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Fixed Point of Interpolative Contraction on Metric Space Endowed with Graph

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Abstract

In this manuscript, a new concept of interpolative contraction, namely interpolative Kannan-type (G-α-μ)-contraction is introduced in metric space endowed with a graph and novel conditions for which the new mapping is a Picard operator are investigated. The preeminence of this type of contraction is that it complements and subsumes a few corresponding notions in the literature. Substantial examples are constructed to validate the assumptions of our obtained results and to show their distinction from the existing ones.

Keywords: Metric space, fixed point, Kannan-type contraction, interpolation, connected graph, Picard operator.
MSC2010: 47H10; 54H25; 46T99; 46N40; 05C40.

1 Introduction and Preliminaries

The prominent Banach contraction in metric space has laid a solid foundation for fixed point theory in metric space. The applications of fixed point range across inequalities, approximation theory, optimization and so on. Researchers in this area have introduced several new concepts in metric space and obtained a great deal of fixed point results for linear and nonlinear contractions (see, e.g. [1–6]). Recently, Karapınar [7] introduced a new notion of interpolative contraction which is an extension of the famous Kannan contraction in metric space in the following manner.

Definition 1.1. *[7] Let (X, d) be a metric space. A self-mapping $T : X \longrightarrow X$ is called an interpolative Kannan-type contraction if there exist $\mu \in [0, 1)$ and $\alpha \in (0, 1)$ such that*

$$d(Tx, Ty) \leq \mu d(x, Tx)^{\alpha} \cdot d(y, Ty)^{1-\alpha} \tag{1.1}$$

for all $x, y \in X$ with $x \neq Tx$.

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Theorem 1.2. *[7]. Let (X, d) be a complete metric space and let $T : X \longrightarrow X$ be an interpolative Kannan-type contraction. Then T has a unique fixed point in X.*

However, [8] observed that the fixed point obtained in the above Theorem 1.2 is not necessarily unique. Hence, a robust version of the results in [7] is provided therein.
For some extensions of the idea of interpolative contractions in fixed point theory, we refer to [9,10] and the references therein.

Following Petruşel and Rus [11], a self-mapping T of a metric space (X, d) is said to be a Picard operator (abbr., PO) if T has a unique fixed point x^* and $\lim_{n\to\infty} T^n x = x^*$ for all $x \in X$ and T is said to be a weakly Picard operator (abbr. WPO) if the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges, for all $x \in X$ and the limit (which may depend on x) is a fixed point of T.

Jachymski [12] introduced the notion of contraction in metric space endowed with a graph G. Accordingly, let (X, d) be a metric space and let Δ denote the diagonal of the Cartesian product $X \times X$. Consider a directed graph G such that the set $V(G)$ of its vertices coincides with X, and the set $E(G)$ of its edges contains all loops, i.e., $E(G) \supseteq \Delta$. It is assumed that G has no parallel edges, so G can be identified with the pair $(V(G), E(G))$. Moreover, G may be treated as a weighted graph (see [[13], p. 376]) by assigning to each edge the distance between its vertices. Denote by G^{-1}, the conversion of a graph G, i.e., the graph obtained from G by reversing the direction of edges. Therefore,

$$E(G^{-1}) = \{(x, y) \in X \times X | (y, x) \in E(G)\}.$$

The letter $\tilde{G}$ denotes the undirected graph obtained from G by ignoring the direction of edges, or more conveniently, by treating G as a directed graph for which the set of its edges is symmetric. Under this convention,

$$E(\tilde{G}) = E(G) \cup E(G^{-1}). \tag{1.2}$$

The pair (V', E') is said to be a subgraph of G if $V' \subseteq V(G)$, $E' \subseteq E(G)$ and for any edge $(x, y) \in E'$, $x, y \in V'$. If x and y are vertices in a graph G, then a path in G from x to y of length $N \in \mathbb{N}$ is a sequence $\{x_i\}_{i=0}^N$ of $N+1$ vertices such that $x_0 = x$, $x_N = y$ and $(x_{n-1}, x_n) \in E(G)$ for all $i = 1, 2, ..., N$. A graph G is connected if there is a path between any two vertices. G is weakly connected if $\tilde{G}$ is connected.

Subsequently, fixed point results for Lipschitzian-type contractions in metric spaces endowed with graph have been obtained by several authors (see, e.g. [14–20]). In particular, Bojor [20] obtained the following result.

Definition 1.3. *[20] Let (X, d) be a metric space endowed with a graph G. A self-mapping $T : X \longrightarrow X$ is called a G-Kannan mapping if:*

(i) $\forall x, y \in X\big((x, y) \in E(G) \Rightarrow (Tx, Ty) \in E(G)\big)$*;*

(ii) there exists $\mu \in \left[0, \frac{1}{2}\right)$ *such that*

$$d(Tx, Ty) \leq \mu[d(x, Tx) + d(y, Ty)] \tag{1.3}$$

for all $(x, y) \in E(G)$.

Definition 1.4. *[20] Let (X, d) be a metric space endowed with a graph G and $T : X \longrightarrow X$ be a self-mapping. Then G is said to be T-connected if for all vertices x, y of G with $(x, y) \notin E(G)$, there exists a path in G, $\{x_i\}_{i=0}^N$ from x to y such that $x_0 = x$, $x_N = y$ and $(x_i, Tx_i) \in E(G)$ for all $i = 1, 2, ..., N-1$. A graph G is weakly T-connected if $\tilde{G}$ is T-connected.*

Theorem 1.5. *[20] Let (X, d) be a complete metric space endowed with a graph G and $T : X \longrightarrow X$ be a G-Kannan mapping. Assume further that:*

(i) G is weakly T-connected;

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(ii) for any sequence $\{x_n\}_{n\in\mathbb{N}}$ *in* X*, if* $x_n \longrightarrow x$ *and* $(x_n, x_{n+1}) \in E(G)$ *for* $n \in \mathbb{N}$*, then there is a subsequence* $\{x_{k_n}\}_{k\in\mathbb{N}}$ *with* $(x_{k_n}, x) \in E(G)$ *for* $n \in \mathbb{N}$*.*

Then T *is a PO.*

Following the existing literature, we realize that fixed point results for interpolative contractions in metric space endowed with graph have not been adequately investigated. Hence, motivated by the ideas in [7, 12, 19], we introduce a new concept of interpolative Kannan-type (G-α-μ)-contraction in metric space equipped with graph and prove some related fixed point results. Comparative examples are constructed to demonstrate that our obtained results are valid and distinct from the existing results in the literature.

2 Main Results

In this section, we introduce the notion of interpolative Kannan-type (G-α-μ)-contraction in metric space endowed with a graph G.

Definition 2.1. *Let* (X, d) *be a metric space endowed with a graph* G*. A self-mapping* $T : X \longrightarrow X$ *is called an interpolative Kannan-type* (G-α-μ)*-contraction if:*

(i) $\forall x, y \in X\big((x, y) \in E(\tilde{G}) \Rightarrow (Tx, Ty) \in E(\tilde{G})\big)$*;*

(ii) there exist $\mu, \alpha \in (0, 1)$ *such that*

$$d(Tx, Ty) \leq \mu d(x, Tx)^{\alpha} \cdot d(y, Ty)^{1-\alpha} \tag{2.1}$$

for all $(x, y) \in E(\tilde{G})$ *with* $x \neq Tx$*.*

Example 2.2. *Let* $X = \{1, 2, 3, 4\}$ *with the Euclidean metric* $d(x, y) = |x - y|$ $\forall x, y \in X$*. Define* $T : X \longrightarrow X$ *by*

$$Tx = \begin{cases} 2x, & \text{if } x = 1; \\ x, & \text{if } x = 2; \\ 1, & \text{if } x = 3; \\ \frac{x}{2}, & \text{if } x = 4. \end{cases}$$

Then T *is an interpolative Kannan-type* (G-α-μ)*-contraction with* $\mu = \frac{4}{5}$ *and* $\alpha = \frac{1}{5}$*, where the graph* G *is defined by* $V(G) = X$ *and*

$$E(G) = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 4), (3, 4), (4, 4)\},$$

but T *is not an interpolative Kannan-type contraction defined in [7], since* $d(T3, T1) = 1$ *while* $\frac{4}{5}d(3, T3)^{\frac{1}{5}} \cdot d(1, T1)^{1-\frac{1}{5}} = \frac{23}{25}$*.*

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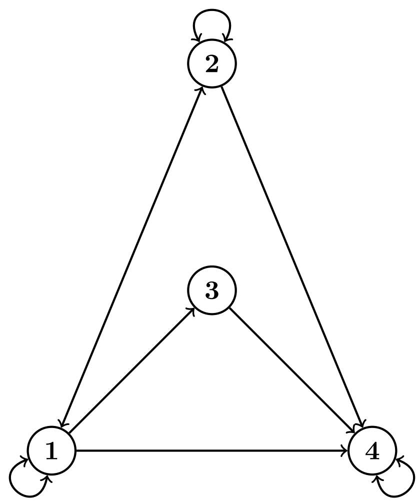

Figure 1: Graph G defined in Example 2.2

We now present the following technical lemmas.

Lemma 2.3. *Let (X,d) be a metric space endowed with a graph G and $T : X \longrightarrow X$ be an interpolative Kannan-type $(G$-α-$\mu)$-contraction. If there exists $x \in X$ such that $(x, Tx) \in E(\tilde{G})$, then*

$$d(T^n x, T^{n+1}x) \leq \mu^n d(x, Tx)$$

for all $n \in \mathbb{N}^* = \mathbb{N} \cup \{0\}$.

Proof. Let $x \in X$ be such that $(x, Tx) \in E(\tilde{G})$. Then by induction, we have $(T^n x, T^{n+1}x) \in E(\tilde{G})$. Hence, by (2.1), we obtain

$$d(T^{n+1}x, T^n x) \leq \mu d(T^n x, T^{n+1}x)^{\alpha} \cdot d(T^{n-1}x, T^n x)^{1-\alpha},$$

implying that

$$d(T^n x, T^{n+1}x)^{1-\alpha} \leq \mu d(T^{n-1}x, T^n x)^{1-\alpha},$$

so that

$$d(T^n x, T^{n+1}x) \leq \mu^{\frac{1}{1-\alpha}} d(T^{n-1}x, T^n x) \leq \mu d(T^{n-1}x, T^n x).$$

Continuing inductively, we have

$$d(T^n x, T^{n+1}x) \leq \mu^n d(x, Tx)$$

for all $n \in \mathbb{N}^*$. □

Remark 2.4. *Note that the symmetry of the graph G is necessary for the above lemma to hold.*

Lemma 2.5. *Let (X,d) be a metric space endowed with a graph G and $T : X \longrightarrow X$ be an interpolative Kannan-type $(G$-α-$\mu)$-contraction such that G is weakly T-connected. Then for all $x \in X$, the sequence $\{T^n x\}_{n\in\mathbb{N}}$ is a Cauchy sequence.*

Proof. Let $x \in X$ be fixed. We consider the following cases:

1. If $(x, Tx) \in E(\tilde{G})$, then by Lemma 2.3, we have
$$d(T^n x, T^{n+1}x) \leq \mu^n d(x, Tx) \tag{2.2}$$
for all $n \in \mathbb{N}^*$. Since $\mu < 1$, then we have
$$\sum_{n=0}^{\infty} d(T^n x, T^{n+1}x) \leq \frac{1}{1-\mu} d(x, Tx) < \infty.$$
By standard argument, we see that $\{T^n x\}_{n\in\mathbb{N}}$ is a Cauchy sequence.

2. If $(x, Tx) \notin E(\tilde{G})$, then there exists a path in $\tilde{G}$, $\{x_i\}_{i=0}^N$ from x to Tx such that $x_0 = x$, $x_N = Tx$ with $(x_{i-1}, x_i) \in E(\tilde{G})$ for all $i = 1, 2, ..., N$ and $(x_i, Tx_i) \in E(\tilde{G})$ for all $i = 1, 2, ..., N-1$. Hence, by triangle inequality and (2.1), we have
$$d(T^n x, T^{n+1}x) \leq \sum_{i=1}^{N} d(T^n x_{i-1}, T^n x_i) \leq \mu \sum_{i=1}^{N} d(T^{n-1}x_{i-1}, T^n x_{i-1})^{\alpha} \cdot d(T^{n-1}x_i, T^n x_i)^{1-\alpha}.$$
Let $d(T^{n-1}x_{i-1}, T^n x_{i-1}) = \frac{d(T^{n-1}x_i, T^n x_i)}{\mu^{\frac{1}{\alpha-1}}}$. Then
$$d(T^n x, T^{n+1}x) \leq \mu^{\frac{1}{1-\alpha}} \sum_{i=1}^{N} d(T^{n-1}x_i, T^n x_i) \leq \sum_{i=1}^{N} d(T^{n-1}x_i, T^n x_i) < \infty.$$

□

The following is our main result.

Theorem 2.6. *Let (X, d) be a complete metric space endowed with a graph G and $T : X \longrightarrow X$ be an interpolative Kannan-type (G-α-μ)-contraction. Assume further that:*

(i) G is weakly T-connected;

(ii) for any sequence $\{x_n\}_{n\in\mathbb{N}}$ in X, if $x_n \longrightarrow x$ and $(x_n, x_{n+1}) \in E(\tilde{G})$ for $n \in \mathbb{N}$, then there is a subsequence $\{x_{k_n}\}_{k\in\mathbb{N}}$ with $(x_{k_n}, x) \in E(\tilde{G})$ for $n \in \mathbb{N}$.

Then T is a PO.

Proof. By Lemma 2.5, $\{T^n x\}_{n\geq 0}$ is a Cauchy sequence for all $x \in X$ and from hypothesis, we have $\{T^n x\}_{n\geq 0}$ is convergent.
Let $x, y \in X$. Then $\{T^n x\}_{n\geq 0} \longrightarrow x^*$ and $\{T^n y\}_{n\geq 0} \longrightarrow y^*$ as $n \longrightarrow \infty$. We now consider the following two cases:

1. If $(x, y) \in E(\tilde{G})$, then $(T^n x, T^n y) \in E(\tilde{G})$ for all $n \in \mathbb{N}$. Therefore,
$$d(T^n x, T^n y) \leq \mu d(T^{n-1}x, T^n x)^{\alpha} \cdot d(T^{n-1}y, T^n y)^{1-\alpha}$$
for all $n \in \mathbb{N}$. Letting $n \to \infty$, we obtain
$$d(x^*, y^*) \leq \mu d(x^*, x^*)^{\alpha} \cdot d(y^*, y^*)^{1-\alpha},$$
implying that $d(x^*, y^*) \leq 0$, that is, $x^* = y^*$.

2. If $(x, y) \notin E(\tilde{G})$, then there exists a path in $\tilde{G}$, $\{x_i\}_{i=0}^N$ from x to y such that $x_0 = x$, $x_N = y$ with $(x_{i-1}, x_i) \in E(\tilde{G})$ for all $i = 1, 2, ..., N$ and $(x_i, Tx_i) \in E(\tilde{G})$ for all $i = 1, 2, ..., N-1$.

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Therefore, $(T^n x_{i-1}, T^n x_i) \in E(\tilde{G})$ for all $i = 1, 2, ..., N$ and for all $n \in \mathbb{N}$. Hence, by triangle inequality, we have

$$d(T^n x, T^n y) \leq \sum_{i=1}^{N} d(T^n x_{i-1}, T^n x_i) \leq \mu \sum_{i=1}^{N} d(T^{n-1} x_{i-1}, T^n x_{i-1})^{\alpha} \cdot d(T^{n-1} x_i, T^n x_i)^{1-\alpha}. \tag{2.3}$$

By Lemma 2.5 and hypothesis, the sequence $\{T^n x\}_{n \geq 0}$ is convergent and by the continuity of d, we have that the sequence $\{d(T^n x_{i-1}, T^n x_i)\}_{n \in \mathbb{N}}$ is convergent.
Let $\lim_{n \to \infty} d(T^n x_{i-1}, T^n x_i) = L_i$ for all $i = 1, 2, ..., N$. Then letting $n \to \infty$ in (2.3), we obtain $L_i = 0$ for all $i = 1, 2, ..., N$, that is, $d(x^*, x^*) \leq 0$, implying that $x^* = y^*$.

Hence, for all $x \in X$, there is a unique point $x^* \in X$ such that

$$\lim_{n \to \infty} T^n x = x^*.$$

We now prove that $x^* \in Fix(T)$. Since G is weakly T-connected, then there is at least a point $x_0 \in X$ such that $(x_0, Tx_0) \in E(\tilde{G})$, and so $(T^n x_0, T^{n+1} x_0) \in E(\tilde{G})$ for all $n \in \mathbb{N}$. But $\lim_{n \to \infty} T^n x_0 = x^*$. Hence, by hypothesis, there is a subsequence $\{T^{k_n} x_0\}_{k \in \mathbb{N}}$ with $(T^{k_n} x_0, x^*) \in E(\tilde{G})$ for all $n \in \mathbb{N}$. Therefore,

$$\begin{aligned} d(x^*, Tx^*) \leq & d(x^*, T^{k_n+1} x_0) + d(T^{k_n+1} x_0, Tx^*) \\ \leq & d(x^*, T^{k_n+1} x_0) + \mu d(T^{k_n} x_0, T^{k_n+1} x_0)^{\alpha} \cdot d(x^*, Tx^*)^{1-\alpha}. \end{aligned}$$

Letting $n \to \infty$, we obtain

$$d(x^*, Tx^*) \leq d(x^*, x^*) + \mu d(x^*, x^*)^{\alpha} \cdot d(x^*, Tx^*)^{1-\alpha},$$

implying that $d(x^*, Tx^*) \leq 0$, that is, $x^* = Tx^*$. Hence, $x^* \in Fix(T)$.
If there exists some $y \in X$ such that $Ty = y$, then from the above, we must have that $T^n y \longrightarrow x^*$, implying that $y = x^*$.
Therefore, T is a PO. □

Example 2.7. *Let $X = \{1, 2, 3, 4, 5, 6\}$ be endowed with the metric $d : X \times X \longrightarrow \mathbb{R}_+$ defined by*

$$d(x, y) = |x - y|, \quad \forall x, y \in X.$$

Then (X, d) is a complete metric space.
Define a self-mapping $T : X \longrightarrow X$ by

$$Tx = \begin{cases} \frac{x}{2}, & \text{if } x \in \{2, 4, 6\}; \\ 1, & \text{if } x \in \{1, 3, 5\} \end{cases}$$

for all $x \in X$.
Consider the symmetric graph $\tilde{G}$ defined by $V(\tilde{G}) = X$ and

$$E(\tilde{G}) = \{(1, 2), (1, 3), (1, 5), (2, 3), (2, 4), (3, 4), (3, 6), (4, 5), (4, 6), (5, 6)\} \cup \Delta.$$

Then it is clear that T preserves edges and G is weakly T-connected.
To see that T is an interpolative Kannan-type $(G$-α-$\mu)$-contraction, let $\mu = \frac{9}{10}$ and $\alpha = \frac{2}{5}$. We then consider the following cases:

Case 1: $x, y \in \{2, 4, 6\}$, $x = y$;

Case 2: $x, y \in \{2, 4, 6\}$, $x \neq y$;

Case 3: $x, y \in \{1, 3, 5\}$, $x = y$;

Case 4: $x, y \in \{1, 3, 5\}$, $x \neq y$;

Case 5: $x \in \{2, 4, 6\}$ and $y \in \{1, 3, 5\}$;

Case 6: $x \in \{1, 3, 5\}$ and $y \in \{2, 4, 6\}$.

We demonstrate using the following Table 1 that inequality (2.1) is satisfied for each of the above cases.

Table 1: Table of values for cases 1-6.

Cases	x	y	$d(Tx, Ty)$	$\mu d(x, Tx)^{\alpha} \cdot d(y, Ty)^{1-\alpha}$	
Case 1	2	2	0	0.9	[t]
	4	4	0	1.8	[t]
	6	6	0	2.7	[b]
Case 2	2	4	1	1.36414	[t]
	4	2	1	1.18755	[b]
	4	6	1	2.29576	[t]
	6	4	1	2.11694	[b]
Case 3	1	1	0	0	[t]
	3	3	0	1.8	[t]
	5	5	0	3.6	[b]
Case 4	1	3	0	0	[t]
	1	5	0	0	[t]
	3	1	0	0	[t]
	5	1	0	0	[b]
Case 5	2	1	0	0	[b]
	2	3	0	1.36414	[b]
	4	3	1	1.8	[b]
	4	5	1	2.72828	[b]
	6	3	2	2.11694	[b]
	6	5	2	3.20868	[b]
Case 6	1	2	0	0	[t]
	3	2	0	1.18755	[t]
	3	4	1	1.8	[t]
	3	6	2	2.29576	[t]
	5	4	1	2.37511	[t]
	5	6	2	3.02927	[t]

In the following Figures 2 and 3, we present the symmetric graph $\tilde{G}$ defined in Example 2.7 and illustrate the validity of contractive inequality (2.1) using Example 2.7.

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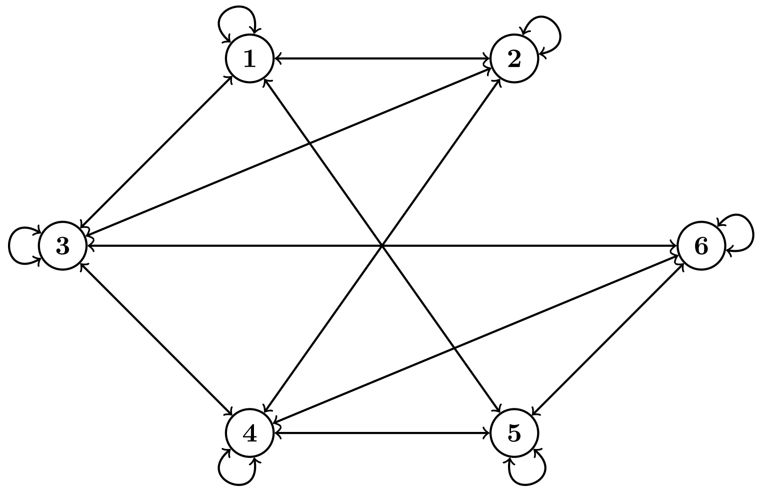

Figure 2: Symmetric graph $\tilde{G}$ defined in Example 2.7

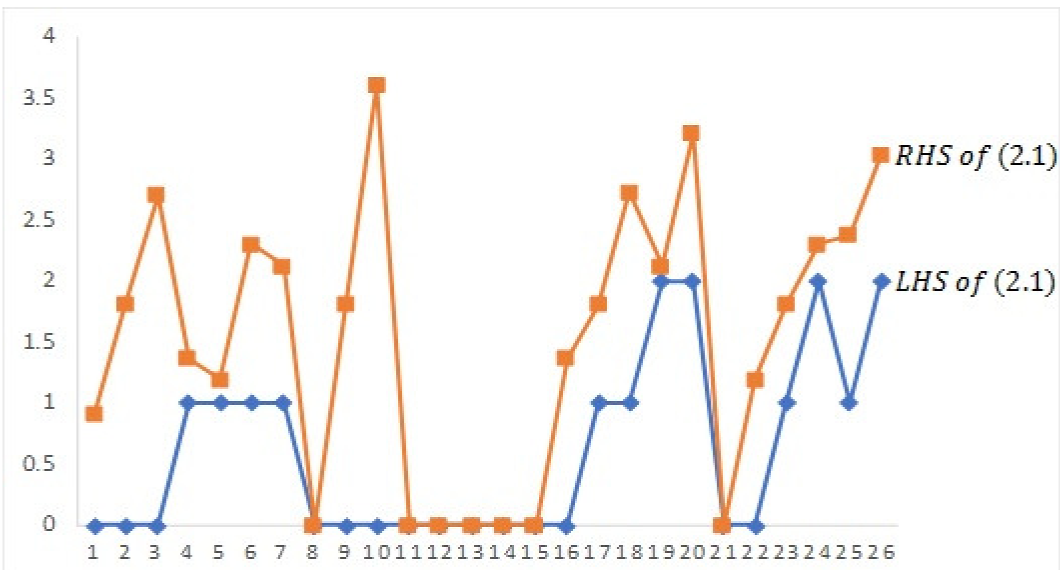

Figure 3: Illustration of contractive inequality (2.1) using Example 2.7

Therefore, all the hypotheses of Theorem 2.6 are satisfied, T has a unique fixed point, $x = 1$ and $\lim_{n\to\infty} T^n x = 1$ for all $x \in X$. Consequently, T is a PO.

Remark 2.8. *Note that an inherent property of interpolative contractions is that the fixed point is not necessarily unique (e.g. see [[21], Example 1]). However, in the case of graphic-type interpolation as introduced in this manuscript, the fixed point is unique.*

3 Conclusion

In this note, the notion of interpolative Kannan-type (G-α-μ)-contraction in metric space endowed with graph is introduced (Definition 2.1). Sufficient conditions under which the new mapping is a Picard operator are examined (Theorem 2.6). To authenticate the hypotheses and indicate the generality of our new ideas, comparative examples are constructed with graphical illustration (Examples 2.2 and 2.7). In particular, the obtained results herein are inspired by and compared with [7, 12, 19].

Competing Interests

The authors declare that they have no competing interests.

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