International Journal of Mathematical Analysis and Optimization: Theory and Applications
Vol. 2018 , pp. 261 - 275

Modeling crude oil spot price as an Ornstein - Uhlenbeck process

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Article Info
Received: 17 December 2017 Revised: 10 April 2018
Accepted: 8 May 2018 Available online: 14 June 2018

Abstract

Unexpected downturn in crude oil price in recent years has led to recession in economies of countries like Nigeria, and Venezuela. Search for a stochastic model that could give a good description of the movement of crude oil price led to the use of Ornstein Uhlenbeck process, since mean reversion is exhibited by the price of a number of commodities. We consider crude oil price series for four Niger-Delta crude types, over a five year period, and analyse same using the Ornstein-Uhlenbeck process. Parameters of the Ornstein-Uhlenbeck process were estimated for the data set using regression approach in R. These parameters were employed to simulate the Ornstein-Uhlenbeck process using an R-computational scheme. In-sample and Out-of-sample forecast were done. It is found that in the absence of the unusual price movements (jumps), the O-U model can be used to model crude oil price movement.

Keywords: Crude oil price, Downturn, Mean reversion, Movement simulation, Orstein-Uhlenbeck.
MSC2010: 63P05

1 Introduction

Crude oil is an important commodity, which is considered both a consumption and investment commodity. [1]. Records from the past few years have shown that movement in price of crude oil has been very rapid, leading to severe consequences for economies and companies. We study crude oil price using empirical data obtained from DPR Lagos for four Niger Delta crude types, for period, 2005- 2009.

By oil price, we mean the spot price of a barrel of bench mark crude oil. There are different types of crude in the crude oil market, which includes West Texas intermediate crude, (WTI), Brent Ice, Bonny Light, Western Canadian Select (WCS) etc. Crude oil price fluctuates rapidly over time. Trends of this movement have been of interest to researchers, market participants such as speculators, buyers, sellers and end -users.

The study in this paper is motivated by effort to understand crude oil price trend. Apart from non-economic, non-stochastic factors such as political arm flexing of OPEC giant countries and other conflicts, in mathematical finance, effort is made at identifying a model that describes oil

http://ijmao.unilag.edu.ng/article/view/25

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price dynamics. Here we consider a mean reverting process. Mean reversion theory was introduced by Ornstein and Uhlenbeck [2] and has been used by other researchers to model commodities and asset prices. Gibson and Schwartz [3], studied financial and real assets contingent on price of oil. They used mean reverting two factor model. Convenience yield has been studied largely using mean reverting process. Convenience yield is the effect which evolves from the ownership of physical commodity compared to ownership of futures contract. It is also seen as a reflection the market's expectation concerning the future availability of commodity. This points to the fact that much work has been done on oil-linked financial instruments such as leases, contracts and futures than on spot. Bessembinder et al [4] studied some agric products such as wheat, sugar, and metals- gold, silver, platinum. They used their test to detect mean reversion in prices arising from correlation between convenience yield and prices, and correlation between I-rate and prices. They proved that there is mean reversion in spot asset prices of these wide range of commodities. Gibson and Schwarz compared three models of commodity prices that takes mean reversion into account. Dixit A.K. and Pindyck R.S [5] used some statistical tests, to study price of crude oil and copper. They confirmed that the prices are mean reverting. These studies motivate the study in this paper to analyse oil spot price with the Ornstein- Uhlenbeck process. The remainder of the paper is organised as follows; in section 2 we present the data used for the work, the O-U process is discussed and the model for crude oil spot price formulated in section 3. The estimated parameters and computational scheme for in-sample and out-of - sample forecasts are presented in section 4. Results are presented, analysed, and conclusion drawn in section 5.

2 Ornstein-Uhlenbeck mean reverting process

In recent work by Roger et al [6],the Ornstien-Uhlenbeck (O-U) process which is a volatility process, has been used in finance as a model of volatility of asset price process. Since the crude oil price is considered as both consumption commodity and an investment (asset) commodity, we analyse its price fluctuations using the O-U process. Let $\{S_t : t \geq 0\}$ denote the crude oil price process.

A process $\{S_t : t \geq 0\}$ is an Ornstien-Uhlenbeck process if S_t satisfies the following O-U stochastic differential equation given as

$$dS_t = \alpha S_t dt + \sigma dW_t \tag{2.1}$$

where σ and α are constants and represent volatility and rate of reversion of the process respectively. W_t is the Wiener process.

The Ornstien-Uhlenbeck process, is the most popular of the type of mean reverting processes and are generally used in finance to model interest rates and commodities. The idea behind the mean reverting process (Ornstien-Uhlenbeck), the economic principle that when prices become too high, forces of demand and supply will "act" such that demand will fall as supply increases, producing a counter balancing effect that yields equilibrium. Also when price is too low, the opposite event will occur, prices are then pushed towards equilibrium which may be described as a long run mean. Mean reversion is also induced by negative relationships between I-rate and prices.

When S_t in equation 2.1 is described to include long run mean value of price denoted μ, then we have equation

$$dS_t = \alpha(\mu - S_t)dt + \sigma dW_t \tag{2.2}$$

The O-U equation has a closed form solution. There are three parameters for the O-U process, namely mean, μ, volatility, σ, and rate of reversion, α. These parameters were estimated in this paper for the crude oil price data. The speed of reversion is proportional to the distance between the current position and the equilibrium level. So the variance grows at first but then stabilizes.

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The O-U process is driven by a Brownian motion.

The Ornstein and Uhlenbeck, process has only two major parameters that influence results, the mean and volatility. The O-U process sometimes generates both positive and negative values (oil prices) over time. Negative values may be obtained sometimes for the crude oil price S_t from the computational scheme of the O-U model. This limitation can be made up for, by modifying the process away from pure O-U process, that is, modulating the volatility parameter as S tends towards zero. This can be seen in the modification of Cox et al [10] which expresses variations for interest rate r . Also the log of the spot price can be used instead. However negative values were not obtained here.

3 The model

Let S_t be the spot price of crude oil at time t. Crude oil price fluctuates over time with volatility $\sigma > 0$, and is expected to return to a long-run mean value μ at a rate (mean reversion rate) denoted $\alpha > 0$.

Ignoring random fluctuations in the process, dW_t, then S_t has an overall drift towards a mean value μ. The process reverts to μ at a rate α, with a magnitude proportional to distance between current value of S_t and μ.

The crude oil price process S_t is expected to satisfy the following equation

$$dS_t = \alpha S_t dt + \sigma dW_t, \qquad S(0) = S_0 \tag{3.1}$$

W_t is a standard Brownian motion, $t \in [0, \infty]$.

Choosing a suitable function

$$F(t,s) = e^{-\alpha t} S$$
$$Y_t = F(t, S_t) \tag{3.2}$$

$$\begin{aligned} dY_t &= -\alpha e^{-\alpha t} S_t dt + e^{-\alpha t} dS_t && (3.3)\\ &= -\alpha e^{-\alpha t} S_t dt + e^{-\alpha t}(\alpha S_t dt + \sigma dW_t) \\ dY_t &= \sigma e^{-\alpha t} dW_t && (3.4) \end{aligned}$$

From 3.2

$$dY_t = d(e^{-\alpha t} S_t) \tag{3.4a}$$

Equating 3.4 and 3.4a we have

$$d(e^{-\alpha t} S_t) = \sigma e^{-\alpha t} dW_t \tag{3.4b}$$

Integrating on both sides of 3.4 from s to t we have

$$Y_t - Y_s = \sigma \int_s^t e^{-\alpha t} dW_t$$

From 3.4b integrating from 0 to t

$$e^{\alpha t} S_t - e^{\alpha \dot{0}} S_0 = \sigma \int_0^t e^{\alpha s} dW_s$$

$$e^{-\alpha t} S_t = S_0 + \sigma \int_0^t e^{-\alpha s} dW_s$$

dividing through by $e^{-\alpha t}$

$$S_t = e^{\alpha t} S_0 + \sigma \int_0^t e^{\alpha(t-s)} dW_s \tag{3.5}$$

Equation 3.5 is the closed form solution of equation 3.1
Second term of equation 3.5 is a Wiener integral, and from definition by Roger et al [6]

$$\begin{aligned}\int_0^t e^{\alpha(t-s)} dW_s &\sim N\left(0, \int_0^t e^{2\alpha(t-s)} dS\right)\\ &\approx N\left(0, \frac{e^{2\alpha t}-1}{2\alpha}\right)\end{aligned}$$

Equation 3.1 can be rewritten as

$$dS_t = \alpha(\mu - S_t)d_t + \sigma dW_t \tag{3.6}$$

(in which case S_t in 3.1 is deviated from long run mean value μ)
Now for

$$Y_t = e^{\alpha t} S_t \tag{3.7}$$

$$\begin{aligned}dY_t &= \alpha e^{\alpha t} S_t dt + e^{\alpha t} dS_t\\ &= \alpha e^{\alpha t} S_t dt + e^{\alpha t}[\alpha(\mu - S_t)dt + \sigma dW_t]\\ &= \alpha e^{\alpha t} S_t dt + \alpha e^{\alpha t}\mu dt - \alpha e^{\alpha t} S_t dt + e^{\alpha t}\sigma dW_t\\ &= \alpha e^{\alpha t}\mu dt + e^{\alpha t}\sigma dW_t \qquad (3.7a)\end{aligned}$$

From equation 3.7

$$dY_t = d\left(e^{\alpha t} S_t\right) \tag{3.7b}$$

From equation 3.7a

$$dY_t = \alpha e^{\alpha t}\mu dt + e^{\alpha t}\sigma dW_t \tag{3.7c}$$

equating 3.7b and 3.7c

$$\begin{aligned}d\left(e^{\alpha t} S_t\right) &= \alpha e^{\alpha t}\mu dt + e^{\alpha t}\sigma dW_t\\ d\left(e^{\alpha t} S_t\right) &= e^{\alpha t}[\alpha\mu dt + \sigma dW_t] \qquad (3.8)\end{aligned}$$

integrating equation 3.8 and evaluating from 0 to t, we have

$$\begin{aligned}e^{\alpha t} S_t - e^{\alpha \cdot 0} S_0 &= \alpha \int_0^t e^{\alpha t}\mu dt + \int_0^t e^{\alpha s}\sigma dW_s\\ e^{\alpha t} S_t - S_0 &= e^{\alpha s}\mu\Big|_0^t + \int_0^t e^{\alpha s}\sigma dW_s\\ e^{\alpha t} S_t - S_0 &= e^{\alpha t}\mu - e^0\mu + \int_0^t e^{\alpha s}\sigma dW_s\\ e^{\alpha t} S_t - S_0 &= \mu(e^{\alpha t} - 1) + \int_0^t e^{\alpha s}\sigma dW_s\end{aligned}$$

dividing through by $e^{\alpha t}$, we have

$$S_t = S_0 e^{-\alpha t} + \mu(1 - e^{-\alpha t}) + \sigma \int_0^t e^{-\alpha(t-s)} dW_s \tag{3.8a}$$

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For purpose of simulation, discretizing equation 3.8a we use

$$S_t = e^{-\alpha\Delta t}S_{t-1} + \left(1 - e^{-\alpha\Delta t}\right)\mu + \sigma\sqrt{\frac{(1 - e^{-2\alpha\Delta t})}{2\alpha}}dW_t \tag{3.9}$$

$$W_t \sim N(0,1)$$

3.1 Callibration of data for the parameters of the oil price process

Callibration of data for the parameters of the Ornstein-uhlenbeck (O-U) process was done for empirical crude oil spot price S_t using Regression approach by Smith [7]. The R-Code for the estimation process is attached.

Crude type	**Mean (μ)**	**Volatility(σ)**	**Mean reversion rate(α)**
BL	77.990269	28.320770	1.054858
BB	78.881217	27.948929	0.973454
PL	80.0346053	27.8207744	0.9473295
ANTAN	76.430276	28.987445	1.003883

Table 1: The estimated O-U parameters for oil price

In this work actual prices are used because actual price S_t is used in the model formulation. Most financial studies involve returns instead of price for two reasons. For the average investor return on an asset is a complete and scale free summary of investment opportunity. By the work of Tsav [8] return series are easier to handle than price series because the former have more attractive statistical properties. However an investor in a field project may be more concerned about analysis using actual price of crude oil. We therefore used price series in the analysis.

3.2 Computational scheme for theoretical price data

O-U process is simulated using estimated parameters for mean, volatility, and rate of reversion i.e. μ, σ, and α, respectively given in Table 1. The computational scheme written in R-programming language follows an algorithm adapted from the work of Smith. It is an iterative procedure, that starts with initial values S_0. Initial price values S_0 and the estimated parameters are imputed. Depending on the value of α, we compute $dW_t = \sqrt{\Delta t}W_t$. Here, W_t is generated as a standard normal random variable i.e. $W_t \sim N(0,1)$. If $\alpha = 0$, the limit is used, i.e. $\sqrt{\Delta t}N(0,1)$, otherwise we compute $\sigma\sqrt{\frac{(1 - e^{-2\alpha\Delta t}}{2\alpha}}N(0,1)$. The foregoing procedure yields S_1, the process is iterated to obtain S_2 from S_1, S_3 based on $S_2 \cdots S_n$ based on S_{n-1}.

$S_t = e^{-\alpha\Delta t}S_{t-1} + dS_t,$ where dS_t interprets the O-U SDE in the algorithm. i.e.
$S_t = e^{-\alpha\Delta t}S_{t-1} + \alpha(\mu - S_{t-1})\Delta t + \sigma dW_t$

This approach for simulation of price paths is as given in Gillespie [9]. Codes for the Computational Scheme is given in Appendix B.

For initial values 44.1667, 44.2267, 44.2967 and 42.7717 for BL, BB, PL, ANTAN, S_t values are randomly generated from this computational scheme, for each crude type.

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3.3 Computation of crude oil price using the model; In- sample forecast and Out-of sample forecast

The R-program computational scheme for O-U simulation described above, was used to compute S_t values as given in equation $3.8a$. The values obtained for the in-sample forecast, for the process are given in Table 1. The table displays together the empirical and computed values obtained from the O-U process, for quick comparison. These computed price values are also referred to as theoretical price values.

Results

We may write the model equations in line with equation $3.8a$ as follows;

For Bonny Light (BL), the model equation for the crude oil spot price is

$$S_t = 44.1667e^{-1.0548t} + 78.881(1 - e^{-1.0548t}) + 27.948\int_0^t e^{-1.0548u} dW_u \qquad (a)$$

For Brass Blend (BB), the model equation for the crude oil spot price is:

$$S_t = 44.2267e^{-0.9734t} + 77.990(1 - e^{-0.9734t}) + 28.320\int_0^t e^{-0.9734u} dW_u \qquad (b)$$

For Pennington Light (PL), the model equation for the crude oil spot price is:

$$S_t = 44.2967e^{-0.9473t} + 80.034(1 - e^{-0.9473t}) + 27.820\int_0^t e^{-0.9473u} dW_u \qquad (c)$$

For Antan (ANTAN), the model equation for the crude oil spot price is:

$$S_t = 42.7717e^{-1.0038t} + 76.430(1 - e^{-1.0038t}) + 28.987\int_0^t e^{-1.0038u} dW_u \qquad (d)$$

Equations a, b, c and d are solved and the results are shown in Tables 2,3 and 4 and Figure 1

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Month	BL		BB		PL		ANTAN	
	Empirical	Computed	Empirical	Computed	Empirical	Computed	Empirical	Computed
1	44.1667	41.66700	44.2267	44.22670	44.297	44.29670	42.7717	42.77170
2	45.2401	47.43367	45.3001	49.90632	45.37	43.37090	45.874	49.90391
3	52.9998	52.23230	53.0598	49.35153	53.13	52.90592	51.6048	48.61238
4	51.8679	49.72159	51.9279	47.43308	51.998	46.94257	50.4729	53.06749
5	48.936	56.52354	48.996	50.11351	49.066	52.05549	47.541	58.27146
6	54.898	57.83155	54.958	69.20631	55.028	70.55239	53.503	63.39716
7	58.0465	54.52253	58.1065	63.33078	58.177	78.48543	56.6515	69.90806
8	65.945	60.69544	66.005	74.65364	66.075	76.80124	64.55	59.91308
9	65.1395	50.45600	65.1995	69.24294	65.27	72.16463	63.7445	47.29118
10	60.1407	49.73182	60.2007	76.45841	60.271	61.48250	58.7457	45.48306
11	55.8968	48.92676	55.9568	100.16742	56.027	61.23749	54.5018	55.64199
12	57.154	56.70354	57.214	95.86338	57.284	69.78391	55.759	72.06037
13	63.3527	61.17013	63.4127	94.63160	63.483	71.85430	61.9567	71.53486
14	60.74	63.67812	60.8	107.91350	60.87	84.08268	59.344	60.82734
15	63.2524	59.81300	63.3124	97.34751	63.382	86.21255	61.8564	64.66190
16	72.1166	54.92595	72.1766	88.41918	72.247	92.46888	70.3206	51.82987
17	71.164	61.35969	71.224	77.93862	71.294	87.35507	69.571	47.53672
18	69.5066	70.48500	69.5666	75.69837	69.637	89.13135	69.989	44.36505
19	75.2569	86.04181	75.3169	70.65420	75.387	79.93386	73.8609	42.57357
20	74.4721	78.99376	74.5321	80.71907	74.602	63.90042	73.0761	56.93465
21	61.94	76.60859	62	81.13169	62.07	70.35043	60.544	46.08053
22	58.757	77.72866	58.817	81.56856	58.887	60.85054	57.361	42.29501
23	60.3197	72.95170	60.3797	86.50901	60.45	54.24445	58.9237	45.32685
24	64.2745	72.57435	64.3345	92.53920	64.405	63.38928	62.8785	50.43963
25	55.5622	59.36490	55.6222	88.79971	55.692	62.96146	54.12	56.78713
26	59.395	66.07659	59.455	98.52625	59.525	58.89604	58.005	57.09746
27	64.058	73.80190	64.118	93.60172	64.188	69.02160	58.331	63.32086
28	70.332	79.35955	70.392	95.89198	70.462	67.30953	64.73	68.59395
29	70.104	77.76714	70.164	86.00331	70.234	77.09301	66.584	70.41388
30	73.801	72.10914	73.861	77.02278	73.931	70.07364	68.401	71.10848

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	BL		BB		PL		ANTAN	
Month	Empirical	Computed	Empirical	Computed	Empirical	Computed	Empirical	Computed
31	79.456	77.03370	79.516	84.82500	79.586	74.29352	73.421	83.83560
32	73.344	80.77479	73.404	100.84791	73.474	74.73757	73.344	89.64958
33	79.465	81.67353	79.525	106.60526	79.595	74.66236	79.465	90.40844
34	84.584	68.25922	84.644	97.33675	84.714	68.51354	84.584	93.01963
35	94.461	80.46156	94.521	101.39300	94.591	72.74077	94.461	97.79276
36	92.8548	91.35458	92.9148	92.24391	92.985	86.55666	92.8548	95.55891
37	94.0321	94.43542	93.4394	95.57106	92.851	83.30716	87.75	111.83627
38	99.2161	85.90044	97.5285	99.86483	98.09	91.38968	96.252	104.65552
39	95.5473	94.12988	106.817	90.55823	105.26	87.17604	101.39	111.90591
40	114.0519	88.81098	113.093	87.85896	114.36	104.70790	113.178	103.56445
41	129.1207	85.22240	128.595	89.91474	129.57	123.13966	126.2017	101.99229
42	130.4259	93.90251	140.727	87.18653	137.96	113.53481	140.249	89.37027
43	132.6695	91.69036	133.428	93.51601	137.07	115.47080	130.885	83.94756
44	115.1328	86.17292	113.94	92.80068	117.89	112.08761	109.7788	87.54776
45	98.9226	82.34907	96.8924	97.94952	98.673	118.89777	89.6836	97.67127
46	66.7688	86.38346	70.3236	91.90131	72.584	107.69950	59.3712	88.99821
47	52.1634	88.52181	52.8436	91.40409	54.212	105.25454	45.404	90.95963
48	43.1876	101.93840	41.944	98.69697	44.569	102.32254	37.2078	87.02977
49	45.54142	106.90631	45.3806	98.04400	46.158	110.10463	41.3517	99.64655
50	45.02294	95.40848	45.9318	86.37208	48.348	94.46201	39.8582916	95.32308
51	51.85602	92.16217	49.8717	80.26828	48.444	101.89059	46.5591827	85.22161
52	52.3576	87.32683	53.2512	76.69073	48.444	106.13368	49.7834	70.82256
53	62.7257	80.39869	61.1513	83.50707	62.677	103.56543	59.2899	77.80016
54	69.0588	75.38852	69.9299	75.65975	70.87	93.25381	66.4178	79.14109
55	68.23453	84.54474	66.4206	69.62545	69.728	99.78968	66.0467896	78.13330
56	71.19531	77.21906	72.1659	64.87678	74.086	101.90740	71.5394438	89.62917
57	68.79465	80.60658	68.4615	66.33830	70.483	87.90602	66.8443707	92.32398
58	75.66168	100.17531	77.1753	58.84420	72.966	94.79761	74.918345	88.92973
59	78.03899	99.51410	78.0614	67.13717	79.492	87.55082	76.4878708	88.02889
60	76.87649	107.52623	77.0659	51.15576	79.942	95.40997	76.4167254	95.29407

Table 2: Empirical and computed S_t values for the crude types (2005-2009): in-sample values.

Month	BL	BB	PL	ANTAN
61	107.52620	51.15570	95.40990	95.29400
62	93.38448	65.30055	92.74331	83.51294
63	84.89283	74.01729	98.89090	85.38763
64	85.81999	75.90186	100.19331	87.04141
65	89.09493	80.12267	85.48451	89.86838
66	80.27032	83.16689	93.70087	81.80172
67	88.41165	75.11808	90.22491	78.66721
68	78.17387	91.47837	88.23089	78.21191
69	72.20130	86.49417	86.38484	93.60420
70	66.73577	73.76189	100.97459	99.61995
71	70.20471	63.62807	98.79806	91.42792
72	67.86930	61.41196	89.47490	72.96200

Table 3: Computed S_t Values for 2010(Out-of-sample forecast)

	PL		ANTAN	
Month	**Empirical**	**Computed**	**Empirical**	**Computed**
61	75.213	95.40990	71.503	95.29400
62	76.495	92.74331	77.819	83.51294
63	83.228	98.89090	81.6748	85.38763
64	84.3782	100.19331	84.331772	87.04141
65	81.681	85.48451	70.531	89.86838
66	74.2882	93.70087	73.591	81.80172
67	83.194	90.22491	74.690512	78.66721
68	77.6853	88.23089	72.649	78.21191
69	80.6194	86.38484	78.76617	93.60420
70	84.4912	100.97459	84.143179	99.61995
71	88.687	98.79806	83.841	91.42792
72	92.8277	89.47490	92.446032	72.96200

Table 4: Empirical and Computed S_t values for 2010 (Out-of-sample forecast)

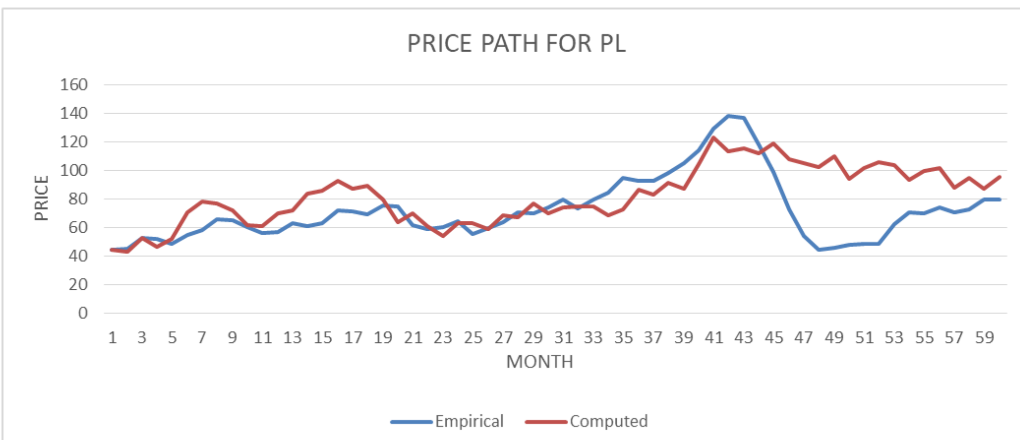

Figure 1:

Figure 1 [1] showing price path for PL crude type

4 Discussion

4.1 In-sample Forecasts

From the results on Table 1 , it can be seen that theoretical price values approximate empirical price values well only over certain periods of time (some months together). For other periods the theoretical and empirical prices are not close. The periods of fit basically account for those periods when empirical price differences were not much from month to month, that is periods of price stability for a few months. These periods are on the average of five to six months, and occur intermittently through the data set. For all crude types about the last twenty months were out of fit. These are the months where unusual, unexpected, price movements/deviations occurred. From Table 1 , we also observe that high S_t values were generated but not exactly where the prices were

[1] Empirical price path coincides with computed price path except for periods exhibiting unusual price shifts(jumps).

high in the empirical data set. This indicates that the O-U model does well, in modeling crude oil spot price process for periods of steady, continuous, normal price change, but not for periods of unexpected abnormal price changes. However considering trend of price movement rather than price fit, the crude oil spot price trends well as a mean reverting process.

4.2 Out-Of-Sample Forecast

Computation of future values for crude oil price S_t ,using the O-U model

Crude oil price for December 2009 (60th month) was used as initial price, S_0 in the program to generate S_t values for 2010. The program generated values iteratively for 12 months for the mean reverting O-U model. The values obtained are displayed on Tables 2 and 3. In order to compare results, we used crude oil price data for the year 2010. We obtained 2010 complete data for only ANTAN and PL crude types. Remarks about out-of-sample forecast results, is made in comparison with these two data sets. Crude oil price data for 2005-2010 obtained from DPR compares very well with NYMEX crude oil price, WTI, which is available on Yahoo finance.

The model was used to forecast oil price data for 2010, results are shown on Tables 2 and 3. For the out-of - sample forecast results, we also observe that theoretical price values approximate empirical price values well only for some for months. Outside these months the theoretical and empirical prices are not close, as the computed values differ from the empirical values by an average of fifteen to twenty dollars. For PL crude type, the 65th, 69th, and 72nd months have good fit. Computed values for 63rd, 64th as well as 66th to 68th months are close in the case of ANTAN crude type. The out-of-sample forecast has not done well, this can be accounted for by the last month values of the in-sample forecast, which were out of fit.

5 Conclusion

Since only periods in the price series with absence of abnormal values, seem to fit the model we conclude that the model will do well to capture crude oil pattern in the absence of unusual price movement. Though the model has done well with other commodity prices and crude oil contingents, it is not perfect for crude oil spot price, because such unusual or abnormal price movement occur from time to time. The result signifies existence of abnormal price shifts (jumps) in the crude oil price movement, an indication that a Lévy model may be considered. Since the price trends mimics the Ornstein-Uhlenbeck mean reverting trend, considering a model of crude oil price as an Ornstein-Uhlenbeck process driven by a Lévy process could yield more interesting results.

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Acknowledgments

I thank my Ph.D supervisor Prof G.O.S Ekhaguere of the department of Mathematics, University of Ibadan, and Dr Richard Minkah of the Department of Statistics, University of Ghana for his assistance with R-programs and also DPR Lagos for supplying the data used in this work.

Competing financial interests

The author declares no competing financial interests.

Appendix A

```
##################################################################
OU_Reg<-function(S,deltat)
{
  end<-length(S)
SLr<-lm(S[2:end]~S[1:(end-1)]);
a = coef(SLr)[1]; b = coef(SLr)[2];
alpha = -log(b)/deltat;
mu = a/(1-b);
sigma = sd(residuals(SLr)) * sqrt( 2* alpha/(1-b^2) );
return(as.vector(c(mu,alpha,sigma)))
}
OUREG<-sapply(Price,function(x)OU_Reg(S=x,deltat=1/12))
rownames(OUREG)<-c("mu","alpha","sigma")
OUREG
```

Table 5: R-Code for Parameter Estimation

Appendix B

```
##########################################################################################
OUSim<-function(S0,mu, sigma, lambda, deltat,n,Title)
{
S<-numeric(n)
S[1]<-S0
if(lambda == 0){
  #dWt<-sqrt(deltat)*rnorm(n,mean = 0,sd=sqrt(deltat))
  dWt<-sqrt(deltat)*rnorm(n,mean = 0,sd=1)
}else {
#  dWt<-sqrt((1-exp(-2*lambda* deltat))/(2*lambda))*
rnorm(n,mean = 0,sd=sqrt(deltat))
  dWt<-sqrt((1-exp(-2*lambda* deltat))/(2*lambda))*rnorm(n,mean = 0,sd=1)
  }
for(i in 2:n)S[i]<-S[i-1]*exp(-lambda*deltat)+mu*(1-exp(-lambda*deltat))
\\+sigma*dWt[i]
#Price<-sapply(2:n,function(i) S[i-1]*
exp(-lambda*deltat)+mu*(1-exp(-lambda*deltat))+sigma*dWt[i])
plot(S,type="b",ylab="Price",main=Title)
print(S)
return(S)
}

BL<-OUSim(41.667,71.493,21.236,0.00062,1/12,60,"BL")
BB<-OUSim(44.2267,71.827,21.895,0.00063,1/12,60,"BB")
PL<-OUSim(44.2967,72.207,22.079,0.00063,1/12,60,"PL")
ANT<-OUSim(42.7717,69.443,22.054,0.00062,1/12,60,"ANTAN")
```

Table 6: R-Code for Computational Scheme generating S_t values

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Appendix C

Oil price data for the 4 Crude types

MONTH / YEAR	BL (US$)	BB (US$)	PL (US$)	AT (US$)
2005				
JAN	44.1667	44.2267	44.2967	42.7717
FEB	45.2401	45.3001	45.3701	45.874
MAR	52.9998	53.0598	53.1298	51.6048
APR	51.8679	51.9279	51.9979	50.4729
MAY	48.936	48.996	49.066	47.541
JUN	54.898	54.958	55.028	53.503
JUL	58.0465	58.1065	58.1765	56.6515
AUG	65.945	66.005	66.075	64.55
SEP	65.1395	65.1995	65.2695	63.7445
OCT	60.1407	60.2007	60.2707	58.7457
NOV	55.8968	55.9568	56.0268	54.5018
DEC	57.154	57.214	57.284	55.759
2006	BL (US$)	BB (US$)	PL (US$)	AT (US$)
JAN	63.3527	63.4127	63.4827	61.9567
FEB	60.74	60.8	60.87	59.344
MAR	63.2524	63.3124	63.3824	61.8564
APR	72.1166	72.1766	72.2466	70.3206
MAY	71.164	71.224	71.294	69.571
JUN	69.5066	69.5666	69.6366	69.989
JUL	75.2569	75.3169	75.3869	73.8609
AUG	74.4721	74.5321	74.6021	73.0761
SEP	61.94	62	62.07	60.544
OCT	58.757	58.817	58.887	57.361
NOV	60.3197	60.3797	60.4497	58.9237
DEC	64.2745	64.3345	64.4045	62.8785
2007	BL (US$)	BB (US$)	PL (US$)	AT (US$)
JAN	55.5622	55.6222	55.6922	54.12
FEB	59.395	59.455	59.525	58.005
MAR	64.058	64.118	64.188	58.331
APR	70.332	70.392	70.462	64.73
MAY	70.104	70.164	70.234	66.584
JUN	73.801	73.861	73.931	68.401
JUL	79.456	79.516	79.586	73.421
AUG	73.344	73.404	73.474	73.344
SEP	79.465	79.525	79.595	79.465
OCT	84.584	84.644	84.714	84.584
NOV	94.461	94.521	94.591	94.461
DEC	92.8548	92.9148	92.9848	92.8548

MONTH / YEAR	BL (US$)	BB (US$)	PL (US$)	AT (US$)
2008				
JAN	94.0321	93.4394	92.8512	87.75
FEB	99.2161	97.5285	98.0903	96.252
MAR	95.5473	106.8165	105.256	101.39
APR	114.0519	113.0931	114.3591	113.178
MAY	129.1207	128.5945	129.574	126.2017
JUN	130.4259	140.727	137.959	140.249
JUL	132.6695	133.4284	137.0709	130.885
AUG	115.1328	113.9404	117.885	109.7788
SEP	98.9226	96.8924	98.673	89.6836
OCT	66.7688	70.3236	72.584	59.3712
NOV	52.1634	52.8436	54.212	45.404
DEC	43.1876	41.944	44.569	37.2078
2009	**BL (US$)**	**BB (US$)**	**PL (US$)**	**AT (US$)**
JAN	45.54141886	45.38057343	46.158	41.35170087
FEB	45.02294382	45.93184663	48.348	39.85829155
MAR	51.85602396	49.87174229	48.444	46.55918275
APR	52.3576	53.2512	48.444	49.7834
MAY	62.7257	61.1513	62.6772	59.2899
JUN	69.0588	69.9299	70.87	66.4178
JUL	68.2345291	66.42060408	69.728	66.04678958
AUG	71.19531051	72.16594376	74.086	71.53944376
SEP	68.79464947	68.4614642	70.483	66.84437075
OCT	75.66167712	77.17534434	72.966	74.91834501
NOV	78.03899299	78.06138685	79.492	76.48787075
DEC	76.87649492	77.06592552	79.94190281	76.41672541

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