\beta-\alpha}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\frac{1}{Y_t+j+2}$$

Variance:

$$E(\tilde{\lambda})^2 - [E(\tilde{\lambda})]^2$$

$$E(\tilde{\lambda})^2 = \frac{1}{\beta-\alpha}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\frac{1}{Y_t+j+3}$$

So, $V(\tilde{\lambda}) = \left[\frac{1}{\beta-\alpha}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\frac{1}{Y_t+j+3}\right] - \left[\frac{1}{\beta-\alpha}\sum_{j=0}^{\infty}\binom{n-Y_t}{j}\frac{1}{Y_t+j+2}\right]^2$

Hence, the modified EWMA statistic is now described, for each model, as

$$EWMA_t - \tilde{\lambda}Y_t + (1-\tilde{\lambda})EWMA_{i-t} \qquad for\ t = 1, 2, \ldots, n$$

While the estimated variance of the EWMA statistic is approximately:

$$\sigma^2 = \left(\tilde{\lambda}/(2-\tilde{\lambda})\right)\sigma^2$$

This means that after the EWMA control chart has been running for several time periods, the control limits will approach steady – state values.

$$UCL = \mu + l\sigma\sqrt{\lambda/(2-\lambda)}$$

$$CL = \mu$$

$$LCL = \mu - l\sigma\sqrt{\lambda/(2-\lambda)}$$

3.0 Data

The data for this work was collected from a tyre producing company on the weight of radial car tyre of sizes 185, rim 14 Elite. A random sample of 30 batches was taken, with each batch containing five observations. The workers were involved in shift work for morning, afternoon and night on eight hours per day. The tyres produced have the following qualities: size= 185/80/R14, dimension (overall diameter), load capacity (664mm), inflation pressure (2.8) and tolerance limit (10.30kg-11.03kg). The weights of tyres were taken at random and were recorded. The two EB models presented were applied to the radial car tyre data collected after a simulation study of 10000 MCMC samples of the weighted factor λ drawn from the posterior distributions using the Gibbs sampler in Open BUGS windows software application. The historical values were first determined. Then the empirical Bayesian posterior estimates of weighting factor from beta-Bernoulli and uniform-Bernoulli models are applied to each batch of five, to determine the observed values for the new estimates of EWMA statistic. The individual estimates are presented in Tables 2 and 3 while the new estimates EWMA, for the two models including the classical EWMA are presented in Table 1. Figure 1 indicates the kernel density and trace plots from the two proposed models. The kernel density plots showed uniformity, indicating that the samples were actually drawn from the required distributions and convergence is well established, as in the trace plots, since the sampling error or Markov chain error (MC error) is less than 5%, respectively.

4.0 Results and Discussion

Table 1 Presents the EWMA estimates from beta-Bernoulli and uniform-Bernoulli empirical Bayesian weighting factors. The results show that the uniform-Bernoulli model and the beta-Bernoulli model provide almost identical values of the weighting factor λ. The estimates of the weighting factor from the EB models were in turn used to estimate the EWMA statistic and the process chart drawn. The EWMA estimates from classical approach were also obtained and so was its process chart in Figure 3. The control chart plots in Figures 2 indicate that the EWMA charts of weighting factor from beta-Bernoulli and uniform-Bernoulli models are also reliable. The results for EWMA Chart of EB models in Figure 2 showed that there are some points more than 3.00 standard deviations from center line which are out of control as shown in points 1, 2, 3 while the results for classical EWMA Chart in Figure 3 showed that there are some points that are more than 3.00 standard deviations from center line that are out of control as shown in points 1, 2, 3, 5.

5.0. Conclusion

The use of empirical Bayesian methods in most statistical applications is becoming more important in statistical process control chart due to instability and errors associated with the estimates from classical approaches (Okafor and Mbata 2012). As a result of this, it is deemed fit to employ EB models to estimate EWMA statistics for process control charts in quality control studies. The proposed beta-Bernoulli and uniform-Bernoulli empirical Bayesian models were suitable in modifying the EWMA statistic weighting factor (λ), and subsequently, optimizing the EWMA estimates for the quality control chart. The range of estimated values of (λ) from the beta-Bernoulli model is obtained as 0.493 ≤ λ ≤ 0.506, while the uniform-Bernoulli model gives 0.494 ≤ λ ≤ 0.506. This results shows that the application of EB models gives a better improvement for both EWMA statistic estimation and the EWMA control chart.

Table 1: EWMA values and Optimized EWMA Values

Classical Method						*Beta-Bernoulli Model*					*Uniform-Bernoulli Model*				
Sample	*EWMA*	*SD*	*LCL*	*UCL*	*CV%*	*OptEWMA*	*SD*	*LCL*	*UCL*	*CV%*	*OptEWMA*	*SD*	*LCL*	*UCL*	*CV%*
1	10.56	0.519	10.44	10.68	4.91	10.68	0.784	9.90	11.47	7.34	10.68	0.790	9.89	11.47	7.40
2	10.61	0.108	10.58	10.64	1.02	10.71	0.789	9.92	11.49	7.37	10.71	0.784	9.92	11.49	7.32
3	10.68	0.335	10.60	10.76	3.14	10.74	0.790	9.95	11.53	7.35	10.74	0.788	9.95	11.53	7.34
4	10.85	0.061	10.84	10.86	0.56	10.83	0.786	10.04	11.61	7.26	10.83	0.794	10.03	11.62	7.34
5	10.65	0.328	10.57	10.73	3.08	10.73	0.787	9.94	11.51	7.34	10.73	0.784	9.94	11.51	7.30
6	10.79	0.102	10.77	10.81	0.95	10.80	0.792	10.00	11.59	7.34	10.80	0.786	10.01	11.58	7.28
7	10.81	0.082	10.79	10.83	0.76	10.81	0.791	10.02	11.60	7.32	10.81	0.792	10.01	11.60	7.33
8	10.82	0.097	10.80	10.84	0.90	10.81	0.786	10.02	11.60	7.27	10.81	0.782	10.03	11.59	7.24
9	10.89	0.042	10.88	10.90	0.39	10.85	0.790	10.06	11.64	7.29	10.85	0.788	10.06	11.63	7.27
10	10.92	0.045	10.91	10.93	0.41	10.86	0.791	10.07	11.65	7.28	10.86	0.790	10.07	11.65	7.28
11	10.80	0.112	10.77	10.83	1.04	10.80	0.782	10.02	11.58	7.24	10.80	0.786	10.02	11.59	7.27
12	10.85	0.117	10.82	10.88	1.08	10.83	0.786	10.04	11.61	7.26	10.83	0.784	10.04	11.61	7.24
13	10.81	0.042	10.80	10.82	0.39	10.81	0.792	10.01	11.60	7.33	10.81	0.792	10.01	11.60	7.33
14	10.78	0.057	10.77	10.79	0.53	10.79	0.790	10.00	11.58	7.32	10.79	0.787	10.00	11.58	7.29
15	10.79	0.042	10.78	10.80	0.39	10.80	0.789	10.01	11.59	7.31	10.80	0.787	10.01	11.58	7.29
16	10.80	0.061	10.79	10.81	0.56	10.80	0.788	10.01	11.59	7.29	10.80	0.787	10.01	11.59	7.28
17	10.82	0.057	10.81	10.83	0.53	10.81	0.785	10.03	11.60	7.26	10.81	0.784	10.03	11.59	7.25
18	10.82	0.057	10.81	10.83	0.53	10.81	0.794	10.02	11.61	7.35	10.81	0.789	10.02	11.60	7.30
19	10.89	0.065	10.87	10.91	0.60	10.85	0.783	10.06	11.63	7.22	10.85	0.785	10.06	11.63	7.23
20	10.77	0.091	10.75	10.79	0.84	10.79	0.789	10.00	11.57	7.31	10.79	0.783	10.00	11.57	7.26
21	10.89	0.065	10.87	10.91	0.60	10.85	0.780	10.06	11.63	7.20	10.85	0.787	10.06	11.63	7.25
22	10.89	0.065	10.87	10.91	0.60	10.85	0.787	10.06	11.63	7.25	10.85	0.785	10.06	11.63	7.24
23	10.90	0.071	10.88	10.92	0.65	10.85	0.791	10.06	11.64	7.29	10.85	0.787	10.06	11.64	7.25
24	10.82	0.120	10.79	10.85	1.11	10.81	0.787	10.02	11.60	7.28	10.81	0.786	10.03	11.60	7.27
25	10.85	0.127	10.82	10.88	1.17	10.83	0.786	10.04	11.61	7.26	10.83	0.790	10.04	11.62	7.30
26	10.92	0.027	10.91	10.93	0.25	10.86	0.791	10.07	11.65	7.28	10.86	0.790	10.07	11.65	7.27
27	10.72	0.104	10.70	10.74	0.97	10.76	0.788	9.97	11.55	7.32	10.76	0.785	9.98	11.55	7.29
28	10.86	0.074	10.84	10.88	0.68	10.83	0.790	10.04	11.62	7.29	10.83	0.786	10.04	11.62	7.26
29	10.63	0.091	10.61	10.65	0.86	10.72	0.791	9.92	11.51	7.38	10.72	0.786	9.93	11.50	7.33
30	10.87	0.091	10.85	10.89	0.84	10.84	0.784	10.05	11.62	7.23	10.84	0.789	10.05	11.62	7.28
Overall	***10.80***	***0.094***	***10.78***	***10.82***	***0.87***	***10.80***	***0.002***	***10.800***	***10.804***	***0.020***	***10.80***	***0.002***	***10.800***	***10.804***	***0.020***

Tolerance Level: (10.38 – 11.30)kg

Table 2: Posterior Estimates of Beta-Bernoulli Model by MCMC

	λ	sd	MC_error	val2.5pc	Median	val97.5pc	start	Sample
theta[1]	0.4965	0.2886	0.003212	0.02533	0.4968	0.9742	1	10000
theta[2]	0.5020	0.2893	0.002906	0.02448	0.5038	0.9742	1	10000
theta[3]	0.5021	0.2888	0.002815	0.02488	0.5025	0.9763	1	10000
theta[4]	0.4970	0.2865	0.002837	0.02723	0.4959	0.9732	1	10000
theta[5]	0.5006	0.2901	0.002622	0.02214	0.5014	0.9767	1	10000
theta[6]	0.5050	0.2898	0.002816	0.02495	0.5125	0.9768	1	10000
theta[7]	0.5022	0.2872	0.002714	0.02705	0.5060	0.9737	1	10000
theta[8]	0.4983	0.2879	0.002899	0.02452	0.4971	0.9764	1	10000
theta[9]	0.5019	0.2879	0.002961	0.02680	0.5000	0.9757	1	10000
theta[10]	0.5027	0.2877	0.002899	0.02295	0.5035	0.9778	1	10000
theta[11]	0.4949	0.2884	0.002891	0.02196	0.4931	0.9753	1	10000
theta[12]	0.4979	0.2876	0.002550	0.02812	0.4956	0.9754	1	10000
theta[13]	0.5046	0.2892	0.002697	0.02586	0.5007	0.9785	1	10000
theta[14]	0.5024	0.2896	0.002846	0.02409	0.5059	0.9756	1	10000
theta[15]	0.5007	0.2871	0.002797	0.02525	0.5004	0.9739	1	10000
theta[16]	0.4997	0.2884	0.002705	0.02276	0.5041	0.9773	1	10000
theta[17]	0.4965	0.2872	0.002891	0.02643	0.4916	0.9730	1	10000
theta[18]	0.5060	0.2880	0.002796	0.02802	0.5074	0.9739	1	10000
theta[19]	0.4948	0.2881	0.002930	0.02393	0.4955	0.9743	1	10000
theta[20]	0.5010	0.2883	0.002907	0.02574	0.5009	0.9758	1	10000
theta[21]	0.4930	0.2884	0.003082	0.02569	0.4917	0.9710	1	10000
theta[22]	0.4980	0.2871	0.002960	0.02698	0.4983	0.9722	1	10000
theta[23]	0.5023	0.2871	0.003173	0.02467	0.4985	0.9759	1	10000
theta[24]	0.4996	0.2883	0.002755	0.02261	0.5016	0.9743	1	10000
theta[25]	0.4967	0.2866	0.002679	0.02537	0.4948	0.9737	1	10000
theta[26]	0.5022	0.2873	0.003068	0.02599	0.5036	0.9750	1	10000
theta[27]	0.5006	0.2897	0.002703	0.02353	0.5031	0.9740	1	10000
theta[28]	0.5005	0.2867	0.002754	0.02749	0.4968	0.9754	1	10000
theta[29]	0.5015	0.2860	0.002933	0.02527	0.4999	0.9719	1	10000
theta[30]	0.4958	0.2883	0.002837	0.02637	0.4920	0.9756	1	10000

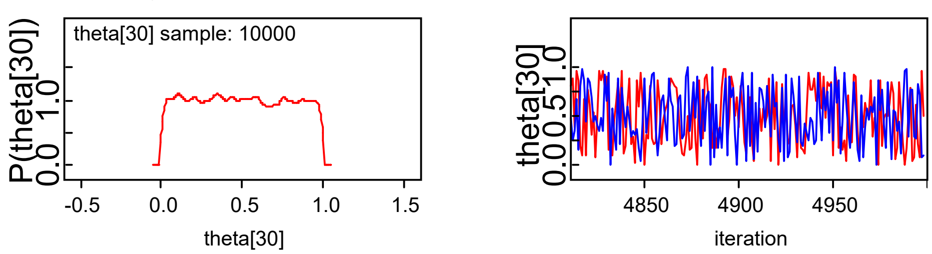
Minimum and Maximum value of λ: (0.493, 0.506)

Table 3: Posterior Estimates of Uniform-Bernoulli Model by MCMC

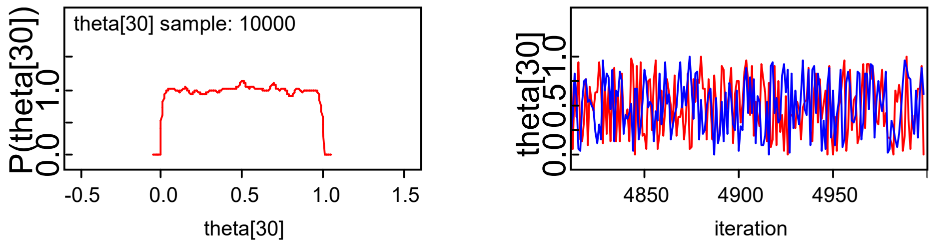
	λ	sd	MC_error	val2.5pc	median	val97.5pc	start	Sample
theta[1]	0.5026	0.2895	0.002723	0.02583	0.5057	0.9743	1	10000
theta[2]	0.4965	0.2887	0.002770	0.02208	0.4904	0.9751	1	10000
theta[3]	0.5007	0.2894	0.002702	0.02813	0.4966	0.9757	1	10000
theta[4]	0.5064	0.2888	0.003258	0.02658	0.5107	0.9764	1	10000
theta[5]	0.4958	0.2882	0.002716	0.02520	0.4953	0.9742	1	10000
theta[6]	0.4998	0.2903	0.002750	0.02662	0.4982	0.9784	1	10000
theta[7]	0.5038	0.2886	0.002954	0.02384	0.5056	0.9762	1	10000
theta[8]	0.4936	0.2864	0.002905	0.02592	0.4918	0.9740	1	10000
theta[9]	0.5007	0.2887	0.003013	0.02658	0.5014	0.9752	1	10000
theta[10]	0.5013	0.2868	0.003064	0.02622	0.5027	0.9759	1	10000
theta[11]	0.4986	0.2895	0.002608	0.02636	0.4961	0.9750	1	10000
theta[12]	0.4953	0.2872	0.002850	0.02384	0.4914	0.9744	1	10000
theta[13]	0.5026	0.2864	0.002927	0.02849	0.4992	0.9740	1	10000
theta[14]	0.4982	0.2875	0.002633	0.02684	0.4919	0.9756	1	10000
theta[15]	0.4983	0.2868	0.003003	0.02554	0.4983	0.9750	1	10000
theta[16]	0.4999	0.2898	0.002890	0.02442	0.4943	0.9771	1	10000
theta[17]	0.4966	0.2891	0.002778	0.02505	0.4910	0.9756	1	10000
theta[18]	0.5001	0.2870	0.002903	0.02637	0.4996	0.9750	1	10000
theta[19]	0.4979	0.2900	0.002789	0.02472	0.4995	0.9765	1	10000
theta[20]	0.4957	0.2888	0.002861	0.02553	0.4902	0.9747	1	10000
theta[21]	0.4998	0.2899	0.002710	0.02439	0.4978	0.9761	1	10000
theta[22]	0.4961	0.2861	0.003153	0.02502	0.4949	0.9729	1	10000
theta[23]	0.4994	0.2892	0.003049	0.02421	0.5008	0.9747	1	10000
theta[24]	0.4997	0.2912	0.002667	0.02427	0.5009	0.9757	1	10000
theta[25]	0.5021	0.2879	0.002557	0.02529	0.5073	0.9741	1	10000
theta[26]	0.5039	0.2915	0.002843	0.02673	0.5024	0.9768	1	10000
theta[27]	0.4982	0.2903	0.003073	0.02529	0.4997	0.9738	1	10000
theta[28]	0.4977	0.2870	0.002961	0.02675	0.4953	0.9724	1	10000
theta[29]	0.4978	0.2884	0.002621	0.02400	0.4960	0.9737	1	10000
theta[30]	0.5002	0.2872	0.002995	0.02378	0.5031	0.9742	1	10000

Minimum and Maximum value of λ: (0.494, 0.506)

Figure 1: Density and Trace plots of the EB models



Beta-Bernoulli Model



Uniform-Bernoulli Model

Figure 2: EWMA Charts of Beta- Bernoulli and Uniform-Bernoulli EB Models

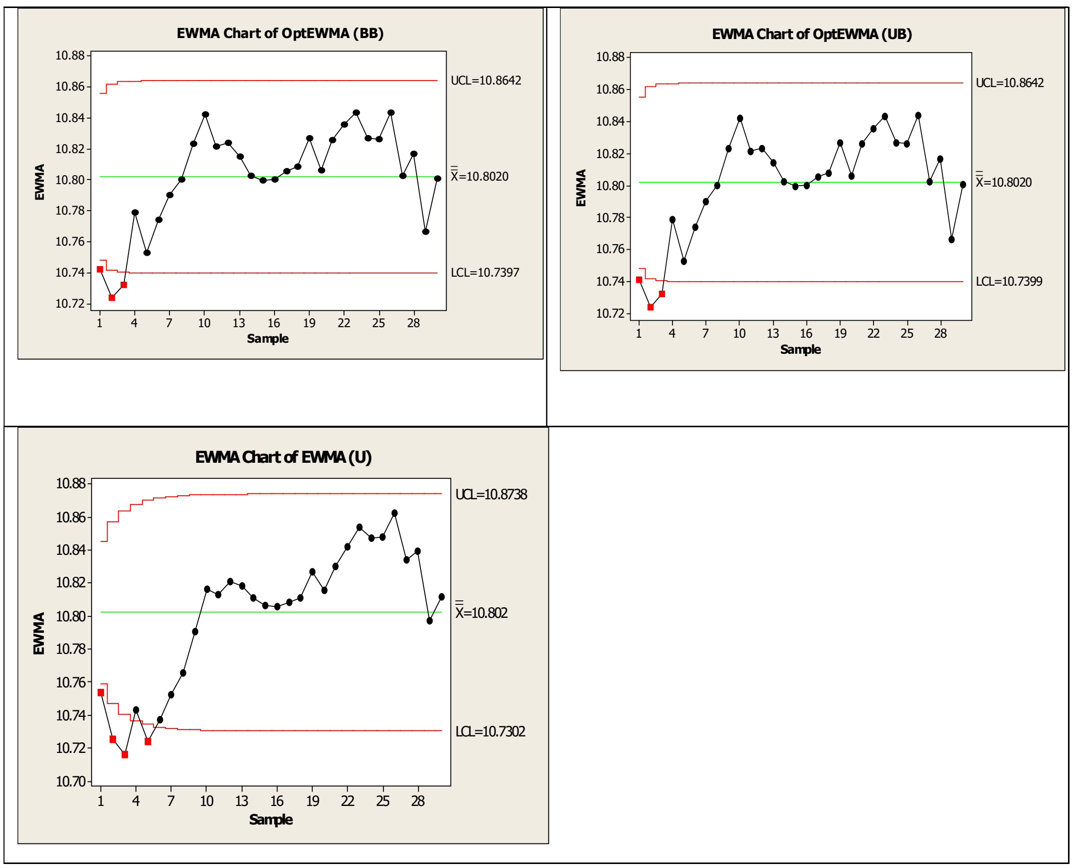


Figure 3: EWMA Charts of Classical EWMA

References

[1] Aerne, L. A., Champ, C. W. and Rigdon, S. E. (1991). Evaluation of control charts under linear trend. *Communications in Statistics-Theory and Methods,* 20, 3341-3349.

[2] Cowden, D. J. (1957). *Statistical Methods in Quality Control*. Englewood Cliffs, New Jersey: Prentice-Hall.

[3] Box, G.E., Jenkins, G.M., and Reinsel, G.C. (1994). *Time Series Analysis, Forecasting and Control*. 3rd Ed. Englewood Cliffs, New Jersey: Prentice-Hall.

[4] Crowder, S.V. and Hamilton, M. (1992). An EWMA for Monitoring a Process Standard Deviation. *Journal of Quality Technology*, 24 (1), 12-21.

[5] Duncan, A.J. (1986). *Quality control and Industrial statistics,5th edition,* Chicago, Illinois: Homewood.

[6] Fazeli, A.R. and Sharifi, E. (2011). Statistical Control and Investigation of Capability of Process and Machine in Wire Cut EDM Process of Gas Turbine Blade Airfoil Tip *Engineering*, 3, 260-265 doi:10.4236/eng.2011.33030

[7] Gershman, S. J. and Blei, D. M. (2012). A tutorial on Bayesian nonparametric models. *Journal of Mathematical Psychology,* 56 (1), 1-12.

[8] Hunter, J.S. (1986). The Exponentially Weighted Moving Average, *Journal of Quality Technology,* 18, 203-210.

[9] Lucas, J.M. and Saccucci, M.S. (1990). Exponentially weighted moving average control schemes: properties and enhancements, Technometrics, 32, 1-12.

[10] *Khalaf, S. and Rahim, M.A. (1997). Optimization in Quality Control - Page 280 - Google Books Result books.google.com.ng/books isbn 0792398890.*

[11] Montgomery, D.C. (2001). Introduction to Statistical Quality Control. (4th ed.). Canada: John Wiley & Sons, Inc.

[12] Montgomery, D.C. (2009). *Introduction to Statistical Quality Control, 6th Edition.* New York: John Wiley & Sons, Inc.

[13] Okafor, R.O. and Mbata, U.A. (2012). A Bayesian model for inference on population proportions, *WIREs Comput Stat*, 4, 482–488.

[14] *Page, E. S. (June 1954). "Continuous Inspection Scheme". Biometrika 41 (1/2): 100–115. doi:10.1093/biomet/41.1-2.100. JSTOR 2333009.*

[15] Roberts, S.V. (1959). *Control chart tests based on geometric moving averages. Technometrics,* 1, 239-250.