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# Stress-Work and Chemical Reaction Effects on MHD Forced Convection Heat and Mass Transfer Slip-Flow towards a Convectively Heated Plate in a Non-Darcian Porous Medium with Surface Mass-Flux

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**Abstract**

Analysis is conducted numerically on forced-convective heat and reactive solute mass transfer of a steady incompressible, electrically conducting, chemically reacting and Joule dissipating viscous fluid streaming towards a stationary porous planar surface embedded in a saturated non-Darcian porous medium in the presence of surface mass flux, pressure stress-work and velocity slip. The governing coupled nonlinear boundary layer partial differential equations are transformed by existing similarity variables into a set of nonlinear, ordinary differential equations in conjunction with the accompanying boundary conditions and then solved using a shooting quadrature along with fourth order Runge-Kutta integration scheme. The features of the flow, heat and mass transfer characteristics as per the skin friction coefficient, heat and mass transfer rates subject to simulated values of the Darcian number, Forchheimer number, Prandtl number, Eckert number, Schmidt number, reaction rate and order, mass flux (transpiration) and Biot number are analysed and discussed by means of tables while the dimensionless temperature, velocity and concentration profiles are captured through graphs. All these basic flow parameters bear significant influences on the flow.

**Keywords:** Forced convection, higher order chemical reaction, heat and mass transfer, Magnetohydrodynamics (MHD), stress work, non-Darcy model, nonlinear Roseland radiation.
**MSC2010: 76S05, 76W05, 76R05**

## 1 Introduction

Since a system under appreciable pressure change has potential to perform work on its surroundings, transport through or past engineering components such as compressors, turbines, thermal energy collectors/distributors, abrasive blasters, high-performance rotor engines, arterial vasodilators and vasoconstrictors, etc are of greater use in not only technological and industrial processes but also biomedical and surgical purposes amongst others. Convection in general is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. Forced convection takes place when the fluid flow is being driven over the surface by virtue of external means, such as pumps, fans, atmospheric winds, and so forth. Forced convection heat transfer is one of the most commonly employed modes of heat transfer in the processing industries. Cold and hot fluids, separated by baffles, are often pumped through specialized heat transfer equipments such as gas waste heat recovery processors, air cooled exchangers, boilers etc. Quite a number of authors have investigated various aspects of forced convection heat transfer flows over heated or cooled surfaces under various conditions : Chaudhary and Merkin [1], Andersson *et al.* [2], Weidman *et al.* [3], Kechil and Hashim [4], Merkin and Pop [5], Uddin *et al.* [6], to mention but a few among many others.

The systematic study of viscous stress work influence due to laminar convective flow over semi-infinite flat surfaces appears to be initiated by Gebhart [7] and later on Gebhart and Mollendorf [8] investigated the effects of viscous and stress work on the flow model earlier considered. In their investigations, the latter has shown that the viscous dissipation procures significant influence on flow and heat transfer pertinent characteristics incorporating the stress work effect as per several engineering devices subject to reasonably large decelerations or operating under high rotational speeds as well as strong thermal and/or gravitational field and geological processes. Zakerullah [9] tolled in line similar investigation for axially symmetric natural convective flow whilst Ackroyed [10] examined the effects of both viscous and pressure stress work in laminar natural convection on plates and observed that the pressure stress effects are in general, rather more important for fluids undergoing small changes in temperature and pressure. Joshi and Gebhart [11] also observed the effect of pressure stress work and viscous dissipation on natural convection and heat transfer along an isothermal and uniform heat flux for four different vertical flows and concluded that for most practical circumstances, the pressure stress work in its effect is well pronounced as compared with that of viscous dissipation.

All the above-mentioned studies were carried out by means of regular perturbation method, except Pantokratoras [[11],[13]] who analyzed numerically on the previous investigations and found that the variations of wall shear stress and wall heat transfer along the plate quite differed from the results obtained through the use of perturbation method. Considering the heat conduction due to the slab-wall of a finite thickness, Alam *et al.* [14] numerically investigated the effect of pressure stress work and viscous dissipation on free convection flow along a flat plate. Other aspects of the flow as per flow over surfaces with cylindrical geometry and various conditions have been recently considered by Bhuiyan et el. [15] and Eldabe *et al.* [16] who examined and accounted for the influence of variability of the fluid property besides others.

Magnetic field has inherent properties that control the motion of electrically conducting fluids embedded in its fields. Of late, however Islam *et al.* [17] examined via the numerical integration the combined effects of stress work, heat generation and viscous dissipation on MHD natural convection flow over a vertical plate with power law temperature variation. The applications of hydromagnetic incompressible viscous flow in science and engineering involving heat and mass transfer under the influence of chemical reaction are of great importance to many areas of science and engineering. There is an often occurrence in petro-chemical industry, power and cooling systems, chemical vapor and deposition on surfaces, power and cooling of nuclear reactors, heat exchange design, forest fire dynamics and geophysics as well as in magnetohydrodynamic (MHD) power generation and hydrometallurgical systems. Makinde [18] examined theoretically hydromagnetic boundary layer flow, heat and mass transfer with convective heat exchange at surface with the surroundings.

On the control of the quality of penultimate products from industrial processes such as polymer and shingling processing, annealing and thinning of copper wires, oven backed painting, etc. the effects of surface mass flux and thermal radiation play an incredible role in controlling how heat is transferred or exchanged at the surface with the surroundings. The combined influence of magnetic Lorentz forces and thermal radiation on boundary layer flow and heat transfer has received increasingly, very serious attention of quite a number of investigators in the recent past consequent upon the much needed engineering devices such as high-performance space vehicles and produc-

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tion processes in metal and alloy castings, high-performance forced air-cooling in microelectronics. Babu *et al.* [19], considered MHD natural convection laminar flow over a shrinking vertical surface with suction or injection and observed considerable influence of the emerging flow parameters on heat and mass transfer characteristics. Many analytical and numerical studies have been conducted to explain the various aspects of boundary layer flow with heat and mass transfer over flat surfaces using Darcian or non-Darcian models via porous medium drag effects for Newtonian and non-Newtonian fluids.

Thermal radiation as a part of electromagnetic emission of a substance is characterized by heat transfer when the temperature difference exists between two bodies and it is associated with high temperatures. Apart from the convective heat transfer in fluids, thermal radiation proffers considerable effect on these- days engineering innovations and advanced technology such as in geothermal and astrophysical flows, aircraft and spacecraft devices, high temperature industrial processes, next-generation solar film collectors, cryogenic insulation radiation inhibitors and many others. Thusly, many researchers in the recent past have considered the inclusion of thermal radiation in their studies. Radiation effects are salient under the context of space technology and processes involving high temperatures, Ozisik [20], Sparrow and Cess [21] and Adeniyan [22] happen to be among early pioneers to study the interaction of thermal radiation awash in a semi-infinite plate. The influence of thermal radiation on heat and mass transfer of a viscous fluid past a vertical porous plate permeated by a transverse magnetic field has been reported by Makinde and Ogulu [23], Olanrewaju *et al.* [24] using linear radiation model. The problem of heat and mass transfer boundary layer flow of hydromagnetic electrically conducting fluid over a vertical plate in a porous medium with thermal radiation and uniform heat flux was considered by Makinde [25]. Rashidi *et al.* [26] carried out heat and mass transfer study for a two-dimensional flow past a convectively heated stretching plate taking into cognizance the effects of radiation and buoyant forces. Omowaye and Ajayi [27] investigated numerically, using a shooting technique, the combined influence of Rosseland thermal radiation and viscous heating on stagnation flow and heat transfer due to a convectively heated stretching sheet immersed in an electrically conducting fluid awash with a uniform transverse magnetic field. Very recently, Akinbobola and Okoya [28] investigated the non-Newtonian second grade boundary layer flow over a stretching plate due to temperature dependent fluid properties.

In astrophysical regimes, the presence of planetary debris, cosmic dust etc. creates a suspended porous medium saturated with plasma fluids. As in other porous media problems such as geo-mechanics and insulation engineering, the convectional approach is to stimulate the pressure drop across the porous regime using Darcy linear model. Using Peclet based similarity transformation, Salem [29] studied the radiation and mass transfer effects in Darcy-Forchheimer mixed convection from a vertical plate embedded in a fluid-saturated porous medium. Many a researcher has investigated as well, other various aspects of convection flows in various porous media. Rashidi *et al.* [30] via group theory analysis investigated mixed convective chemical reaction, heat and mass transfer characteristics using DTM-Padé approximants technique alongside the shooting scheme. Later on, Makinde [31], Makinde *et al.* [32], Olanrewaju and Adeniyan [33] examined and observed the double diffusive characteristics on the flow, mass and heat transfer. Of late, Hayat *et al.* [34] analyzed using homotopy analysis method (HAM), the heat transfer characteristics of a viscous nanofluid flow induced by a convectively heated shrinking sheet with the surface mass flux while Adeniyan and Aroloye [35] investigated the case of convective heat transfer in an anisitropic porous medium.

A cursory account of studies above unravels that sparse work has been done to date under this intriguing subject in the area of pressure stress work interaction on convective heat transfer after the pioneering studies of Gebhart and his contemporaries in spite of its utmost importance as regards wide range of applications as mentioned. Additionally, all the aforementioned studies incorporating the effect of thermal radiation assumed that the temperature difference between the surface and the ambient stream is small enough as to neglect higher powers of the difference, as seen in Adeniyan [36] and Fatumbi and Fenuga [37]. However, in some circumstances, as seen in the combustion temperatures needed for the launching of rockets to space, such assumption is inappropriate most especially when temperature differences are not small. Of course, the former and the latter form the bases of our motivation for the present communication.

This present study aims at examining the effects of pressure stress work, chemical reaction on MHD stagnation point flow past a horizontal, stationary porous planer surface permeated by a uniform magnetic field which is transversely placed to it and considering Darcy and Forchheimer body forces in the presence of Ohmic dissipation and dual convective boundary conditions. In addition, this model incorporates the Rosseland radiation diffusion effect without the use of linear approximation to Taylor series expansion for the fourth power absolute temperature about the ambient temperature. The authors within their available facilities are convinced that this study is a rear find and completely anew.

## 2 Mathematical Model

Consider a steady two-dimensional forced laminar magnetohydrodynamic boundary layer flow of an n-th order homogeneous and chemically reactive fluid towards a semi-infinite porous plate moving with velocity $U_w(x)$ and permeated by a uniform magnetic field of strength $B_0$ perpendicular to it (see Fig. 1). The porous medium is fully saturated with an incompressible Newtonian fluid of infinite extent, and also the induced magnetic field due to the motion of the electrically conducting fluid is negligible. It is further assumed that the external electrical field is zero and that the electric field due to the polarization of charges is negligible. Assumption is made that both the isothermal wall temperature of the plate and the uniform concentration at the wall $C_w$ (to be later determined) are respectively in consequence to convective heating process and mass transfer process characterized by a temperature $T_f$ and solute concentration $C_s$ with accompanying respective heat and mass transfer coefficients $h_f$ and $h_s$. In addition, the porous plate allows slippage velocity $U_s$, where $T_f > T_w$ and $C_f > C_w$. The chemical reaction processes take place under isothermal and uniform solute concentration conditions. Under the usual boundary layer and constant fluid property approximations in conjunction with all the pre-stated premises, the dimensional form of continuity, momentum, energy and solute diffusion equations describing the non-Darcian flow can be written as (pressure stress term: Hughes and Gaylord [39], Jaluria [40]) follows

Under the stated assumptions and the boundary layer approximations, the governing boundary layer continuity, momentum, microrotation and energy are respectively given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\rho\left(\frac{\partial u^2}{\partial x} + \frac{\partial (uv)}{\partial y}\right) = u_\infty \frac{du_\infty}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \sigma_e B_0^2 \left(u - u_\infty\right) - \frac{\rho}{K_p}\left[\gamma + b\left(u + u_\infty\right)\right]\left(u - u_\infty\right), \tag{2.2}$$

$$\rho C_p\left(\frac{\partial (uT)}{\partial x} + \frac{\partial (vT)}{\partial y}\right) = T\beta\left(\frac{\partial (uP)}{\partial x} + \frac{\partial (vP)}{\partial y}\right) + k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \sigma_e B_0^2\left(u - u_\infty\right)^2, \tag{2.3}$$

$$\left(\frac{\partial (uC)}{\partial x} + \frac{\partial (vC)}{\partial y}\right) = Dm\frac{\partial^2 C}{\partial y^2} - K_r\left(C - C_\infty\right)^n, \tag{2.4}$$

for which the free stream condition proffer the pressure gradient as

$$\frac{1}{\rho}\frac{dp}{dx} = u_\infty\frac{du_\infty}{dx} + \left(\frac{\sigma_e B_0^2}{\rho} + \frac{\gamma}{K_P}\right)u_\infty + \frac{b}{K_p}u_\infty^2 \tag{2.5}$$

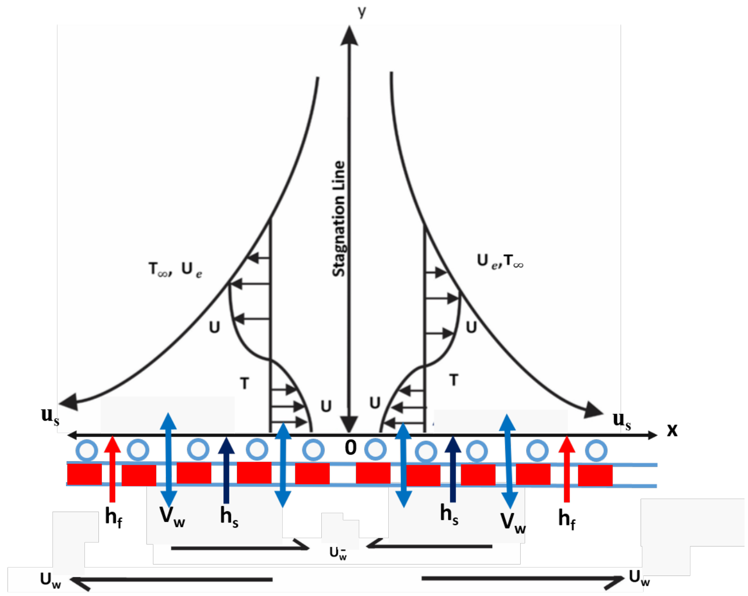


Figure 1: Physical model and coordinate system.

and the transverse gradient of radiative heat transfer is posited by Rosseland approximation:

$$\frac{4\sigma^\star}{3k^\star}\frac{\partial T^4}{\partial y} = -q_r, \tag{2.6}$$

where $u(x, y)$ and $\upsilon(x, y)$ are non-Darcian velocity components along and normal to the plate respectively. $T(x, y)$ is the temperature field, $\beta$ is fluid thermal expansivity, $\mu$ is dynamic viscosity, $K_r$ is the reaction rate constant of the *n-th* order homogeneous and irreversible reaction, n is the order of the chemical reaction, $\nu$ is the kinematic viscosity, a is the thermal diffusivity, $C_p$ is the specific heat capacity at constant pressure, $K_p$ is the porous medium permeability coefficient, $b$ is a positive empirical constant, $P$ is the fluid pressure, $\rho$ is the fluid density, $\gamma = \mu/\rho$ is the kinematic viscosity, $k$ is the thermal conductivity, $\sigma_e$ is the electrical conductivity of the fluid, $\boldsymbol{B} = (0, B_0)$ ,is the imposed magnetic field, $D_m$ is the solute concentration diffusivity wherein the hydrodynamic pressure of the fluid has been modeled as $P = p_0 - \frac{1}{2}\rho a^2 x^2$, and in consequence the characteristic pressure $p_0$ may be taken as that at the leading edge. The first term on the right of eq. (3) indicates the pressure stress work, and it is easily verifiable that $\beta T = 1$ for a calorically perfect gas convective flow.

The boundary conditions for the above partial differential equations (PDEs) $(1 - 4)$ are posited as

$$\begin{gathered} u\left(x,0\right) = \sigma u_w + u_s,\;\; \upsilon\left(x,0\right) = V_w(x),\; u\left(x,\infty\right) = u_\infty = ax,\;\; T\left(x,\infty\right) = T_\infty,\quad C\left(x,\infty\right) = C_\infty \\ -k\frac{\partial T}{\partial y}\left(x,0\right) = h_f\left(T_f - T\left(x,0\right)\right),\; -D_m\frac{\partial C}{\partial y}(x,0) = h_S\left(C_S - C(x,0)\right), \end{gathered} \tag{2.7}$$

where $\sigma$ is stretching or shrinking parameter, $a > 0,\ b \geq 0$ are strains (inverse time constants) capturing the wall speed $u_w = ax$ and stream ambient speed $u_\infty = bx$ ($x$ is a distance along the plate measured from leading edge, O), $u_S = N\frac{\partial u}{\partial y}$ is the slip velocity, $V_w\ (x)$is surface mass flux (i.e. dimensional suction or injection). It may be noted here that $V_w < 0$ signifies suction and $V_w > 0$ signifies injection and the velocity slip length scale is N. The ambient temperature and solute concentration are respectively $T_\infty (< T_W)$ and $C_\infty (< C_W)$.

The stream function $\psi(x, y)$, satisfies the continuity equation (1) automatically with Cauchy-Riemann conditions:

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x}. \tag{2.8}$$

In a bid to simplify the mathematical analysis of the problem, and under the assumption that the temperature and concentration differences may be small but without being negligible, we introduce the following non-similar variables:

$$\eta = y\sqrt{\frac{a}{\gamma}},\quad \psi\left(x, y\right) = x\sqrt{a\gamma} f\left(\eta\right),\ \theta\left(\eta\right) = \frac{T - T_\infty}{T_f - T_\infty},\ \phi\left(\eta\right) = \frac{C - C_\infty}{C_f - C_\infty}, \tag{2.9}$$

where $F_W > 0$ expresses suction whereas $F_W < 0$ expresses injection. Substituting eq. (9) into eqs. (2-7), we obtain:

$$f''' + \ ff'' - \ (f')^2 - (M + Da)\left(f' - \lambda\right) - DaF_s\left(\left(f'\right)^2 - \lambda^2\right) + \lambda^2 = \ 0, \tag{2.10}$$

$$\frac{1}{\Pr}\left\{\left(1 + \frac{4}{3}Rd\left[1 + \left(\theta_R - 1\right)\theta\right]^3\right)\theta'\right\}' + f\theta' - \in\left(\frac{1}{\theta_R - 1} + \theta\right)f' + MEc\left(f' - \lambda\right)^2 = 0, \tag{2.11}$$

$$\frac{1}{Sc}\phi'' + f\phi' - K\phi^n = 0, \tag{2.12}$$

The boundary conditions become

$$\begin{gathered} f(0) = F_W,\ f'(0) = \sigma + \varsigma f''(0),\ f'(\infty) = \lambda,\ \ \theta(\infty) = 0,\ \phi(\infty) = 0, \\ -\ \theta'\left(0\right) = Bi\left(\ 1\ -\ \theta\left(0\right)\right),\ -\phi'(0) = Bs\left(1 - \phi(0)\right), \end{gathered} \tag{2.13}$$

where $\theta_R \geq 2$ features strong nonlinear radiation effect Gebhart [7], the prime symbolizes the derivative with respect to $\eta$, $\sigma$ is stretching/shrinking parameter and the following :

$$\left.\begin{array}{cccc} \theta_R = \frac{T_f}{T_\infty}, & Sc = \frac{\gamma}{D_m}\ , & \Pr = \frac{\gamma}{\alpha}, & M = \frac{\sigma_e B_0^2}{\rho a}, \\ \in = \frac{\beta u_\infty^2}{C_P}, & Bi = \frac{h_f}{k}\sqrt{\frac{\gamma}{a}}, & Bs = \frac{h_S}{D_m}\sqrt{\frac{\gamma}{a}}, & Da = \frac{\gamma}{k_P}, \\ \varsigma = N\sqrt{\frac{a}{\gamma}}, & F_S = \frac{bu_w}{\gamma}, & K = \frac{K_r}{a}\left[\left(C_f - C\infty\right)\phi\right]^{n-1}, & Ec = \frac{u_\infty^2}{C_P(T_f - T_\infty)}, \\ F_W = -\frac{V_W}{\sqrt{a\gamma}}, & Rd = 4\frac{\sigma^* T_\infty^3}{kk^*}, & \lambda = \frac{b}{a}, & \sigma^*,\ k^*, \end{array}\right\} \tag{2.14}$$

which are respectively the temperature ratio, Schmidt number, Prandtl number, Magnetic parameter, stress work parameter(or Gebhart number), heat transfer Biot number, mass transfer Biot number, Darcy number, slip parameter, Forchheimer number, chemical reaction parameter, Eckert number, wall mass flux parameter, radiation parameter, velocity ratio, Stefan-Boltzmann constant and Rosseland mean absorption coefficient. In addition, the chemical reaction order $n = 1, 2, 3$. It is noteworthy to mention here that use is made of moderate Prandtl and Schmidt numbers.
The reduced forms of local physical quantities of pertinent interest, in terms of the ambient velocity based local Reynolds number $Re_x = u_w x/\gamma$ , are :

$$\left.\begin{array}{c} Cfr = f''(0), \quad Shr = -\phi'(0) = B_S\left(1 - \phi(0)\right) \\ Nur = -\left(1 + \frac{4}{3}Rd\left[1 + \left(\theta_R - 1\right)\theta(0)\right]^3\right)\theta'(0) = \left(1 + \frac{4}{3}Rd\left[1 + \left(\theta_R - 1\right)\theta(0)\right]^3\right)B_i\left(1 - \theta(0)\right) \end{array}\right\} \tag{2.15}$$

respectively for skin-friction coefficient, Sherwood and Nusselt numbers. Where

$$Cfr = C_f Re_x^{\frac{1}{2}}, Shr = Sh_x Re_x^{-,\frac{1}{2}}, Nur = Nu_x Re_x^{-,\frac{1}{2}} \tag{2.16}$$

# 3 Numerical simulation

The set of equations $10 - 12$ under the boundary conditions in equation (13) have been solved numerically using a shooting algorithm with a fourth order Runge-Kutta integration scheme. A systematic guessing of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ have been accessed until upstream (far-field) boundary conditions are gotten asymptotically. Use has been made of the step-size $\Delta\eta = 10^{-3}$ as per the numerical computations ensuring the accuracy up to seventh decimal place $(10^{-7})$ and found to be sufficient for convergence criterion. Based on this technique, a finite value $\eta_\infty$ has been adopted in place of $\eta\ to\infty$ which depends on the parametric values. Also, the computations are carried out by a program coded in a symbolic and computational computer language, Maple-18. From the process of numerical computations, the reduced skin friction coefficient, Nusselt number and Sherwood number, which are defined in eq. (15) are all sorted out and their numerical values are presented in a tabular form. The accuracy of this numerical codes has been attested by direct comparison with the numerical results reported by Grubka and Bobba [40] as indicated in Tables 1, which agree exactly to four decimal places and where parameter large value $\infty = 2000$. Also the computed skin-friction parameter $f''(0) = -1.000008$ agrees with those obtained by Crane [41] correct to four decimal places.

Table 1: Comparison of the values of the local Nusselt number $-\theta'(0)$, with Grubka & Bobba [41] in the absence of concentration when $\sigma = 1,\ \theta_R \to \infty,\ Bi \to \infty,\ Bs \to \infty$ and$M = Da = Fs = Rd = \lambda = \varsigma = 0$

| Flow parameters with different values | | $-\theta'(0)$ | |
|---|---|---|---|
| Pr | $\in$ | Grubka & Bobba [40] | Present Study |
| 0.72 | -3 | -2.2500 | -2.2349 |
| | -2 | -0.7200 | -0.7166 |
| | -1 | 0.0000 | 0.0011 |
| | 0 | 0.4631 | 0.4635 |
| | 1 | 0.8086 | 0.8088 |
| | 2 | 1.0885 | 1.0886 |
| | 3 | 1.3270 | 1.3270 |
| 10 | -3 | 5.0000 | 5.0001 |
| | -2 | 10.0000 | 9.9998 |
| | -1 | 0.0000 | 0.0000 |
| | 0 | 2.3080 | 2.3080 |
| | 1 | 3.7207 | 3.7206 |
| | 2 | 4.7969 | 4.7968 |
| | 3 | 5.6934 | 5.6933 |

# 4 Results and discussion

The numerical results are demonstrated in Tables 2-4 and plotted in Figures 2(a)-1(a). In order to have a greater insight into the qualitative analysis of the results, we have taken the values of various controlling flow parameters as $0.72(\text{Air}) \leq Pr \leq 7.10$ (water), $0.24\ (H_2) = Sc =$

0.78 $(NH_3)$ , $0.1 = Bi, Bs = 1.0$, $0.1 = M = 5.0$, $0.1 = K = 1.0$, $0.5 = Ec = 3.0$, $-0.5 = F_w = 5.0$, $0.1 = Da = 3.0$, $0.1 < \varsigma < 0.5$, $0.1 < Ra < 1.0$, $3.0 < \theta_R < 5.0$, $0.01 = F_s = 1.0$ , $1.0 = n = 5.0$, $\lambda = 1$, $0 \leq \in < 0.1$ and $\sigma = 0$ *or* 1.
Tables 2-4 illustrate the effects of magneto-thermophysical parameters on the skin friction coefficient, wall temperature, Nusselt number, wall concentration and Sherwood number. Table 2 demonstrates that the Lorentz force magnitude magnification leads to increase in the skin friction at the surface. Appreciable levels of increment are also observable in the level of friction drag within the flow system as the parameter of porousity, $Da$ is increased. Nonetheless, reverse is the case for Forchheimer force and the velocity slip parameter. In Table 3, we observe that the rate at which heat is transferred within the flow system increases with increasing values of Prandtl number, stress work parameter and Eckert number. The heat transfer Biot number also can increase the rate of heat transfer due to the convective heat exchange at the surface of the plate. However, a contrary phenomenon has been observed in the rate of heat transfer for increasing values of the radiation parameter and temperature ratio. As unveiled in Table 4, the temperature at the plate surface, $\theta(0)$ considerably increases with increasing values of $Ra$, $M, Bi$ and $\theta_R$ while the Prandtl and Eckert numbers are both seen to douse it. As unveiled in Table 4, the temperature at the plate surface, $\theta(0)$ considerably increases with increasing values of $Ra$, $M, Bi$ and $\theta_R$ while the Prandtl and Eckert numbers are both seen to douse it. A noteworthy observation in Table 5 indicates that the surface mass transfer rate, $-\phi'(0)$ is increased when there is a rise in the values of $Sc, K$ and solutal Biot number, $B_s$ but decreases with increasing order of the chemical reaction, n. As observed the concentration at the surface rises up with an increase in $n$ and $B_s$ but downsizes with increase in $Sc$ and $K$.

**Table 2**: Values of Skin-friction coefficient $f''(0)$ for various values of controlling parameters

| $Da$ | $M$ | $F_s$ | $\varsigma$ | $F_w$ | $f''(0)$ |
|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 1.28618324 |
| 0.5 | | | | | 1.39070886 |
| 1 | | | | | 1.50622176 |
| 0.1 | 0.5 | | | | 1.39126483 |
| | 1 | | | | 1.50731079 |
| | 2 | | | | 1.70391256 |
| | 0.1 | 0.02 | | | 1.28602473 |
| | | 0.04 | | | 1.2857076 |
| | | 0.05 | | | 1.28554898 |
| | | 0.01 | 0.2 | | 1.16219335 |
| | | | 0.4 | | 0.96377802 |
| | | | 0.8 | | 0.70742984 |
| | | | 0.1 | -0.2 | 1.10652522 |
| | | | | -0.1 | 1.15009957 |
| | | | | 0 | 1.19461492 |
| | | | | 0.2 | 1.28618324 |
| | | | | 0.5 | 1.42879879 |

**Table 3**: Values of Nusselt number and plate surface temperature for various values of controlling parameters

| $Pr$ | $Ra$ | $M$ | $Ec$ | $\epsilon$ | $Bi$ | $\theta_R$ | $F_w$ | $\theta(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.72 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.10892239 | 0.08910777 |
| 2 | | | | | | | | 0.03552247 | 0.09644776 |
| 7.1 | | | | | | | | -0.03399759 | 0.10339976 |
| 0.72 | 0.2 | | | | | | | 0.12096709 | 0.0879033 |
| | 0.5 | | | | | | | 0.15681321 | 0.08431868 |
| | 0.7 | | | | | | | 0.17846944 | 0.08215306 |
| | 0.1 | 0.5 | | | | | | 0.04260051 | 0.09573995 |
| | | 1.5 | | | | | | -0.09753505 | 0.10975351 |
| | | 2 | | | | | | -0.15519996 | 0.11552 |
| | | 0.1 | 0.2 | | | | | 0.11915652 | 0.08808435 |
| | | | 0.5 | | | | | 0.10796311 | 0.08920369 |
| | | | 0.8 | | | | | 0.09674126 | 0.09032588 |
| | | | 0.5 | 0.02 | | | | 0.08466237 | 0.09153377 |
| | | | | 0.05 | | | | 0.01839172 | 0.09816083 |
| | | | | 0.09 | | | | 0.06198859 | 0.10619886 |
| | | | | 0.01 | 0.2 | | | 0.23157665 | 0.15368468 |
| | | | | | 0.3 | | | 0.33077678 | 0.20076697 |
| | | | | | 0.4 | | | 0.4110276 | 0.23558897 |
| | | | | | 0.1 | 3 | | 0.12578384 | 0.08742162 |
| | | | | | | 4 | | 0.13864674 | 0.08613533 |
| | | | | | | 5 | | 0.15394598 | 0.08460541 |
| | | | | | | 2 | -0.2 | 0.14787104 | 0.0852129 |
| | | | | | | | -0.1 | 0.13677499 | 0.08632251 |
| | | | | | | | 0 | 0.12681678 | 0.08731833 |
| | | | | | | | 0.2 | 0.10982149 | 0.08901786 |
| | | | | | | | 0.5 | 0.09008745 | 0.09099126 |

## 4.1 Effects of Parameter Variation on Velocity Profiles

The illustrations of the velocity profiles with respect to the transverse distance are displayed in Figs. 2(a)-4(b). Generally, the fluid velocity is lowest at the plate surface and increases gradually to its free stream values asymptotically satisfying the far field boundary condition. For the set of simulated parameter values, it is interesting to note that the far-field boundary conditions are significant at dimensionless transverse distance $\eta = 3$ in all cases. In Figs. 1(a) & 1(b), however, the effects of increasing the magnetic field strength and porous medium permeability on the momentum boundary-layer thickness are demonstrated. Previous studies on boundary-layer flow past a stretching surface incorporating basic parameters devoid of stress work, establish fervently that the magnetic field and porous medium characteristics proffer a damping influence on the velocity field due to drag-like forces that oppose the fluid motion, causing the velocity to decrease. This study unveils very different results. Two different Maple codes written, however,

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**Table 4**: Values of Sherwood Number and plate surface concentration for various values of controlling parameters

| Sc | Bs | K | n | Fw | $\phi(0)$ | $-\phi(0)$ |
|---|---|---|---|---|---|---|
| 0.24 | 0.1 | 0.2 | 1 | 0.1 | 0.1924804 | 0.080752 |
| 0.62 | | | | | 0.1295063 | 0.0870494 |
| 2.64 | | | | | 0.0643598 | 0.093564 |
| 0.24 | 0.2 | | | | 0.4169369 | 0.1749189 |
| | 0.5 | | | | 0.5437541 | 0.228123 |
| | 0.7 | | | | 0.6252605 | 0.2623177 |
| | 0.1 | 0.3 | | | 0.1834967 | 0.0816503 |
| | | 0.4 | | | 0.1756425 | 0.0824358 |
| | | 0.5 | | | 0.168705 | 0.0831295 |
| | | 0.2 | 1 | | 0.1924804 | 0.080752 |
| | | | 2 | | 0.2114407 | 0.0788559 |
| | | | 3 | | 0.2144726 | 0.0785527 |
| | | | 1 | -0.2 | 0.2189409 | 0.0781059 |
| | | | | -0.1 | 0.2118532 | 0.0788147 |
| | | | | 0 | 0.2050898 | 0.079491 |
| | | | | 0.2 | 0.1924801 | 0.080752 |
| | | | | 0.5 | 0.1756496 | 0.0824351 |

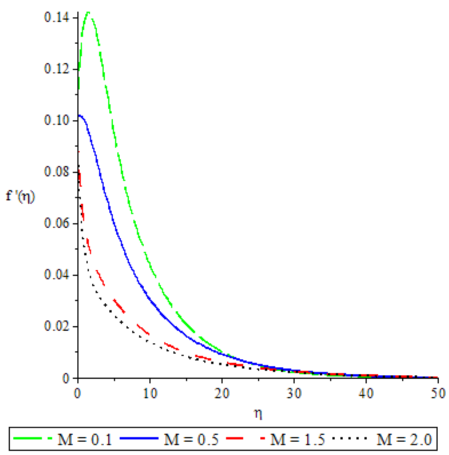
confirm that the fluid can be accelerated due to increase in both M and Da in the presence of the stress work. A reverse trend is observed with an increase in slip parameter as seen in Fig. 4(a). An increase in fluid suction ($F_w > 0$) retards the rate of transport and reduces the boundary layer thickness. An opposite trend features for fluid injection ($F_w < 0$) as seen in Fig 4.

## 4.2 Effects of Parameter Variation on Temperature Profiles

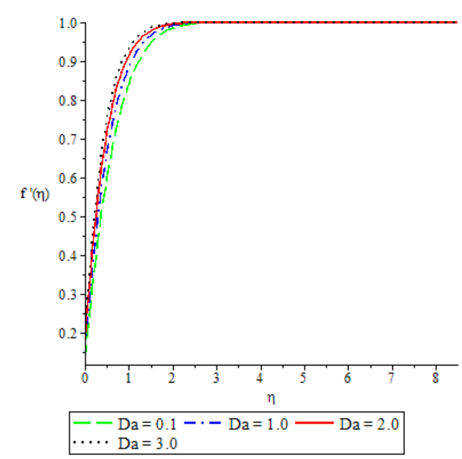
Figs. 4(b)-8(b) depict the influence of the controlling flow parameters on the temperature profile. It is noteworthy that increase in heat transfer Biot number makes the fluid temperature to rise as a result of the convective heat exchange between the cold fluid at the upper surface of the plate and the hot fluid at the lower surface of the plate. The temperature boundary layer also gets thicker. The same is observed for the radiation parameter. However, high values of Prandtl number creates a retardation in the convective movement of fluid particles and a lower thermal diffusivity as a direct consequence; a rapid fall in fluid temperature. The magnetic (M) and porous medium permeability (Da) parameters however, feature uncommon effects on the dimensionless fluid temperature due to the presence of the stress work characteristics. Strengthening either of M or Da leads to decimation of the temperature field. Also as revealed in the temperature plots, increase in viscous dissipation parameter (Ec) and stress work parameter ($\in$) all exhibit opposite trends for the temperature distribution. Lastly, fluid suction retards the rate of thermal diffusion and the thermal boundary layer gets thinner while the opposite trend features for increasing the fluid injection.

## 4.3 Effects of Parameter Variation on Concentration Profiles

Figs. (7(a)-10(a)) depict chemical species concentration profiles against dimensionless span-wise coordinate $\eta$ for varying values of basic physical parameters in the boundary layer. It is observed that an increase in fluid suction, Schmidt number and reaction rate parameter cause a decrease in the chemical species concentration within the boundary layer leading to a decaying concentration boundary layer thickness. Reverse is the case for the mass transfer Biot number ($Bs$)and order of chemical reaction (n). They both make the solute boundary layer thicken.

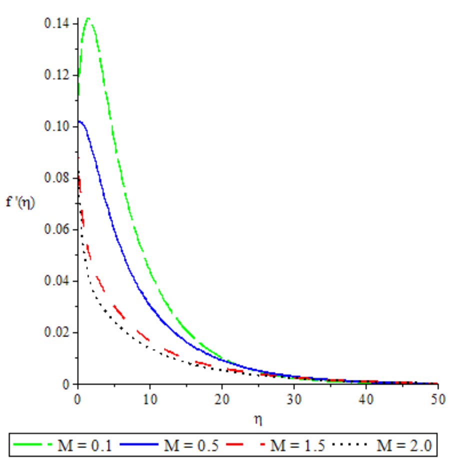



(a) Effect of magnetic parameter on the velocity profiles of the fluid

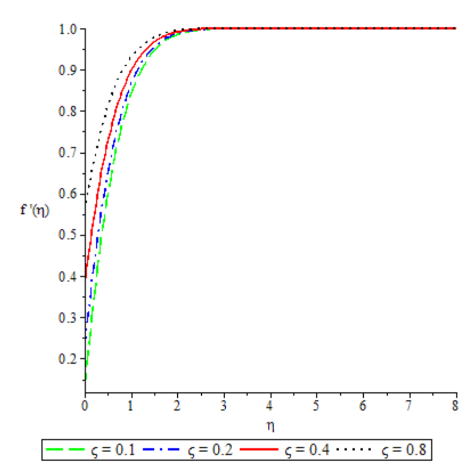



(b) Effect of porousity parameter on the velocity profiles of the fluid

Figure 2




(a) Effect of suction/injection parameter on the velocity profiles of the fluid




(b) Effect of slip parameter on the velocity profiles of the fluid

Figure 3

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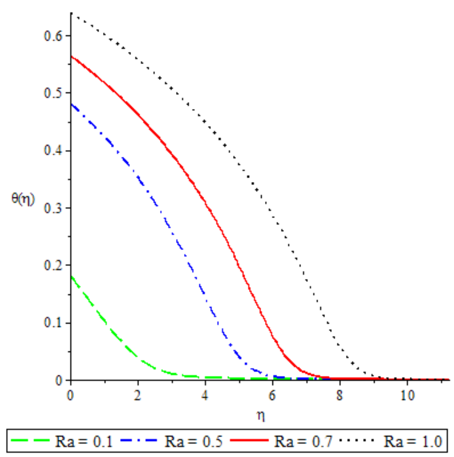
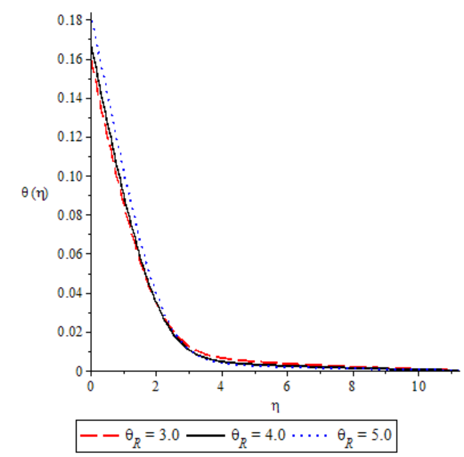
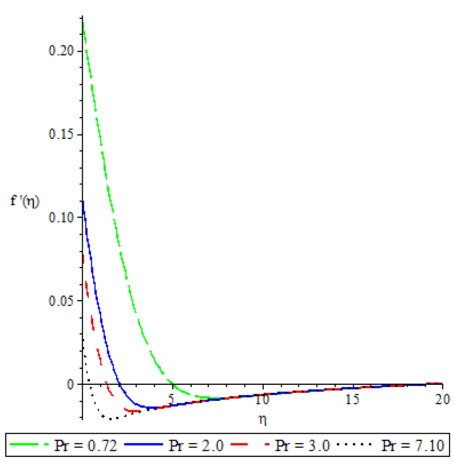



(a) Effect of radiation parameter on the temperature profiles of the fluid

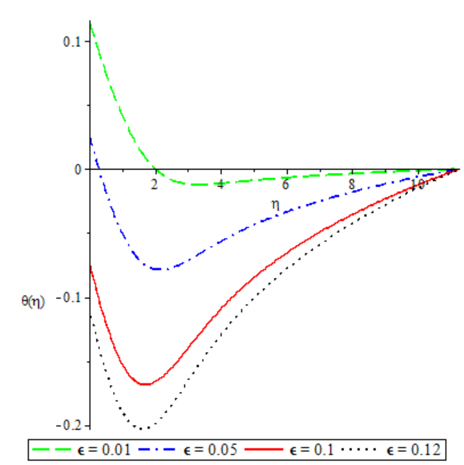



(b) Effect of Temperature ratio on the temperature profiles of the fluid

Figure 4

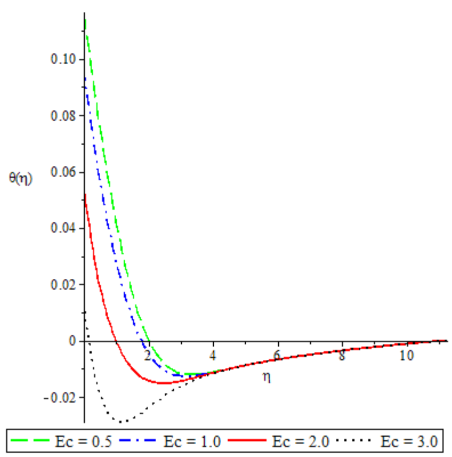



(a) Effect of Prandtl number parameter on the temperature profiles of the fluid

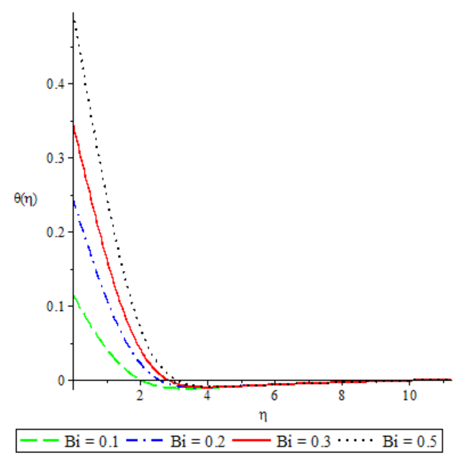



(b) Effect of stress work parameter on the temperature profiles of the fluid

Figure 5

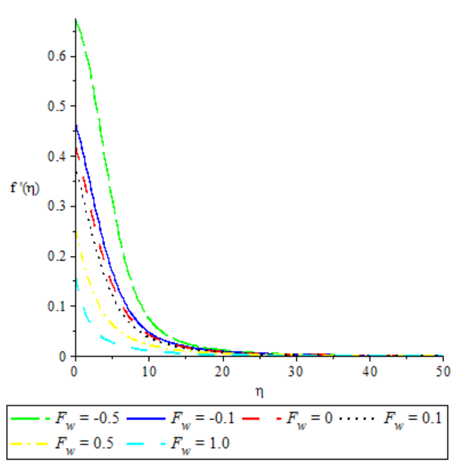



(a) Effect of Eckert number parameter on the temperature profiles of the fluid

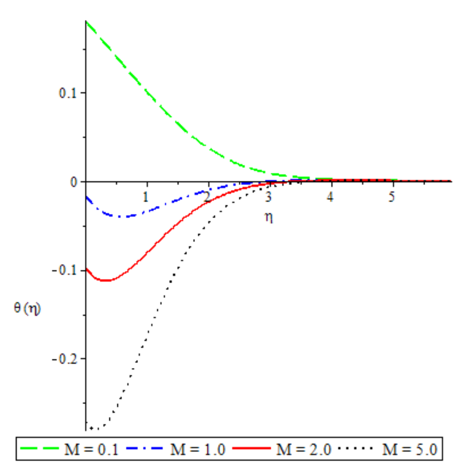



(b) Effect of Biot number parameter on the temperature profiles of the fluid

Figure 6

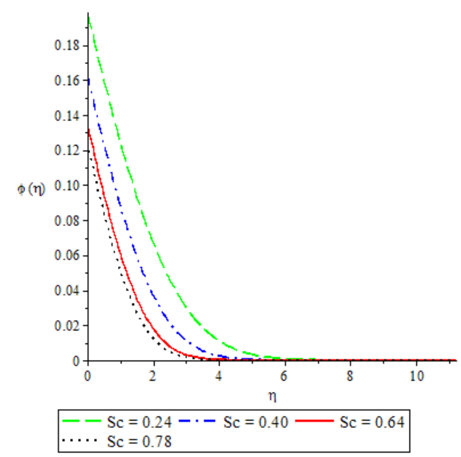



(a) Effect of suction/injection parameter on the temperature profiles of the fluid

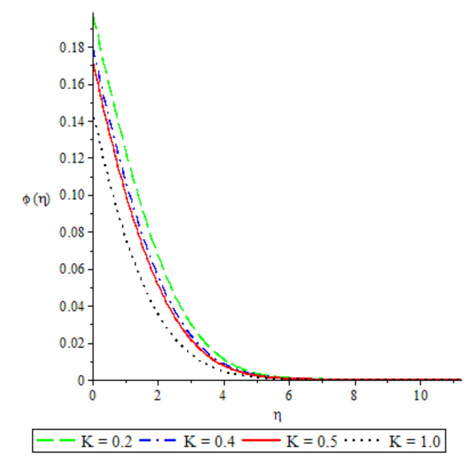



(b) Effect of magnetic parameter on the temperature profiles of the fluid

Figure 7

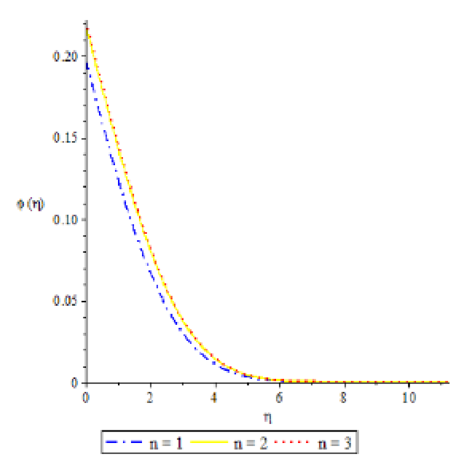



(a) Effect of schmidt number parameter on the concentration profiles of the fluid

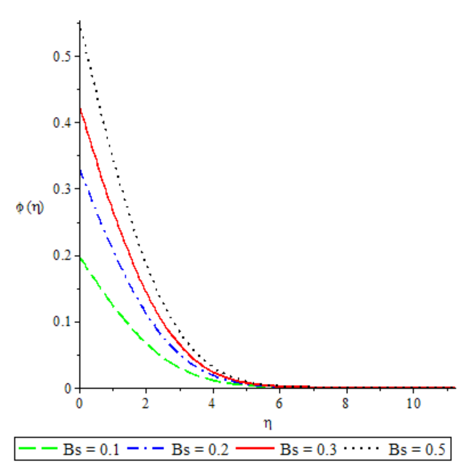



(b) Effect of reaction rate constant parameter on the concentration profiles of the fluid

Figure 8

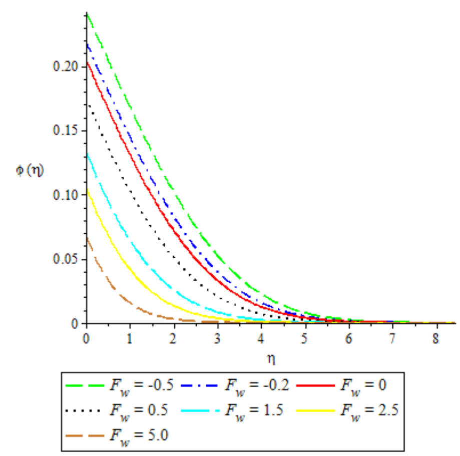



(a) Effect of order of chemical reaction parameter on the concentration profiles of the fluid




(b) Effect of mass transfer biot number parameter on the concentration profiles of the fluid

Figure 9




(a) Effect of suction/injection parameter on the concentration profiles of the fluid

Figure 10

## 5 Conclusions

The effects of pressure stress-work, velocity slip, Darcy and Forchheimer forces along with suction/injection mass flux on a stagnation flow of a fluid which is chemically irreversibly reacting and moving over a porous plate have been studied quantitatively. A systematic study of the effects of various parameters on the flow, heat and mass transfer characteristics is carried out and presented graphically to illustrate the special features of the similarity solutions. The particular conclusions drawn from the present study are as listed:

1. Increasing viscous dissipation and pressure stress-work leads to a decrease of plate surface temperature and an increase of heat transfer rate.
2. Radiation parameter boosts not only the fluid motion but also the surface wall temperature.
3. Mass transfer and heat transfer Biot numbers cause a rise in the level of species concentration and fluid temperature within the boundary-layer flow regime.
4. Suction stabilizes both velocity and thermal boundary-layer growth and may serve as to delay boundary layer separation.
5. Intensification of both the magnetic and porous medium permeability parameters respectively leads to accelerating the fluid flow and decimating the fluid temperature.

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## Competing financial interests

The author(s) declare no competing financial interests.

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