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Volatility Modeling and Forecasting Using Range-Based GARCH Models

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Abstract

This paper investigates the forecast performance of symmetric and asymmetric GARCH models in comparison with symmetric and asymmetric range-based GARCH models. Specifically, we explore whether including the range and assuming asymmetry in the conditional variance equation significantly impacts the forecast performance of range-based GARCH models. The models examined in this study include GARCH(1,1), TARCH(1,1), RGARCH(1,1,1), and RTARCH(1,1,1). Our evaluation of these models utilizes different loss functions.Using daily, weekly and monthly opening, closing, highest and lowest all-share historical prices of the Nigeria Stock Exchange from 2014 to 2024, the results of data analysis reveal that incorporating the range and accounting for asymmetry in the conditional variance equation enhances the forecast performance of range-based GARCH models. Importantly, this finding holds for daily, weekly, and monthly forecast horizons.

Keywords: Volatility, Conditional Variance, Range-based GARCH, Leptokurtic Distribution, Loss Function and Forecast Horizon.
MSC2010: 03C50.

1 INTRODUCTION

Volatility modeling and forecasting has received a considerable attention in the literature due to its crucial role in financial market such as portfolio selection, option pricing and value at risk applications among others. Prior to Parkinson [1], Garman-Klass [2], Ball and Torous [3], Alizadeh et al. [4] , and Brandt and Jones [5], the majority of volatility modeling studies relied solely on daily closing prices of asset returns for estimating volatility. Shaik amd Maheswaran [6]; Padmakumari and Maheswaran [7]; Shaik and Maheswaran [8]; Shaik and Maheswaran [9]; indicated that volatility and co-movement estimations derived from extreme values exhibit enhanced efficiency, thereby contributing to more precise volatility modeling. These findings demonstrate the feasibility of developing more efficient volatility estimators based on high-low prices, often referred to as range-based volatility estimators. This is particularly beneficial when high-frequency intraday data is not readily accessible. Nieto [10] conducted research indicating that the forecasting outcomes

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are influenced by both out-of-sample observations and the time horizon. Consequently, it was observed that only asymmetric models such as EGARCH, assuming a skewed Student's t-distribution, demonstrated relatively favorable performance. Ma et al. [11] demonstrated that integrating low- and high-frequency volatility forecasts can enhance the predictive accuracy for both the Shanghai Stock Exchange Composite Index and the S&P 500 index. This implies that low-frequency data can offer supplementary insights to high-frequency data. However, Ma et al. [11] focused their research on day-ahead forecasts, suggesting that high-frequency volatility models may have a significant advantage in this context.

Onyeka-Ubaka and Anene [12] used maximum likelihood estimation and estimation function for parameter estimation of asymmetric GARCH models and showed that estimation function has comparative advantage over maximum likelihood estimation since it does not rely on distributional assumption of data. Other researchers like Tabasi et al. [13]; Altun et al. [14] have highlighted the impact of distributional assumptions on forecasting accuracy. They found that employing a Student-t distribution, as opposed to a normal distribution, yielded improved forecasting accuracy and reduced violation ratios. milosevic et al. [15] employed the ARCH and GARCH models to measure the impact of the holiday effect on the rates of return from investment activities in the studied financial markets. Faldzinski et al. [16] utilized the prices of energy commodities to compare the performance of the GARCH model and the support vector regression model. Their findings indicated that the GARCH model is less accurate and effective for analyzing and predicting commodity prices. Shaik and Maheswaran [17] expanded the utilization of such extreme values in volatility estimation to encompass the distributional characteristics of asset returns, a crucial aspect in refining volatility modeling. Furthermore, these studies have underscored that daily price range data encapsulate comprehensive market information compared to solely relying on closing price data. Aliyev et al. [18] used univariate asymmetric GARCH models to model and estimate the volatility of the Nasdaq-100. They discovered persistent volatility shocks on index returns, a leveraging effect on the index, and an asymmetric impact of shocks.Onyeka-Ubaka and Anene [19] used asymmetric GARCH models to forecast crude oil price and showed that volatility estimates given by exponential GARCH model exhibit lower forecast error.

Kim et al. [20] employed the standard GARCH model along with various asymmetric GARCH models to calculate the volatility of corporate bond yield spreads.Studies on gold returns volatility revealed that FIGARCH, incorporating a long memory process, outperformed other models with similar findings as documented by Emenogu and Adenomon [21] and Slim et al. [22]. These studies also underscored the importance of accounting for asymmetry, particularly in analysis of emerging economies.Research investigating the predictive precision of VaR models has indicated that variations in market circumstances, Elenjical et al. [23] showed that stock size, liquidity, and other factors can impact the forecasting accuracy of these models. Hongwiengjan and Thongtha [24] assessed an analytical approximation of option prices using the TGARCH model. Baum and Hurn [25], refined volatility models to forecast the fluctuating conditional variance of a price series. This volatility model incorporate historical unpredictable fluctuations in returns to enhance predictive accuracy. Naresh et al. [26] examined the asymmetric volatility of the Bank Nifty Index using the EGARCH model. Their research revealed volatility clustering in Nifty Bank returns over a four-year period, along with asymmetrical effects and leverage constants. They concluded that negative news impacts volatility more significantly than positive surprises, and market fluctuations are inversely related to stock market performance. Padmakumari and Shaik [27] conducted an empirical study on Value at Risk (VaR) forecasting using range-based conditional volatility models. Their findings indicate that these range-based models outperformed those based on daily closing prices, as they contain more information, resulting in more precise VaR estimates. This paper aims to evaluate predictive performance of symmetric and asymmetric GARCH models in comparison with symmetric and asymmetric range-based GARCH models.Despite considerable research on volatility models

for forecasting, there have been limited studies assessing the effectiveness of range-based models in comparison to other GARCH models.
Hence, this study seeks to determine whether application of symmetric and asymmetric range-based GARCH models can result in better forecast accuracy compared to their symmetric and asymmetric GARCH counterparts.

2 METHODOLOGY

This paper uses both symmetric and asymmetric GARCH models with symmetric and asymmetric range-based GARCH models. The models examined in this paper are GARCH(1,1), TARCH(1,1), RGARCH(1,1,1) and RTARCH(1,1,1) models. One of the simplest forms for modeling daily returns may be written as follows;

$$r_t = \sigma_t \epsilon_t \tag{2.1}$$

where $r_t = ln(P_t) - ln(P_{t-1})$ is the log returns of the asset at time t. P_t is the price of the asset at time t, ϵ_t is the independent and identically distributed error term, that is, $\epsilon_t \sim$N(0,1) and σ_t is the volatility of the asset.

2.1 Return-based Volatility Models

Symmetric and asymmetric return-based volatility models examined in this paper are GARCH (1,1) and TARCH (1,1) respectively

2.1.1 GARCH (1, 1) Model:

One of the first and most commonly used specifications for time-varying volatility is Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, developed by Bollerslev [28].GARCH model is an extension of the Autoregressive Conditionally Heteroscedastic (ARCH) model, developed by Engle [29], for modeling conditional volatility. These models help to forecast the time-varying conditional variance of a price series by using past unpredictable changes in the returns of that price series. The GARCH (1, 1) model is written as:

$$r_t = \sigma_t \epsilon_t \tag{2.2}$$

$$\sigma_t^2 = w + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2.3}$$

Where $r_t = ln(p_t) - ln(p_{t-1})$ is the log returns of the asset at time t. p_t is the price of the asset at time t, ϵ_t is the independent and identically normally distributed error term, that is, $\epsilon_t \sim$N(0,1) and σ_t is the volatility of the asset. $w > 0$, $\beta > 0$, $\alpha > 0$. w, and α_1 are the parameter of the ARCH model and w, α_1 and β_1 are the parameter of the GARCH model.

2.1.2 TARCH (1, 1) Model:

A symmetric ARCH model is unsuitable as it does not consider the exact variance process. Engle and Ng [30] considered including a news impact curve that will have asymmetric responses to good and bad news in the ARCH process to solve this issue. One such asymmetric model is the Threshold ARCH (TARCH) model proposed by Zakoian [31]. The TARCH (1, 1) model can be written as:

$$r_t = \sigma_t \epsilon_t \tag{2.4}$$

$$\sigma_t^2 = w + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \theta_1 r_{t-1}^2 \lambda_{t-1} \tag{2.5}$$

Where $w > 0$, $\beta > 0$, $\alpha > 0$. w, and α_1 are the parameter of the ARCH model and w, α_1 and β_1 are the parameter of the GARCH model while the good and bad market news have different effects on the model. The asymmetric effect in the model is denoted by $\theta_1 r_{t-1}^2 \lambda_{t-1}$. When negative news appears, $r_{t-i}^2 < 0$ and $\lambda_{t-i} = 1$. When positive news emerges, $r_{t-1}^2 \geq 0$ and $\lambda_{t-1} = 0$. If $\theta > 0$, there is a leverage effect in the sequence.

2.2 Range-Based Volatility Models

Symmetric and asymmetric range-based volatility models examined in this paper are $RGARCH(1,1,1)$ and RTARCH (1,1,1) with the aim of investigating whether inclusion of range and asymmetry in the conditional variance will have impact on forecast performance.

2.2.1 Range-based GARCH model (RGARCH)(1,1,1):

The model is defined as follows:

$$r_t = \sigma_t \epsilon_t \tag{2.6}$$

$$\sigma_t^2 = w + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 R_{t-1}^2 \tag{2.7}$$

Where $w > 0$, $\beta > 0$, $\alpha > 0$. w, and α_1 are the parameter of the ARCH model and w, α_1 and β_1 are the parameter of the GARCH model. while the γ_1 capture the effect of range-based volatility on the volatility process and R_{t-1}^2 is the range of log prices.

2.2.2 Range-based TARCH model (RTARCH)(1,1,1)

This model is defined as:

$$r_t = \sigma_t \epsilon_t \tag{2.8}$$

$$\sigma_t^2 = w + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \theta_1 r_{t-1}^2 \lambda_{t-1} + \gamma_1 R_{t-1}^2 \tag{2.9}$$

Where $w > 0$, $\beta > 0$, $\alpha > 0$. The parameters to be estimated are w, β_1 and α_1 while the good and bad market news have different effects on the model. The asymmetric effect in the model is denoted by $\theta_1 r_{t-1}^2 \lambda_{t-1}$. When negative news appears, $r_{t-i}^2 < 0$ and $\lambda_{t-i} = 1$. When positive news emerges, $r_{t-1}^2 \geq 0$ and $\lambda_{t-1} = 0$. If $\theta > 0$, there is a leverage effect in the sequence and γ_1 capture the effect of range-based volatility on the volatility process and R_{t-1}^2 is the range of log prices.

3 Forecast Evaluation

This paper evaluates volatility forecast using the following forecast accuracy measures: Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE) proposed by Bollerslev et al. [32] along with Mean Absolute Error (MAE) suggested by Hansen and Lunde [33]. The use of multiple metrics is advantageous in the identification of the optimal forecast model, as highlighted by Andersen and Bollerslev [34]. These forecast accuracy measures are defined as follows:

(i) **Mean Square Error(MSE)**:

$$MSE = \frac{1}{n} \sum_{t=1}^{n} (V_c - V_e)^2 \tag{3.1}$$

where n is the number of observations in the data set,V_c is the actual variance and V_e is the forecasted variance at day t.

(ii) **Mean Absolute Error(MAE)**:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|V_c - V_e| \tag{3.2}$$

where n is the number of observations in the data set,V_c is actual variance and V_e is the forecasted variance at day t.

(iii) **Root Mean Square Error(RMSE)**:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(V_c - V_e)^2} \tag{3.3}$$

where n is the number of observations in the data set,V_c is actual variance and V_e is the forecasted variance at day t.

(iv) **Mean Absolute Percentage Error(MAPE)**:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}|\frac{V_c - V_e}{V_e}| \times 100 \tag{3.4}$$

where n is the number of observations in the data set,V_c is the actual variance and V_e is the forecasted variance at day t.

4 Data Used For Analysis

The data used consist of 2476 daily,530 weekly and 122 monthly All-Share historical data (index) from January 3, 2014 to January 3, 2024 collected from Nigerian Stock Exchange (NSE) gathered via the link https://ng.investing.com

5 Results

Comparison of Daily Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with student-t distribution

Table 1: Performance of Symmetric and Asymmetric GARCH Models with student t-distribution

Models	MSE	RMSE	MAE	MAPE
RGARCH (1,1,1)	0.01262802	0.1123745	0.1123084	86.54144
RTARCH (1,1,1)	0.0151704	0.1231682	0.1231632	94.90587
GARCH (1,1)	0.1653829	0.1286013	0.1285051	96.48712
TARCH (1,1)	0.02021876	0.1421927	0.1421822	93.99988

The performance of the range-based and return-based GARCH models is presented in table 1 above using MAE, MSE, RMSE and MAPE loss functions for daily all-share historical data (index).Lower value of loss functions indicates higher performance of the model under consideration. From table 1 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Daily Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with GED distribution

Table 2: Performance of Symmetric and Asymmetric GARCH Models with GED distribution

Models	MSE	RMSE	MAE	MAPE
RGARCH (1,1,1)	0.01475428	0.1214672	0.1214671	93.59887
RTARCH (1,1,1)	0.01534279	0.123866	0.1238633	95.44535
GARCH (1,1)	0.02056576	0.1434077	0.1434077	96.71821
TARCH (1,1)	0.02016041	0.1419874	0.1419701	93.85965

The performance of the range-based and return-based GARCH models is presented in table 2 above using MAE, MSE, RMSE and MAPE loss functions for daily all-share historical data (index).From table 2 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Daily Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with skewed student-t distribution

Table 3: Performance of Symmetric and Asymmetric GARCH Models with skewed student t-distribution

Models	MSE	RMSE	MAE	MAPE
RGARCH (1,1,1)	0.01515481	0.1231049	0.1231049	94.86091
RTARCH (1,1,1)	0.01516281	0.1231373	0.1231359	94.88481
GARCH (1,1)	0.01659503	0.1288217	0.1287172	95.54873
TARCH (1,1)	0.01755684	0.1325022	0.1324311	87.5532

The performance of the range-based and return-based GARCH models is presented in table 3 above using MAE, MSE, RMSE and MAPE loss functions for daily all-share historical data (index).From table 3 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Weekly Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with student t-distribution

Table 4: Performance of Symmetric and Asymmetric GARCH Models with student t-distribution

Models	MAE	MSE	RMSE	MAPE
RGARCH (1,1,1)	0.2923867	0.08598651	0.2932346	79.06156
RTARCH (1,1,1)	0.3462092	0.1198613	0.3462099	93.6152
GARCH (1,1)	0.5737768	0.3292445	0.5737983	87.01911
TARCH (1,1)	0.5549379	0.3079734	0.5549535	93.02772

The performance of the range-based and return-based GARCH models is presented in table 4 above using MAE, MSE, RMSE and MAPE loss functions for weekly all-share historical data (index).From table 4 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Weekly Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with GED distribution

Table 5: Performance of Symmetric and Asymmetric GARCH Models with GED distribution

Models	MAE	MSE	RMSE	MAPE
RGARCH (1,1,1)	0.349759	0.1223314	0.349759	94.57508
RTARCH (1,1,1)	0.35292262	0.1245573	0.3529268	95.4315
GARCH (1,1)	0.588793	0.3465434	0.5886794	96.6368
TARCH (1,1)	0.5622271	0.3161159	0.5622419	94.24966

The performance of the range-based and return-based GARCH models is presented in table 5 above using MAE, MSE, RMSE and MAPE loss functions for weekly all-share historical data (index). From table 5 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Weekly Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with skewed student t-distribution

Table 6: Performance of Symmetric and Asymmetric GARCH Models with skewed student t-distribution

Models	MAE	MSE	RMSE	MAPE
RGARCH (1,1,1)	0.3435835	0.1180497	0.3435835	92.90521
RTARCH (1,1,1)	0.3508247	0.1230781	0.3508249	94.86324
GARCH (1,1)	0.5739889	0.3294902	0.5740124	95.60155
TARCH (1,1)	0.5521996	0.3049474	0.5522205	92.56869

The performance of the range-based and return-based GARCH models is presented in table 6 above using MAE, MSE, RMSE and MAPE loss functions for weekly all-share historical data (index).From table 6 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Monthly Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with student t-distribution

Table 7: Performance of Symmetric and Asymmetric GARCH Models with student t-distribution

Models	MAE	MSE	RMSE	MAPE
RGARCH (1,1,1)	0.637949	0.4081393	0.6388578	78.42112
RTARCH(1,1,1)	0.7617298	0.5802342	0.761731	93.63712
GARCH (1,1)	1.037033	1.075437	1.037033	84.13239
TARCH (1,1)	1.033416	1.067949	1.033416	93.41242

The performance of the range-based and return-based GARCH models is presented in table 7 above using MAE, MSE, RMSE and MAPE loss functions for monthly all-share historical data (index). From table 7 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Monthly Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with GED distribution

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Table 8: Performance of Symmetric and Asymmetric GARCH Models with GED distribution

Models	MAE	MSE	RMSE	MAPE
RGARCH (1,1,1)	0.7448156	0.554752	0.7448167	91.5579
RTARCH(1,1,1)	0.7683284	0.5903295	0.768329	94.44826
GARCH (1,1)	1.03403	1.069219	1.03403	93.79227
TARCH (1,1)	1.036026	1.07335	1.036026	93.64837

The performance of the range-based and return-based GARCH models is presented in table 8 above using MAE, MSE, RMSE and MAPE loss functions for monthly all-share historical data (index). From table 8 above, RGARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RGARCH (1,1,1) model is the best-performing model.

Comparison of Monthly Volatility Forecast Performance of Symmetric and Asymmetric GARCH Models with Symmetric and Asymmetric Range-based GARCH Models with skewed student t-distribution

Table 9: Performance of Symmetric and Asymmetric GARCH Models with skewed student t-distribution

Models	MAE	MSE	RMSE	MAPE
RGARCH (1,1,1)	0.7604241	0.578245	0.7604242	93.47661
RTARCH(1,1,1)	0.7587499	0.5757039	0.7587515	93.27081
GARCH (1,1)	1.039107	1.079743	1.039107	95.20316
TARCH (1,1)	1.032864	1.066809	1.032865	93.36253

The performance of the range-based and return-based GARCH models is presented in table 9 above using MAE, MSE, RMSE and MAPE loss functions for monthly all-share historical data (index).From table 9 above, RTARCH(1,1,1) has the lowest value of MAE, MSE, RMSE and MAPE when compared with other competing models under consideration. Hence, RTARCH (1,1,1) model is the best-performing model.

6 CONCLUSION

Based on the analysis above, it was found that range-based symmetric and asymmetric volatility models: RGARCH (1,1,1) and RTARCH (1,1,1) models out-performed symmetric and asymmetric return-based volatility models: GARCH (1,1) and TARCH (1,1) models. Hence, the result indicate that inclusion of range and asymmetry in the conditional variance equation has significant impact on forecast performance of the range-based GARCH models.

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