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HYDROMAGNETIC BOUNDARY-LAYER FLOW AND HEAT TRANSFER OVER A CONVECTIVELY HEATED PERMEABLE VERTICAL PLATE STRETCHING EXPONENTIALLY IN THERMALLY RADIATING POROUS MEDIUM

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Abstract

The aim of the present study is to investigate the combined effects of magnetic field and porous medium permeability on an hydromagnetic and thermally radiating Newtonian fluid past a permeable exponentially stretching vertical flat plate with convective heat exchange at the surface with the surroundings. Consideration is given to the influence of thermal and Ohmic dissipation, surface mass flux (injection or suction) as well as the velocity ratio of the mainstream with the upstream. The mainstream velocity is assumed to be stretched exponentially and the prescribed wall temperature is also of the exponential form. The governing system of nonlinear partial differential equations is reduced into another system of ordinary differential equations by means of extant transformation, and the resulting equations are then solved numerically by a shooting technique. The features of the flow and heat transfer characteristics within the boundary layer are examined, analyzed and discussed with the help of tables and Maple-codes generated plots. The results unveil that the combined magnetic and porous medium parameter impedes the fluid motion, radiation and Biot parameters enhance the fluid temperature, Grashof number and the velocity ratio accelerate the fluid motion amongst others. The results compare reasonably well with previous studies in the literature.

Keywords: Exponentially stretching, vertical plate, convective heat transfer, magnetic field, porous medium

1. Introduction

In the past few decades, considerable effort has been put increasingly in the study of steady or non-steady flows of a viscous and incompressible fluid engendered by moving solid surfaces in an otherwise still or moving free stream. These types of flows are frequently encountered in many an industrial process such as: cooling of ductile metallic and non-metallic sheets in a cooling bath, aerodynamic extrusion of polymer sheets and nonwoven fabrics extruded continuously from a dye and heat-treated materials transported between feed and wind-up rolls. During the manufacturing processes the extrudates are subsequently stretched into sheets and thermally treated accordingly as to achieve the desired material textural property and thickness. Nonetheless, the stretched surface velocity and wall temperature play a significant role in the cooling process (see Karwe and Jaluria [1, 2]). Sadiakis [3a,b,c] investigated for the first time, the boundary layer flow of viscous incompressible fluid past inextensible continuous surface and cylinder with uniform velocity. Crane [4] initiated boundary layer flow and heat transfer over a sheet linearly stretching with a velocity varying with the axial distance. Kuiken [5] in his study of influence of compression work on flow and free convection considered both wall velocity and temperature to vary linearly with the distance parallel to gravity acceleration. Carragher and Crane [6] extended the earlier work of Crane [4] by considering the case for which the wall temperature varies as a power of the axial distance. The steady viscous boundary layer flow with quadratically stretching sheet was examined analytically by Kumaran et al. [7], while the MHD aspect of it has been investigated by Cortell [8], and more recently Nazar et al. [9] have presented stretching/shrinking model. Weidman and Magyari [10] presented generalized Crane flow due to a general polynomial stretching velocity of an arbitrary order associated with heat transfer convection.

An intriguing extension of the boundary layer flow and heat transfer over continuous stretching surfaces is that which incorporates power exponent as per wall velocity and surface temperature. Several authors have examined various aspects of such flow and heat transfer in the boundary layer regime. Ali [11, 12] investigated analytically flow and heat transfer characteristics for power law variation of axial distance in the wall velocity and temperature.

A lot of studies have been carried out for various stretching situations as afore-mentioned. However, sparse work has been done with yet another pertinent interesting continuous solid surface stretching (different but comparable with axial power-law model) triggered by exponential stretching of surface velocity and temperature distribution, which is frequently encountered in the engineering and manufacturing processes involving high wall speeds. Using a similarity transformation corresponding to power law stretching in wall velocity and temperature distribution, Magyari and Keller [13] investigated discerningly in their pioneering work, heat and mass transfer characteristics in the boundary layers on an exponentially stretching continuous surface for steady Newtonian fluid. Thereafter, many authors inclusive of (Elbashbeshy [14], Partha et al.[15], Al-Odat et al.[16], Sajid and Hayat [17], Bidin and Nazar [18], Pal [19], Ishak [20], Sreenivasulu and Reddy [21], Reddy and Reddy [22], Kameswaran et al. [23], Mukhopadhyay [24], Jat and Chand [25], Yadav and Sharma [26], Sreenivasulu et al. [27] have tackled the problem under various physical circumstances and boundary conditions. Recent works on non-Newtonian models for exponentially stretching refer to Animasaun et al. [28] who studied numerically the influence of variable fluid properties in the boundary layers of a Casson fluid and Hayat et al. [29] for thermally radiating Eyring-Powell fluid with convective heat and mass transfer boundary conditions, amongst others. The boundary layer flow over an exponentially stretching cylindrical surface has not been proved theoretically to admit a self-similar solution. Sulochana and Sandeep [30] erroneously presumed a similarity solution for such a flow in whose transformation similarity variables fail outright to uphold the continuity equation.

A thorough survey of the literature review indicates that the present study extends the studies of all related reported works in archives and in particular those of Partha et al. [15], Jat and Chand [25], Mukhopadhyay [24], Chaudhary et al. [31] and, Adeniyan and Adigun [in press] (see Table 1).

Convective heat exchange at the solid surface plays a significant role in the mechanism of heat and momentum transfer. These special boundary conditions which differ from the traditional prescribed surface temperature (PST) and surface heat flux (PHF), have appeared in pioneering studies of many researchers including Aziz [33], Makinde and Aziz [34], Adeniyan and Adigun [35].

Apart from the convective heat transfer in fluids, thermal radiation proffers considerable effect on more modern engineering innovations and advanced technology such as in geophysical and astrophysical flows, aircraft and spacecraft devices, high temperature industrial processes and many others. Thusly, many researchers in the recent past have considered the inclusion of thermal radiation in their studies. The influence of thermal radiation on heat and mass transfer of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field has been reported in Makinde and Ogulu [36]. The problem of heat and mass transfer boundary layer flow of an unsteady free convection and mass transfer over a moving vertical plate with thermal radiation has been considered by Makinde [37]. Nonetheless, Pal [19] has carried out a study for a two-dimensional MHD stagnation-point flow and heat transfer past a stretching vertical plate taking into cognizance the effects of viscous dissipation, heat generation /absorption and buoyant forces. His similarity transformations are different from those in this study and Ref. [15].

In particular, Table 1 below unveils most recent relevant studies for various hydrodynamic and thermodynamic exponentially stretching models in comparison with the present work, which consequently adduces its (i. e. present work) originality. In this table, ticked and crossed signify respectively the considered and not-considered physical parameters.

Table1: Literature on incompressible hydrodynamic and thermodynamic models for exponentially stretching surface

Authors	Ha	Da	Bi	Fw	Ec/Gb	Rd	S	Ec.Ha	Gr	ε
Partha [15]	×	×	×	×	√	×	×	×	√	×
Reddy & Reddy [22]	√	×	×	×	√	√	×	×	×	×
Kameswaran et al. [23]	√	×	×	×	√	√	×	√	×	×
Jat & Chand [25]	√	×	×	×	√	√	×	√	×	×
Mukhopadhyay [24]	√	×	×	√	×	√	×	×	×	×
Chaudhary et al. [31]	√	×	×	×	√	√	×	×	×	×
Adeniyan & Adigun [in press]	√	√	×	√	√	√	√	√	√	×
Present study	√	√	√	√	√	√	√	√	√	√

The object of this study is to extend the investigations of the referred articles: Partha et al. [15], Reddy and Reddy [22], Kameswaran et al. [23], Jat and Chand [25], Mukhopadhyay [24], Chaudhary et al. [31] and, Adeniyan and Adigun [in press] by incorporating the joint effects of viscous dissipation (Ec or Gb), Ohmic dissipation (Ec.Ha), internal heat generation (S), velocity ratio (ε) and wall mass flux (Fw) on mixed convection (Gr) flow in the boundary layer of convectively heated (Bi) permeable exponentially stretching plate with combined porous medium and magnetic parameter (R=Da+Ha).

2. Model formulation

Consider steady two dimensional hydromagnetic convective flow of a laminar viscous and incompressible electrically conducting fluid along a semi-infinite permeable vertical flat plate stretching upstream in the direction of the x-axis against gravity vector **g,** awash in a saturated porous medium, with an exponential velocity $U_w(x)=U_0e^{\frac{x}{\ell}}$ in an exponentially stretched main stream with velocity

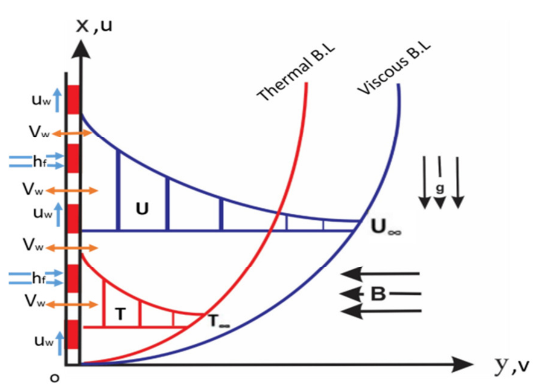

Fig. 1: Model schematics and coordinate system

$U_\infty(x)=U_ee^{\frac{x}{\ell}}$ as depicted in Fig. 1. Here, $U_0>0$ and $U_e\ge 0$ are constants such that quiescent fluid is ascribable to $U_e=0$ and $\ell>0$ is a length scale. It is presumed that the applied magnetic field $B(x)$, an exponential function permeates the plate transversely along a direction parallel to y-axis, and that the wall temperature T_w is isothermal while the sheet is subjected to a convective heat exchange with uniform temperature $T_f\ (>T_w)$, offering heat transfer coefficient h_f. Both the fluid and saturated porous medium are in thermal equilibrium. Furthermore, it is assumed that the induced magnetic field due to fluid motion, electric and magnetic polarizations are infinitesimally small such that the electric field and magnetic Reynolds number are non-applicable in the present scenario. In the context of boundary layer and Boussinesq approximations within the framework of above-stated delimitations, the conservation equations (Bataller [38], Aman and Ishak [39], Seini and Makinde [40]), modified for hydromagnetic mixed convection, exponentially stretching plate in porous media with heat generation are cogently posited as :

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=U_e\frac{dU_e}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\left[\frac{\sigma B^2(x)}{\rho}+\frac{\nu}{K_P(x)}\right](u-U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{\alpha}{k}\frac{\partial q_r}{\partial y}+\frac{\alpha\mu}{k}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\alpha Q(x)}{k}(T-T_\infty)+\frac{\alpha B^2(x)}{k}(u-U_\infty)^2 \tag{3}$$

where u and υ are the components of the velocity in x and y directions respectively, ν is the coefficient of kinematic viscosity, μ is the constant dynamic viscosity, T is the fluid temperature, β is the constant thermal expansion coefficient, σ is the electrical conductivity, g is the acceleration due to gravity, ρ is the fluid density, C_p is the specific heat at constant pressure, k is the constant thermal conductivity, q_r is the radiation heat flux, $\alpha=\dfrac{k}{\rho C_p}$ is thermal diffusivity, Q(x) and $K_P(x)$ are exponentially varying volumetric heat generated and porous medium permeability respectively. The boundary conditions without mass concentration are (Haq et al. [41], Hayat et al. [29]):

$$\left.\begin{aligned} y=0:\quad & u=U_w=U_o e^{\frac{x}{\ell}},\ \mathrm{v}(x,0)=V_w(x), -k\frac{\partial T}{\partial y}(x,0)=h_f\left(T_f-T_w\right) \\ y\to\infty:\quad & u\to U_e e^{\frac{x}{\ell}}, T\to T_\infty \end{aligned}\right\} \tag{4}$$

$V_w(x)$ is surface mass flux (i.e. dimensional suction or injection). It may be noted here that $V_w < 0$ signifies suction and $V_w > 0$ signifies injection. The stretching plate wall temperature T_w (to be determined later) is assumed to be hotter than the ambient fluid temperature T_∞. The radiation heat flux for an optically and non-scattering thick layer is posited by Rosseland approximation as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}. \tag{5}$$

where σ^* and k^* are Stefan-Boltzmann constant and mean absorption coefficient respectively. Following Bidin and Nazar [18], we assume that the temperature difference in the boundary layers is sufficiently small so that T^4 can be expressed as a linear function of temperature, after expressing its Taylor series expansion about the free stream temperature T_∞, neglecting higher-order terms. In consequence

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

As to guarantee the similarity solutions, it is presumed that the magnetic field, porous medium permeability, volumetric heat generated, surface mass flux and wall heat transfer coefficient are of the following form:

$$B(x) = B_0 e^{\frac{x}{\ell}},\ K_p(x) = K_0 e^{\frac{x}{\ell}},\ Q(x) = Q_0 e^{\frac{x}{\ell}},\quad V_w(x) = V_0 e^{\frac{x}{\ell}},\ h_f = h_0 e^{\frac{5x}{2\ell}} \tag{7}$$

such that B_0, Q_0, V_0, h_0 are constants.

Introduce now the following similarity variables (Partha et al., 2005):

$$\eta = y\sqrt{\frac{U_w}{2\nu\ell}},\ \psi(x,y) = \sqrt{2\nu\ell U_w}\, f(\eta),\ T - T_\infty = (T_f - T_\infty)\exp\left(\frac{2x}{\ell}\right)\theta(\eta) \tag{8}$$

Where $\psi(x, y)$ is the streamfunction, defined in the usual form:

$$u = \frac{\partial \psi}{\partial y},\quad \upsilon = -\frac{\partial \psi}{\partial x} \tag{9}$$

upon which eq. (1) is upheld. By means of (8), the axial and transverse velocity components:

$$u = U_w f', \quad v = -\sqrt{\frac{\nu U_w}{2\ell}}\left(\eta f' + f\right), \tag{10}$$

where primes signify differentiation with respect to dimensionless variable η , and f is the non-dimensional Stokes stream function. Substituting (8) and (10) into equations. (2) - (4) yields:

$$f''' + ff'' - 2f'^2 + Gr\theta - R\left(f' - \varepsilon\right) + 2\varepsilon^2 = 0 \tag{11}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}Rd\right)\theta'' + f\theta' - 4\theta f' + Ecf''^2 + S\theta + EcR\left(f'-\varepsilon\right)^2 = 0 \tag{12}$$

$$f(0) = F_w,\ f'(0) = 1,\ \theta'(0) = -Bi\left[1-\theta(0)\right],\ f'(\infty) = \varepsilon,\ \theta(\infty) = 0 \tag{13}$$

In the above set of equations. (11) - (13),

$$\left.\begin{aligned} &Pr=\frac{\mu C_p}{k},\ F_w=-V_0\sqrt{\frac{2\ell}{\nu U_0}},\quad Bi=\frac{h_0}{k}\sqrt{\frac{2\nu\ell}{U_0}}, M=\frac{2\sigma B_0^2\ell}{\rho U_o},\quad Gr=\frac{2g\beta\left(T_f-T_\infty\right)\ell}{U_0^2},\\ &S=\frac{2Q\ell}{\rho C_p U_0}, Rd=\frac{4\sigma^* T_\infty^3}{kk^*},\quad Ec=\frac{U_o^2}{C_p\left(T_f-T_\infty\right)},\quad Da=\frac{2\gamma\ell}{K_p U_o}, \varepsilon=\frac{U_\infty}{U_0}, \end{aligned}\right\} \tag{14}$$

are respectively Prandtl number, wall mass flux ($Fw > 0$ for suction, $Fw < 0$ for injection), Biot number, magnetic parameter, Grashof number, heat generation parameter, radiation parameter, Eckert number, porous medium permeability parameter and velocity ratio. Additionally, the combined magnetic and porous medium permeability parameter, $R = M + Da$.

Surface shear stress and heat transfer rate

The primary physical quantities of engineering interest are the local surface shear stress or local skin-friction coefficient $\left(Cf_x\right)$ and local heat transfer rate at the surface or local Nusselt number $\left(Nu_x\right)$. These may be defined as follows (Makinde [37]):

$$Cf_x = \frac{2\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{xq_w}{k\left(T_f - T_\infty\right)} \tag{15}$$

where

$$\tau_w = \mu \frac{\partial u}{\partial y}(x,0), \quad q_w = -k\left(1+\frac{4}{3}Rd\right)\frac{\partial T}{\partial y}(x,0) \tag{16}$$

are respectively the wall shear stress and surface heat flux. Substituting equations. (8) and (10) into equations. (15) - (16), the result, after simplification yields:

$$Cfr = Cf_x\left(\frac{\mathrm{Re}}{2}\right)^{\frac{1}{2}} = f''(0), \quad Nur = Nu_x\left(\frac{2X\,\mathrm{Re}_\ell^4}{\mathrm{Re}_x^5}\right)^{\frac{1}{2}} = -\left(1+\frac{4}{3}Ra\right)\theta'(0) \tag{17}$$

where Cfr and Nur may be referred to as the reduced skin-friction and reduced Nusselt number respectively (Ferdows et al. [42]). The wall stretching velocity based local Reynolds number $\mathrm{Re}_x = \frac{xU_w}{\nu}$, while $\mathrm{Re}_\ell = \frac{\ell U_w}{\nu}, \mathrm{Re} = \frac{\ell U_0}{\nu}$, $X = \frac{x}{\ell}$. It is worth mentioning that the sudden appearance of variable x in (17) would have destroyed the local similarity structure of the problem. Fortunately, the ratio as expressed by X saves this because it is a dimensionless variable, which may be christened the local position parameter (Ref. [19]).

3. Numerical simulation

The set of coupled nonlinear Equations. (11)-(13) have been solved numerically by means of Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta sixth-order integration method. Full discussion on this technique may be found in Nachtsheim and Swigert [43]. The computations have been performed using Maple software. A step size of $\Delta\eta = 0.001$ is selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of asymptotic transverse distance is found to each iteration loop by the assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of parameters*, Pr*, *Rd*, R, *Gr, S, Ec, Fw, B, and* ε is determined when the values of unknown boundary conditions at $\eta = 0$ do not change to successful loop with error less than 10^{-7}.

4. Results and discussion

From the process of numerical computation, the reduced local skin-friction coefficient and reduced Nusselt number which are respectively proportional to $f''(0)$ and $\left[-(1+4Rd/3)\theta'(0)\right]$ are discerningly sorted out and their numerical values are presented in a tabular form for a wide range of parameter values as indicated.

To validate the accuracy of the present numerical method, the values of the reduced skin friction coefficient are compared with Partha et al. [15], which can be observed in Table 2 while Table 3 delineates the values of the reduced Nuselt number as compared with Mukhopadhyay [24]. The comparisons are found to be in excellent agreement as percentage computed errors are small. Thusly, we are confident that the present numerical method is accurate.

The plots of the dimensionless velocity and temperature distribution against the non-dimensional transverse distance have been presented graphically in Figs. 2-12. Both the velocity and temperature profiles, starting from the prescribed wall values nosedive tacitly to attain their asymptotic values at $\eta_\infty = 10$ in nearly all cases. It is interesting to point out that in Table 4, increase in the radiation parameter (Rd) leads to value decimation of $[-\theta'(0)]$ but the reverse is demonstrated for the reduced Nusselt number (Nur) as well as the wall temperature $[\theta(0)]$ in the presence of surface convective heating. Increase in the velocity ratio between the wall velocity and the mainstream velocity (ε) leads to a rise in Nur and a reduction in the absolute value of skin-friction coefficient $f''(0)$ and wall temperature $\theta(0)$.

It is observed in all cases that the skin friction coefficient is negative. Of course, the negative value of the skin-friction physically means that the dragging force is being exerted on the fluid as opposed to the usual occurrence of dragging force being exerted on the wall by the fluid. The value enhancement of convective heat parameter (Bi) is tantamount to a decrease in $|f''(0)|$ and an increase in Nur while the wall temperature rises.

Table 4 again unveils apparently that increase in combined magnetic and porous medium permeability parameter (R) as well as the suction parameter $(Fw > 0)$ proffers increase in both Nur and $|f''(0)|$ while the opposite response is signposted due to value enhancement of the Grashof number (Gr) and the injection parameter $(Fw < 0)$. Increase in the Prandtl number leads to a decrease in both $\theta(0)$ and Nur but an increase in $|f''(0)|$. The value enhancement in both the Eckert number and the heat generation parameter causes a rise in wall temperature and a decrease in both Nur and $|f''(0)|$.

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Table 2: Comparison with Partha et al. [15] when Ec=0.5 , Pr=5, $Ra = R = Fw = S = \varepsilon = 0$ and $Bi \to \infty$

	Partha et al. [15]		Present Study			
Gr	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	% Error in $f''(0)$	% Error in $[-\theta'(0)]$
-0.1	-1.2989	4.3041	-1.302	4.14542	0.238663	3.686701
-0.3	-1.3333	4.2999	-1.3377	4.09724	0.330008	4.713233
-1	-1.4552	4.2622	-1.4679	3.89853	0.872732	8.532448

Table 3: Comparison with Mukhopadhyay [24] when $Fw = S = Ec = \varepsilon = 0$ and $Bi \to \infty$, in the absence of velocity and temperature slips

			Mukhopadhyay [24]	Present Study	
Pr	Rd	R	$-\theta'(0)$	$-\theta'(0)$	% Error
1	0	0	0.9547	0.954800424996324	0.010519
		1	0.8629	0.854800424996324	0.938645
	0.5	0	0.6765	0.677298032670007	0.117964
	1		0.5311	0.534555627097629	0.345562
2			1.4714	1.471456372641580	0.003831
	0.5	0	1.0734	1.073516745728020	0.010876
	1		0.8626	0.862850143181030	0.028998
3			1.8691	1.869070628670530	0.001571
	0.5		1.3807	1.380747619654330	0.003448

Specifically, the velocity plots as delineated in Fig. 2 for different values of R in the range of values indicated afoot of the legends reveal that the velocity profiles can be impeded due to magnitude strengthening of R near the plate. In Fig. 3, however, the velocity plots show that the fluid velocity is accelerated in consequence to increasing the values of Gr within the specified range of values. The plots in Fig. 4 reveal increasingly that the velocity profiles are strengthened due to increase in the velocity ratio between the wall and the mainstream.

The influence of strengthening the surface mass flux (i.e. suction/injection) on the velocity and temperature distributions in the presence of convective heat exhange at the wall is delineated in Figs. 5 and 12. Both the velocity and temperature depreciate in magnitudes as the suction parameter increases with observable opposite responses when the injection parameter intesifies. Thusly, it can be inferred that suction parameter can serve as a means of delaying both viscous and thermal boundary layer separations in the processes involving a convective heat exchange transfer at the wall.

The influence of the Prandtl number on the fluid temperature within the boundary layer is unravelled with the $\theta-\eta$ plots as demonstrated in Fig. 9. As unveiled, the fluid temperature falls considerable due to intensification of Pr. The Prandtl number can be increased in either of two ways by increasing the viscosity fluid property while the thermal diffusion property is fixed or by fixing the viscosity while the thermal diffusivity is subject to a considerable reduction asgiven in the emperical relation $\left(\mathrm{Pr}=\frac{\nu}{\alpha}\right)$. Liquid metal such as molten cupper or brass possesses low Pr while oil such as glycerol glycol or heavy duty oil is associated with high Prandtle number. Fluid with low Pr, in practice is associated with higher thermal conductivity as confirmed by the emperical relation. Prandtl number can therefore be used in convective heat exhange processes requiring quicker rate of cooling (Ref. [24]).

The plots of the temperature distribution as displayed in Figs. 6-8 and 10-11 show that fluid temperature in the boundary layer rises due to paramer enhacement of Ra, Bi, R, S and Ec in the presence of surface convective heat exchange.

Table 4: Computations showing varying values of the Skin-friction coefficient, wall temperature and Nusselt number for various values of basic flow parameters

R	Gr	ε	Pr	Ra	Ec	Bi	S	F_w	$f''(0)$	$\theta(0)$	Nur
0.1	0.5	0.15	0.72	0.1	0.5	0.1	0.2	0.1	-1.28366901	0.15790047	0.08421
2.0									-1.72070925	0.27093766	0.082627
10.0									-2.92610446	0.61575985	0.043547
0.1	0.5								-1.28366901	0.15790047	0.095438
	1.0								-1.24925912	0.15305731	0.095987
	2.0	0.15							-1.03603842	0.12838284	0.098783
	0.5	0.15							-1.28366901	0.15790047	0.095438
		0.20							-1.26353127	0.15205584	0.0961
		0.50							-1.07580468	0.11639299	0.100142
		0.15	0.72						-1.28366901	0.15790047	0.095438
			2.00						-1.29078579	0.15480262	0.095789
			7.10						-1.29384089	0.16796554	0.094297
			0.72	0.5					-1.27891006	0.16488462	0.094646
				1.0					-1.27346728	0.17440630	0.137599
				2.0					-1.26397641	0.19286273	0.188332
				0.1	0.5				-1.28366901	0.15790047	0.30877
					1.0				-1.26134870	0.24139616	0.085975
					2.0				-1.22102783	0.39385980	0.068696
					0.5	0.1			-1.28366901	0.15790047	0.095438
						0.5			-1.25039476	0.33386326	0.377477
						2.0			-1.19663820	0.62488071	0.85027
						0.1	0.2		-1.28366901	0.15790047	0.095438
							0.5		-1.27804713	0.17144957	0.093902
							1.0		-1.25540845	0.21442266	0.089032
							0.2	-2.0	-0.67638546	0.13790639	0.097704
								-1.0	-0.89907667	0.14246643	0.097187
								0	-1.24124708	0.15616795	0.095634
								0.5	-1.46927650	0.16504521	0.094628
								2.0	-2.36850628	0.18704409	0.092135

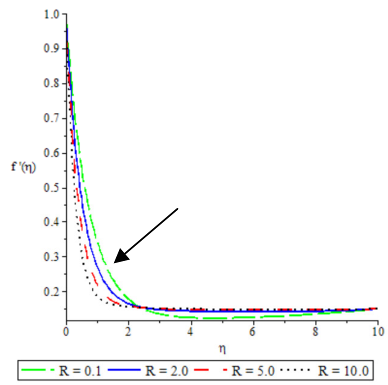

Fig 2: Variation of horizontal velocity f' with η for several values of R

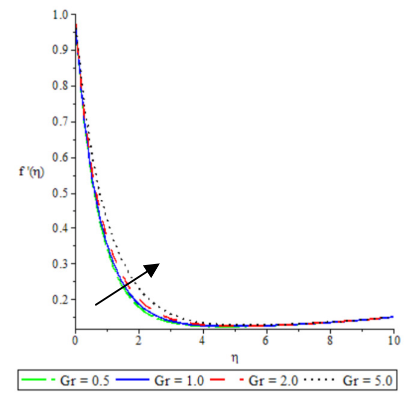

Fig 3: Variation of horizontal velocity f' with η for several values of Grashof number Gr

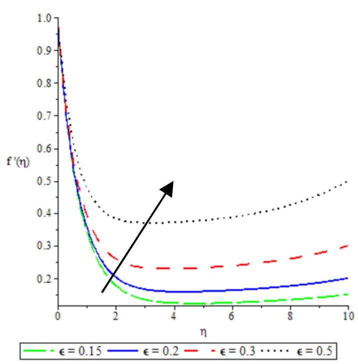

Fig 4: Variation of horizontal velocity f' with η for several values of velocity ratio ε

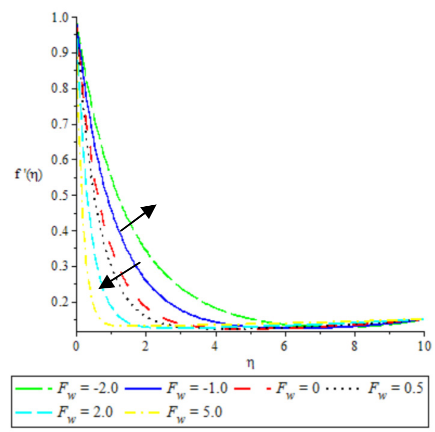

Fig 5: Variation of horizontal velocity f' with η for several values of suction/injection F_w

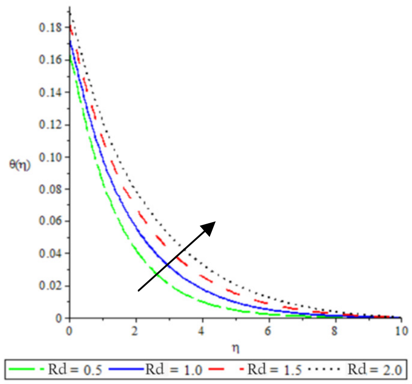

Fig 6: Variation of temperature θ with η for several values of radiation Parameter Rd

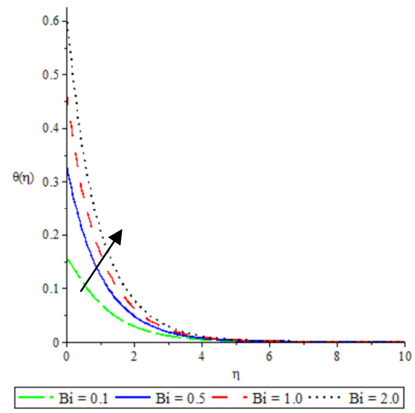

Fig 7: Variation of temperature θ with η for several values of Biot number Bi

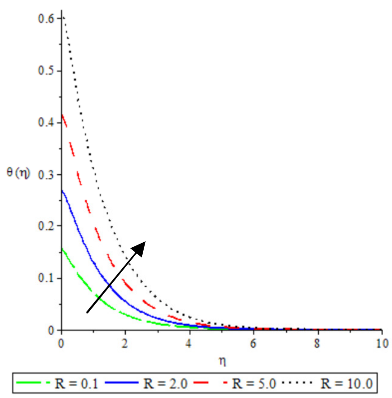

Fig 8: Variation of temperature θ with η for several values of R

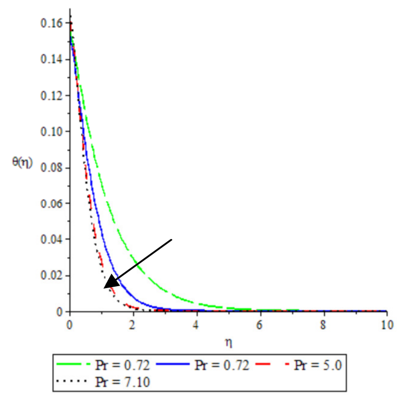

Fig 9: Variation of temperature θ with η for several values of Prandtl number Pr

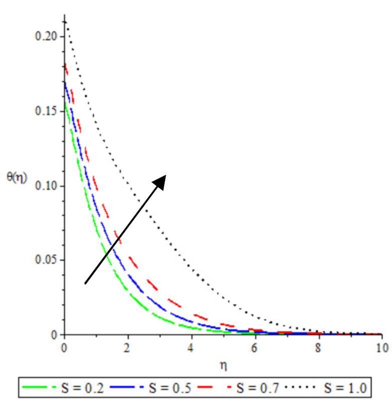

Fig 10: Variation of temperature θ with η for several values of internal heat generation S

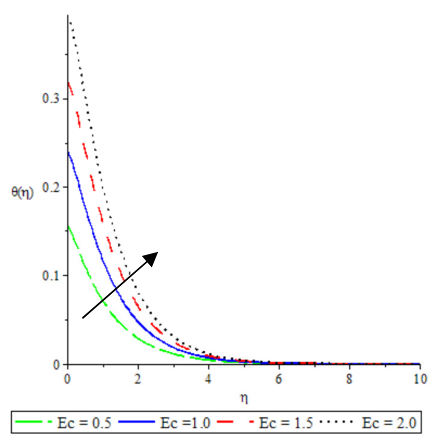

Fig 11: Variation of temperature θ with η for several values of Eckert number Ec

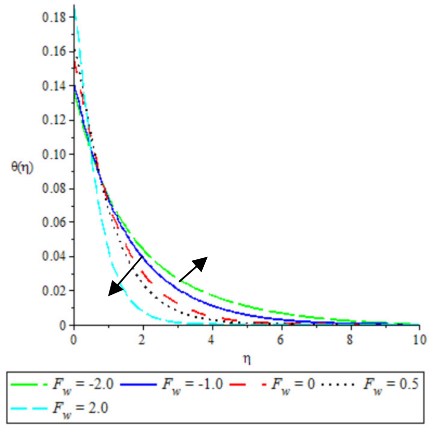

Fig 12: Variation of temperature θ with η for several values of suction/injection F_w

5. Conclusions

The present work investigates numerically the joint effects of viscous and Ohmic dissipation, combined variable magnetic and porous medium parameter on a convectively heated vertical plate for a steady two-dimensional boundary-layer flow. It is assumed that the fluid is thermally radiating, electrically conducting and that the permeable plate stretches exponentially in a mainstream stretched exponentially. The major findings of this study are itemized as follows:

- The surface shear stress parameter and plate temperature are higher in values due to suction as compared with those due to injection.
- The combined magnetic and porous medium parameter impedes the fluid motion and in consequence raises the plate temperature.
- The skin-friction coefficient is significantly decimated whereas, not only the plate temperature but also the entirety of the fluid temperature rises as the convective heat exchange parameter (Bi) intensifies.
- The electrically conducting fluid accelerates; the surface shear stress and the plate temperature reduce considerably due to intensification of the velocity ratio.
- Heat generation parameter can cause a rise in plate and fluid temperatures with a subsequent reduction in the absolute value of the skin-friction coefficient.
- The influence of increasing the mixed convection parameter (Gr) is to accelerate the fluid and stem down not only the fluid temperature but also the absolute skin-friction coefficient.
- The reduced heat transfer rate at the wall depreciates in value as the radiation parameter increases.

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