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A Novel Approach for Normal Parameter Reduction Algorithm of Soft Set Using Unit Similarity Matrix

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Abstract

Soft set has been introduced to deal with uncertainty involved in many real life problems. However, most of the time, these decision-making problems involve less important and redundant parameters, which make the decision making process more complex and challenging.Therefore, in this study the concept of reduct of a soft set is discussed and a new algorithm is developed for normal parameter reduction (NPR) base on the unit similarity matrix. Finally, the propose algorithm is compared with previous parameter reduction algorithms in terms of computational complexity.

Keywords: Soft Set; algorithm; Normal Parameter Reduction; Unit Similarity parameters; Unit Similarity Matrix.
MSC2010: 03E72.

Abbreviations

S-set: Soft Set
F-set: Fuzzy Set
FS-set: Fuzzy Soft Set
FPS-set: Fuzzy Parameterized Soft Set
$FPFS$-set: Fuzzy Parameterized Fuzzy Soft Set
$FFPS$-set: Full Fuzzy Parameterized Soft Set
VS-Set: Vague Soft Set
SR-Set: Soft Rough Set
PRS-Set: Probabilistic Soft Set
$BIVS$-Set: Belief Interval-valued Soft Set
$FPHFLTS$-set: Fuzzy Parameterized Hesitant Fuzzy Linguistic Term Soft Set.

1 Introduction

In an attempt to overcome the problems of uncertainty and vagueness when dealing with data in many fields such as environmental sciences, social sciences, economics, medical sciences and

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engineering, a number of non-classical mathematical theories have been developed such as F-set theory [1] by Zadeh, Rough set theory [2] by Pawlak, Intuitionistic fuzzy set theory [3] by Atanassov and Interval set theory [4] by Gorzalzany. But in [5], Molodtsov pointed out that each of these theory has their associated difficulties. Therefore, in 1999 soft set was introduced by Molodtsov [5] as a new theory to handle these difficulties. Soft set uses adequate parameterization as its tool in its development. Since then soft set have rapidly developed with numerous works by researchers like Maji *et al.,* [6] defined some operations on S-set and used the theory to solve some decision making problems [7]. Chen *et al.,* [8], presented a new definition of S-set as an improvement of [7]. In a related development scholars like Cagman & Enginoglu [9], introduced and investigated soft matrix theory and its application to decision making problem, Aktas *et al.,* [11] introduced the notion of soft group to initiate the study of the algebraic structure of S-set, then Ali *et al.,* [10], in his work noted some errors of previous works and then defined some new operations such as: restricted union, extended intersection, restricted intersection, etc. In [12] Maji *et al.,* introduce the notion of FS-set as a hybridization of S-set to handle more complicated problems that may not be 2-valued. In this direction, researchers have come up with interesting applications of the theory. Roy & Maji [13], investigated some application of FS-set. Yang *et al.,* [14, 15] made some improvement on this concept. As a follow-up Cagman *et al.,* in [16], defined fuzzy soft set theory and its related properties, and fuzzy soft aggregation operator that makes the decision-making process immensely simple and more proficient. In [17] the algebraic property of fuzzy soft sets was studied by Liu & Yan. Consequenctly, Cagman *et al.,* [18, 19], introduced the concepts of FPS-set and gave their related properties, and in [19] the same authors introduced the idea of $FPFS$-set and investigated their associated properties. In the same vein, Alkhazaleh *et al.,* [20] introduced the concept of fuzzy parameterized interval-valued FS-set and gave its application in decision making. FPS-set has continuously developed such that numerous researchers have applied it towards solving more realistic decision making problems. For instance; the work of Rodzi and Ahmad [21] on $FPHFLTS$-set in multi-criteria decision making, which came up by studying the work on hesitant fuzzy linguistic term soft set [22] in a fuzzy parameterized environment. The authors also described some related concepts and consider the fundamental operations of $FPHFLTS$-set, they were able to develop three different algorithm for solving group decision making. In a related development Edeghagba & Muhammad [23] introduced the concept of $FFPS$-set and its related properties. Apart from the combination of S-set theory with F-set theory, researchers have also combined S-set theory with other theories to obtain concepts like: VS-set [24], SR-set [25], PRS-set [26], $BIVS$-Belief interval-valued soft set [27]. Also researchers have also studied soft algebraic structures like soft groups [28], soft quasigroups [29], etc.

The process of parameter reduction is used to remove superfluous and redundant information during decision making problem in S-set theory without changing the decision order of alternatives. Therefore finding a reduction method with less computational complexity have been an important direction for many researchers. In [30] Maji *et al.,* introduced the concept of S-set reduction, Chang in *et al.,* [31, 32] pointed out some problems with the S-set reduction presented in [30] and hence proposed a new method for parameter reduction of S-set. Kong *et al.,* [33] considered the drawbacks of suboptimal choices and parameter addition, and hence proposed a new algorithm for normal parameter reduction which overcame this drawbacks. Ma *et al.,* [34] pointed out that the algorithm presented in [33] is hard and difficulty to understand, and involve a great number of computation, hence proposed a new algorithm. Furthermore, normal parameter reduction algorithm was studied by Danjuma *et al.,* in [35] where the authors propose an alternative approach to normal parameter reduction method to improve computational complexity and reduce running time.

This paper presents a novel approach to the normal parameter reduction method in soft set theory. The proposed method utilizes a unit similarity matrix, leading to improved computational complexity and reduced running time.

The rest of the paper is organized as follows. In section 2 we give a review of basic concepts of S-set theory and information system. In section 3 we give analysis of some previous algorithms. Section 4 presents our new algorithm as an alternative approach of normal parameter reduction. A comparative study among the mentioned algorithms in section 2 is provided in Section 5. Finally,

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In section 6 we conclude the work.

2 Preliminaries

This section reviews some basic notions regarding S-set theory and information system.

(See [5]) (**S-set**): Let U be a universe set, and E be the set of parameters. A pair (F, E) is called a S-set over U if and only if F is a mapping from E into the set of all subsets of the universe set U, i.e., $F\colon E \to P(U)$, where $P(U)$ is the power set of U.
In other words, S-set over U is a parameterized family of subsets of U.
Every set $F(e)$, for every $e \in E$, from this family may be considered as the set of e-elements of the S-set (F, E) or considered as the set of e-approximate elements of the soft set. Accordingly, we can view a soft set (F, E) as a collection of approximations: $(F, E) = \{F(e) : e \in E\}$. As an illustration, we use the following example, to express the meaning of definition 2 Given the following initial universe, $U =$ the set of houses under consideration for sales, E be the set of parameters.
Suppose: $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$
Where we have six houses in the defined universe, and $e_i \in E$ for $i = 1, 2, 3, 4, 5, 6$ stands for the parameters: $e_1 =$ expensive, $e_2 =$ beautiful, $e_3 =$ wooden, $e_4 =$ in green surrounding, $e_5 =$ in serene environment, $e_6 =$ in noisy environment .

Suppose that:
$F(e_1) = \{h_1\}$
$F(e_2) = \{h_2, h_4\}$
$F(e_3) = \{h_3, h_4, h_5\}$
$F(e_4) = \{h_1, h_2, h_3\}$
$F(e_5) = \{h_4,\ h_6\}$
$F(e_6) = \{\}$

Where $F(e_i)$ is a subset of U whose elements match $e_i \in E$. The Boolean-valued table representing the S-set, (F, E) as defined in Example 2, is given by Table 1.

U/E	e_1	e_2	e_3	e_4	e_5	e_6	$f_E(\cdot)$
h_1	1	0	0	1	0	0	2
h_2	0	1	0	1	0	0	2
h_3	0	0	1	1	0	0	2
h_4	0	1	1	0	1	0	3
h_5	0	0	1	0	0	0	1
h_6	0	0	0	0	1	0	1

Table 1: Tabular Representation of S-Set (F, E)

(See [36]) A knowledge representation system can be formulated as a quadruple $K = (U, A, V, f)$, where U is a nonempty finite set of objects and A is a nonempty finite set of attributes, such that $f_a : U \to V_a$ for any $a \in A$, is a knowledge function, where V_a is called the value set of a and $V = \cup_{a\in A} V_a$. Therefore, for a given S-Set (F, E) over the universe U, then $K^{(F,E)} = (U, E, V, f^{(F,E)})$ is a Boolean-valued knowledge representation system induced by (F, E) such that for $V_e = \{0, 1\}$ for all $e \in E$ we have:

$$f_e^{(F,E)}(x) = \begin{cases} 1 & : x \in F(e) \\ 0 & : x \notin F(e) \end{cases}$$

3 Analysis of Some Previous Algorithms

In this section, we analyse the method of normal parameter reductions and their algorithms proposed by Kong *et al.,* [33] and Ma *et al.,* [34].

3.1 Normal parameter reduction of soft sets and its algorithm (See [33])

Assume $U = \{h_1, h_2, ..., h_n\}$, $E = \{e_1, e_2, ..., e_m\}$, then the choice value of each h_i is defined by $f_E(h_i) := \sum_j h_{ij}$, where h_{ij} are entries in the (F, E) table, given as Table 1. (See [33]) Given a S-Set (F, E), with every subset of parameters $B \subseteq E$, an indiscernibility relation IND(B) is defined by

$$\text{IND(B)} = \{(h_i, h_j) \in U \times U : f_e(h_i) = f_e(h_j), \forall e \in B\}.$$

For S-Set (F, E) and $U = \{h_1, h_2, ..., h_n\}$.Then

$$C_E = \{\{h_1, h_2, ..., h_i\}_{f_1}, \{h_{i+1}, h_{i+2}, ..., h_j\}_{f_2}, ..., \{h_k, h_{k+2}, ..., h_n\}_{f_s}\} \tag{3.1}$$

refers to the decision partition of elements in U, which classifies and ranks the elements in U according to the value of $f_E(.)$ based on the indiscernibility relation. For subclass $\{h_u, h_{u+2}, ..., h_{u+w}\}_{f_i}$, $f_E(h_u) = f_E(h_{u+1}) = ... = f_E(h_{u+w}) = f_i$, $f_1 \geq f_2 \geq \cdots \geq f_s$, where s is the number of subclasses.

(See [33]) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, if there exists a subset $A = \{e'_1, e'_2, ..., e'_p\} \subset E$ such that $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$ holds, then A is dispensable, otherwise A is indispensable.

A subset $B \subset E$ is said to be a normal parameter reduction of E, if the following two conditions hold

(i) B is indispensable

(ii) $f_{E-B}(h_1) = f_{E-B}(h_2) = \cdots = f_{E-B}(h_n)$.

(See [33]) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and $U = \{h_1, h_2, ..., h_n\}$, then the decision partition as given in equation 3.1 and the decision partition deleted e_i are respectively given as

$$C_E = \{E_{f_1}, E_{f_2}, ..., E_{f_s}\} \tag{3.2}$$

and

$$C_{E-e_i} = \left\{\overline{E - e'_{if'_1}}, \overline{E - e'_{if'_2}}, ..., \overline{E - e'_{if'_s}}\right\} \tag{3.3}$$

The importance degree of e_i for the decision partition is defined by

$$r_{e_i} = \frac{1}{|U|}(\alpha_{1,e_i} + \alpha_{2,e_i} + \cdots + \alpha_{s,e_i}) \tag{3.4}$$

where $|\cdot|$ refers to the cardinality of set and

$$\alpha_{k,e_i} = \begin{cases} |E_{f_k} - \overline{E - e'_{if'_z}}|, & \exists z' : f_k = f'_z, 1 \leq z' \leq s', 1 \leq k \leq s \\ |E_{f_k}|, & \text{otherwise.} \end{cases}$$

Using the parameter importance degree the authors in [33] presented the algorithm for parameter reduction as in Fig 1.

(1) Input the S-Set (F, E) and its parameter set E ;
(2) Compute the parameter importance degree r_{e_i}, for $1 \leq i \leq m$;
(3) Select the maximum subset $A = \{e'_1, e'_2, ..., e'_p\}$ in E such that the sum of r_{e_i},
for $1 \leq i \leq p$ is a nonnegative integer then put A into the feasible parameter reduction set;
(4) Find A for which $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$ holds,
then E-A is the normal parameter reduction, otherwise delete A from the feasible parameter reduction set;
(5) Find the maximum cardinality of A in the feasible parameter reduction set;
(6) Compute E-A as the optimal parameter reduction.

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Figure 1: Normal parameter reduction algorithm

It is clear that the algorithm presented by Kong *et al.* [33], which utilizes the parameter importance degree, is computationally intensive and quite difficult to understand. In an effort to overcome this computational complexity, Ma *et al.* [34] proposed a new and efficient normal parameter reduction algorithm for soft sets.

3.2 A new efficient normal parameter reduction algorithm

In this section we briefly analyze the new efficient normal parameter reduction algorithm of S-set as was presented by Ma *et al.* in [34].

(See [34]) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and U=$\{h_1, h_2, ..., h_n\}$, we denote $f_E(h_i) = \sum_j h_{ij}$ as an oriented-object sum. (See [34]) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and U=$\{h_1, h_2, ..., h_n\}$, we denote $S(e_j) = \sum_i h_{ij}$ as an oriented-parameter sum. (See [34]) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and U=$\{h_1, h_2, ..., h_n\}$, we denote $S_A = \sum_j S(e_j)$, for $A \subseteq E$ as the overall sum of A. The next two definitions follows from definitions 3.2, 3.2 and 3.2 to check and remove parameters with the same entries. (See [34]) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and U=$\{h_1, h_2, ..., h_n\}$. For $e_j \in E$, if $h_{1j} = h_{2j} = \cdots = h_{nj} = 1$, we denote e_j as e_j^1. (See [34]) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and U=$\{h_1, h_2, ..., h_n\}$. For $e_j \in E$, if $h_{1j} = h_{2j} = \cdots = h_{nj} = 0$, we denote e_j as e_j^0.
Using the oriented-parameter sum the authors in [34] presented the algorithm for parameter reduction as in Fig 2.

(1) Input the S-Set (F, E) and its parameter set E;
(2) If there exist e_j^0 and e_j^1, put them into the reduced parameter set denoted by C, and establish a new S-set (F, E') without e_j^0 and e_j^1, where $U = h_1, h_2, ..., h_n$ and $E' = e_{1'}, e_{2'}, ..., e_{r'}$;
(3) For S-set (F, E'), calculate $S(e_{j'})$ of $e_{j'}$ (i.e., the oriented-parameter sum), for $j' = 1', 2', \cdots, t'$;
(4) Find the subset $A \subseteq E'$ in which S_A is a multiple of $|U|$, and put A into a candidate parameter reduction set;
(5) Check every A in the candidate parameter reduction set. If $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$, keep it; otherwise omit it;
(6) Find the maximum cardinality of A in the candidate parameter reduction set, then compute $E - A - C$ as the optimal parameter reduction.

Figure 2: New efficient normal parameter reduction algorithm

4 Proposed Techniques to Normal Parameter Reduction

In this section, we present our proposed algorithm of a new approach to normal parameter reduction using unit similarity matrix

4.1 Proposed Technique

Given that (F, E) is a S-set with tabular representation, for which $E = \{e_1, e_2, ..., e_m\}$, is the parameter set, U=$\{h_1, h_2, ..., h_n\}$ is the objective set and h_{ij} are the entries in (F, E) table.

(Complementary pairs) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and U=$\{h_1, h_2, ..., h_n\}$. Two parameters e_i and e_j are said to be complementary if they do not have the same value for each of their corresponding h_{ij} entries. That is for all $h_k \in U$, $f_{e_i}^{(F,E)}(h_k) \wedge f_{e_j}^{(F,E)}(h_k) = 0$ and $f_{e_i}^{(F,E)}(h_k) \vee f_{e_j}^{(F,E)}(h_k) = 1$. Then $\{e_i, e_j\}$ is special entry denoted by CP.

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(Unit similarity matrix) Given a S-Set (F, E), with $E = \{e_1, e_2, ..., e_m\}$, and $U = \{h_1, h_2, ..., h_n\}$. Let $K^{(F,E)} = (U, E, V, f^{(F,E)})$ be a Boolean-valued the knowledge representation system induced by (F, E) such that for any $h_i, h_j \in U$ the unit similarity parameter set of h_i and h_j is given as:

$$s_E(h_i, h_j) = \{e \in E : f_e^{(F,E)}(h_i) = 1 = f_e^{(F,E)}(h_j)\}, \tag{4.1}$$

$S((F, E)) = \{s_E(h_i, h_j) : (h_i, h_j) \in U^2\}$ is called the unit similarity matrix of the S-Set (F, E). Using Example 2 and Table 1 we explain Definitions 4.1. Clearly, the unit similarity parameter set of h_i and h_j for all $h_i, h_j \in U$ are:

For $s_E(h_1, h_j)$, $j = 1, 2, \ldots, 6$
$s_E(h_1, h_1) = \{e_1, e_4\}$, $s_E(h_1, h_2) = \{e_4\}$, $s_E(h_1, h_3) = \{e_4\}$, $s_E(h_1, h_4) = \{\}$, $s_E(h_1, h_5) = \{\}$, $s_E(h_1, h_6) = \{\}$;

For $s_E(h_2, h_j)$, $j = 1, 2, \ldots, 6$
$s_E(h_2, h_1) = \{e_4\}$, $s_E(h_2, h_2) = \{e_2, e_4\}$, $s_E(h_2, h_3) = \{e_4\}$, $s_E(h_2, h_4) = \{e_2\}$, $s_E(h_2, h_5) = \{\}$, $s_E(h_2, h_6) = \{\}$

For $s_E(h_3, h_j)$, $j = 1, 2, \ldots, 6$ $s_E(h_3, h_1) = \{e_4\}$, $s_E(h_3, h_2) = \{e_4\}$, $s_E(h_3, h_3) = \{e_3, e_4\}$, $s_E(h_3, h_4) = \{e_3\}$, $s_E(h_3, h_5) = \{e_3\}$, $s_E(h_3, h_6) = \{\}$

For $s_E(h_4, h_j)$, $j = 1, 2, \ldots, 6$ $s_E(h_4, h_1) = \{\}$, $s_E(h_4, h_2) = \{e_2\}$, $s_E(h_4, h_3) = \{e_3\}$, $s_E(h_4, h_4) = \{e_2, e_3, e_5\}$, $s_E(h_4, h_5) = \{e_3\}$, $s_E(h_4, h_6) = \{e_5\}$

For $s_E(h_5, h_j)$, $j = 1, 2, \ldots, 6$ $s_E(h_5, h_1) = \{\}$, $s_E(h_5, h_2) = \{\}$, $s_E(h_5, h_3) = \{e_3\}$, $s_E(h_5, h_4) = \{e_3\}$, $s_E(h_5, h_5) = \{e_3\}$, $s_E(h_5, h_6) = \{\}$

For $s_E(h_6, h_j)$, $j = 1, 2, \ldots, 6$ $s_E(h_6, h_1) = \{\}$, $s_E(h_6, h_2) = \{\}$, $s_E(h_6, h_3) = \{\}$, $s_E(h_6, h_4) = \{e_5\}$, $s_E(h_6, h_5) = \{\}$, $s_E(h_6, h_6) = \{e_5\}$

Therefore the unit similarity matrix is:

$s_E(h_i, h_j)$	h_1	h_2	h_3	h_4	h_5	h_6
h_1	e_1, e_4	e_4	e_4			
h_2	e_4	e_2, e_4	e_4	e_2		
h_3	e_4	e_4	e_3, e_4	e_3	e_3	
h_4		e_2	e_3	e_2, e_3, e_5	e_3	e_5
h_5			e_3	e_3	e_3	
h_6				e_5		e_5

Table 2: Unit Similarity Matrix

The unit similarity parameter set has the following properties. For a S-Set (F, E),
(1) $s_E(h_i, h_j) \subset E$
(2) $s_E(h_i, h_i) \neq \emptyset$
(3) $s_E(h_i, h_j) = s_E(h_j, h_i)$
(4) $s_E(h_i, h_i) \cap s_E(h_j, h_j) = s_E(h_i, h_j)$

Proof. The proof of properties 4.1 follows easily from definition 4.1. □

Using Example 4.1 we explain Properties 4.1. Clearly, all the unit similarity parameter sets are subset of the parameter set E, that is $s_E(h_i, h_j) \subset E$ for $i, j = 1, 2, \ldots, 6$. So property 1 holds. Next, since $s_E(h_1, h_1) = \{e_1, e_4\}$, $s_E(h_2, h_2) = \{e_2, e_4\}$, $s_E(h_3, h_3) = \{e_3, e_4\}$, $s_E(h_4, h_4) = \{e_2, e_3, e_5\}$, $s_E(h_5, h_5) = \{e_3\}$, $s_E(h_6, h_6) = \{e_5\}$, it follows that property 2 holds. Obviously from the unit

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similarity matrix, property 3 holds. Finally, property 4 holds. For example we have that:
$s_E(h_1,h_1)=\{e_1,e_4\}\cap s_E(h_2,h_2)=\{e_2,e_4\}=)=\{e_4\}=s_E(h_1,h_2)$;
$s_E(h_1,h_1)=\{e_1,e_4\}\cap s_E(h_3,h_3)=\{e_3,e_4\}=)=\{e_4\}=s_E(h_1,h_3);\ldots$.

For a S-Set (F,E) with $E=\{e_1,e_2,...,e_m\}$, U=$\{h_1,h_2,...,h_n\}$ and $A=\{e'_1,e'_2,...,e'_p\}\subset E$, then $E-A$ is a normal parameter reduction of E if and only if

$$|\bigcup_{j=1}^{|U|} s_A(h_j,h_i)|=f_A(h_i),\ \forall i=1,2,...,n \tag{4.2}$$

Proof. In one direction assume that $A=\{e'_1,e'_2,...,e'_p\}\subset E$, and $E-A$ is a normal parameter reduction of E. Given that $f_E(h_i):=\sum_j h_{ij}$, then for an arbitrary k if $f_A(h_k)=u$, for a nonnegative integer u, it follows that only u parameters, $\{e''_1,e''_2,...,e''_u\}$ in A are of value 1, corresponding to object h_k in the S-set tabular representation. Therefore, by equation 4.1 it must be the case that for any $e\in\{e''_1,e''_2,...,e''_u\}$ there exist some $s_A(h_j,h_k)$ such that $e\in s_A(h_j,h_k)$, $j\neq k$. Hence $|\bigcup_{j=1}^{|U|} s_A(h_j,h_k)|=u$
The other direction follows easily. □

We give the following definition as an alternative to definition 3.1. Given a S-Set (F,E), with $E=\{e_1,e_2,...,e_m\}$, if there exists a subset $A=\{e'_1,e'_2,...,e'_p\}\subset E$ such that $|\bigcup_{j=1}^{|U|} s_A(h_j,h_1)|=|\bigcup_{j=1}^{|U|} s_A(h_j,h_2)|=\cdots=|\bigcup_{j=1}^{|U|} s_A(h_j,h_n)|$ holds, then A is dispensable, otherwise A is indispensable.
A subset $B\subset E$ is said to be a normal parameter reduction of E, if the following two conditions hold
(i) B is indispensable
(ii) $|\bigcup_{j=1}^{|U|} s_{E-B}(h_j,h_1)|=|\bigcup_{j=1}^{|U|} s_{E-B}(h_j,h_2)|=\cdots=|\bigcup_{j=1}^{|U|} s_{E-B}(h_j,h_n)|$.

Corollary 4.1. *For a S-Set (F,E) with $E=\{e_1,e_2,...,e_m\}$, U=$\{h_1,h_2,...,h_n\}$. If there exists a subset $A=\{e'_1,e'_2,...,e'_p\}\subseteq E$, such that $E-A$ is a normal parameter reduction of E, then we have*

$$\sum_{i}^{|U|}|\bigcup_{j=1}^{|U|} s_A(h_i,h_j)|=xn,\ for\ \ x=1,2,...,m \tag{4.3}$$

where $|\cdot|$ denotes the cardinality of set.

Proof. Assume that $A=\{e'_1,e'_2,...,e'_p\}\subseteq E$, and $E-A$ is a normal parameter reduction of E, then $|\bigcup_{j=1}^{|U|} s_A(h_1,h_j)|=|\bigcup_{j=1}^{|U|} s_A(h_2,h_j)|=\cdots=|\bigcup_{j=1}^{|U|} s_A(h_n,h_j)|$. Therefore the following hold:

$$\begin{aligned}
f_{e'_1}^{(F,A)}(h_1)+f_{e'_2}^{(F,A)}(h_1)+\cdots+f_{e'_p}^{(F,A)}(h_1)&=x\\
f_{e'_1}^{(F,A)}(h_2)+f_{e'_2}^{(F,A)}(h_2)+\cdots+f_{e'_p}^{(F,A)}(h_2)&=x\\
\vdots\qquad\qquad\vdots\qquad\qquad\vdots\qquad\qquad&\vdots\\
f_{e'_1}^{(F,A)}(h_n)+f_{e'_2}^{(F,A)}(h_n)+\cdots+f_{e'_p}^{(F,A)}(h_n)&=x.
\end{aligned}$$

But we have that

$$\sum_{i}^{|U|}|\bigcup_{j=1}^{|U|} s_A(h_i,h_j)|=\sum_{i}^{|U|} f_{e'_1}^{(F,A)}(h_i)+\sum_{i}^{|U|} f_{e'_2}^{(F,A)}(h_i)+\cdots+\sum_{i}^{|U|} f_{e'_p}^{(F,A)}(h_i)$$

Hence It follows easily that

$$\sum_{i}^{|U|}|\bigcup_{j=1}^{|U|} s_A(h_i,h_j)|=\sum_{j}^{|A|} f_{e'_j}^{(F,A)}(h_1)+\sum_{j}^{|A|} f_{e'_j}^{(F,A)}(h_2)+\cdots+\sum_{j}^{|A|} f_{e'_j}^{(F,A)}(h_n)=xn$$

□

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5 Comparison of algorithms

In this section an example is used to analyze the various algorithms in the previous subsections and how they compare to our proposed algorithm.

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	$f(\cdot)$
h_1	1	0	0	1	1	0	1	1	1	0	0	0	0	0	1	0	7
h_2	1	0	0	1	1	1	0	0	1	0	0	0	0	0	1	0	6
h_3	1	1	0	0	0	1	0	1	0	1	1	0	0	0	1	1	8
h_4	0	1	0	1	1	1	0	0	1	0	0	1	0	0	1	1	8
h_5	0	1	1	1	1	1	0	1	0	1	1	0	0	0	1	1	10
h_6	1	0	0	1	1	0	1	0	0	1	0	1	0	0	1	0	7
h_7	1	1	1	0	0	1	1	1	1	0	0	1	0	0	1	0	9
h_8	0	1	0	1	1	1	1	0	0	1	0	1	0	1	1	0	9

Table 3: S-Set (F, E)

Given that (F, E) is the S-set with the tabular representation in Table 3. Suppose $E = \{e_1, e_2, ..., e_{16}\}$ and $U = \{h_1, h_2, ..., h_8\}$.

5.1 Algorithm in [33]

Step 1: The decision partition based on equation 3.1 is:
$C_E = \{\{h_5\}_{10}, \{h_7, h_8\}_9, \{h_3, h_4\}_8, \{h_1, h_6\}_7, \{h_2\}_6\}$.
Thus $s = 5$.
Step 2: The decision partition of deleted e_j based on equation 3.3 are:
$C_{E-e_1} = \{\{h_5\}_{10}, \{h_8\}_9, \{h_4, h_7\}_8, \{h_3\}_7, \{h_1, h_6\}_6, \{h_2\}_5\}$,
$C_{E-e_2} = \{\{h_5\}_9, \{h_7, h_8\}_8, \{h_1, h_3, h_4, h_6\}_7, \{h_2\}_6\}$,
$C_{E-e_3} = \{\{h_5, h_8\}_9, \{h_3, h_4, h_7\}_8, \{h_1, h_6\}_7, \{h_2\}_6\}$,
$C_{E-e_4} = \{\{h_5, h_7\}_9, \{h_3, h_8\}_8, \{h_4\}_7, \{h_1, h_6\}_6, \{h_2\}_5\}$,
$C_{E-e_5} = \{\{h_5, h_7\}_9, \{h_3, h_8\}_8, \{h_4\}_7, \{h_1, h_6\}_6, \{h_2\}_5\}$,
$C_{E-e_6} = \{\{h_5\}_9, \{h_7, h_8\}_8, \{h_1, h_3, h_4, h_6\}_7, \{h_2\}_5\}$,
$C_{E-e_7} = \{\{h_5\}_{10}, \{h_3, h_4, h_7, h_8\}_8, \{h_1, h_2, h_6\}_6\}$,
$C_{E-e_8} = \{\{h_5, h_8\}_9, \{h_4, h_7\}_8, \{h_3, h_6\}_7, \{h_1, h_2\}_6\}$,
$C_{E-e_9} = \{\{h_5\}_{10}, \{h_8\}_9, \{h_3, h_7\}_8, \{h_4, h_6\}_7, \{h_1\}_6, \{h_2\}_5\}$,
$C_{E-e_{10}} = \{\{h_5, h_7\}_9, \{h_4, h_8\}_8, \{h_1, h_3\}_7, \{h_2, h_6\}_6\}$,
$C_{E-e_{11}} = \{\{h_5, h_7, h_8\}_9, \{h_4\}_8, \{h_1, h_3, h_6\}_7, \{h_2\}_6\}$,
$C_{E-e_{12}} = \{\{h_5\}_{10}, \{h_3, h_7, h_8\}_8, \{h_1, h_4\}_7, \{h_2, h_6\}_6\}$,
$C_{E-e_{13}} = C_E$,
$C_{E-e_{14}} = \{\{h_5\}_{10}, \{h_7\}_9, \{h_3, h_4, h_8\}_8, \{h_1, h_6\}_7, \{h_2\}_6\}$,
$C_{E-e_{15}} = \{\{h_5\}_9, \{h_7, h_8\}_8, \{h_3, h_4\}_7, \{h_1, h_6\}_6, \{h_2\}_5\}$,
$C_{E-e_{16}} = \{\{h_5, h_7, h_8\}_9, \{h_4\}_8, \{h_1, h_3, h_6\}_7, \{h_2\}_6\}$.
Step 3: Obtaining the importance degree of e_i based on equation 3.4 are:

$$r_{e_1} = \frac{1}{8}(0+1+1+2+1) = \frac{5}{8}, \quad r_{e_2} = \frac{1}{8}(1+2+2+0+0) = \frac{5}{8},$$

$$r_{e_3} = \frac{1}{8}(1+1+0+0+0) = \frac{2}{8}, \quad r_{e_4} = \frac{1}{8}(1+1+1+2+1) = \frac{6}{8},$$

$$r_{e_5} = \frac{1}{8}(1+1+0+3+1) = \frac{6}{8}, \quad r_{e_6} = \frac{1}{8}(1+2+2+0+1) = \frac{6}{8},$$

$$r_{e_7} = \frac{1}{8}(0+2+0+2+0) = \frac{4}{8}, \quad r_{e_8} = \frac{1}{8}(1+1+1+1+0) = \frac{4}{8},$$

$$r_{e_9} = \frac{1}{8}(0+1+1+1+1) = \frac{4}{8}, \quad r_{e_{10}} = \frac{1}{8}(1+1+1+1+0) = \frac{4}{8},$$

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$r_{e_{11}} = \frac{1}{8}(1+0+1+0+0) = \frac{2}{8}$, $r_{e_{12}} = \frac{1}{8}(0+2+1+1+0) = \frac{4}{8}$,

$r_{e_{13}} = \frac{1}{8}(0+0+0+0+0) = \frac{0}{8}$, $r_{e_{14}} = \frac{1}{8}(0+1+0+0+0) = \frac{1}{8}$,

$r_{e_{15}} = \frac{1}{8}(1+2+2+2+1) = \frac{8}{8}$, $r_{e_{16}} = \frac{1}{8}(1+0+1+0+0) = \frac{2}{8}$.

Step 4: Obtaining the subsets A of E in which the sum of the importance degree of the elements in A are nonnegative integers.
Base on the above computation of parameter importance degree, A could be any of the following: $\{e_{13}\}$, $\{e_{15}\}$, $\{e_9, e_{10}\}$, $\{e_1, e_2, e_4\}$, $\{e_1, e_2, e_4, e_{13}\}$, $\{e_1, e_2, e_4, e_{15}\}$, $\{e_1, e_2, e_5\}$, $\{e_1, e_2, e_5, e_{13}\}$, $\{e_1, e_2, e_5, e_{15}\}$, $\{e_1, e_2, e_6\}$, $\{e_1, e_2, e_6, e_{13}\}$, $\{e_1, e_2, e_6, e_{15}\}$, $\{e_1, e_2, e_3, e_8\}$, $\{e_1, e_2, e_8, e_{11}\}$, $\{e_7, e_8, e_9, e_{10}\}$, $\{e_1, e_2, e_3, e_8\}$, $\{e_1, e_2, e_4, e_7, e_8, e_9, e_{10}\}$, $\{e_1, e_2, e_4, e_{13}, e_{15}\}$, $\{e_7, e_8, e_9, e_{10}, e_{13}, e_{15}\}$, and so on, and put them into a feasible parameter reduction set.
Step 5: Considering the subsets in step 4 we have the following
$\{e_{13}\}$, $\{e_{15}\}$, $\{e_{13}, e_{15}\}$, $\{e_9, e_{10}\}$, $\{e_9, e_{10}, e_{13}\}$, $\{e_9, e_{10}, e_{15}\}$, $\{e_9, e_{10}, e_{13}, e_{15}\}$, $\{e_1, e_2, e_4\}$, $\{e_1, e_2, e_5\}$, $\{e_1, e_2, e_4, e_{13}\}$, $\{e_1, e_2, e_5, e_{13}\}$, $\{e_1, e_2, e_4, e_{15}\}$, $\{e_1, e_2, e_5, e_{15}\}$, $\{e_1, e_2, e_4, e_{13}, e_{15}\}$, $\{e_1, e_2, e_5, e_{13}, e_{15}\}$, $\{e_1, e_2, e_4, e_9, e_{10}\}$, $\{e_1, e_2, e_5, e_9, e_{10}\}$, $\{e_1, e_2, e_4, e_9, e_{10}, e_{13}\}$, $\{e_1, e_2, e_5, e_9, e_{10}, e_{13}\}$, $\{e_1, e_2, e_4, e_9, e_{10}, e_{15}\}$, $\{e_1, e_2, e_5, e_9, e_{10}, e_{15}\}$,
$\{e_1, e_2, e_4, e_9, e_{10}, e_{13}, e_{15}\}$,$\{e_1, e_2, e_5, e_9, e_{10}, e_{13}, e_{15}\}$,
satisfying the condition $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$
Step 6: Obtaining the maximum A from step 5 we have $\{e_1, e_2, e_4, e_9, e_{10}, e_{13}, e_{15}\}$ or $\{e_1, e_2, e_5, e_9, e_{10}, e_{13}, e_{15}\}$. Therefore, the optimal normal parameter reduction set is

$$\{e_3, e_5, e_6, e_7, e_8, e_{11}, e_{12}, e_{14}, e_{16}\}$$

or

$$\{e_3, e_4, e_6, e_7, e_8, e_{11}, e_{12}, e_{14}, e_{16}\}.$$

Figure 1 above presents the normal parameter reduction algorithm which describes the steps involved.

5.2 Algorithm in [34]

Step 1: Pick out the parameters, e_{13}^0 and e_{15}^1 according to definitions 3.2 and 3.2 and put them into the reduced parameter set denoted by C. Therefore, we obtain a new S-set (F, E') without e_{13}^0 and e_{15}^1.
Step 2: Computing the oriented parameter sum $S(e_j)$ for each e_j in $E' = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{14}, e_{16}$
based on definition 3.2 are:
$S(e_1) = \sum_i h_{i1} = 1+1+1+0+0+1+1+0 = 5$,
$S(e_2) = \sum_i h_{i2} = 0+0+1+1+1+0+1+1 = 5$,
$S(e_3) = \sum_i h_{i3} = 0+0+0+0+1+0+1+0 = 2$,
$S(e_4) = \sum_i h_{i4} = 1+1+0+1+1+1+0+1 = 6$,
$S(e_5) = \sum_i h_{i5} = 1+1+0+1+1+1+0+1 = 6$,
$S(e_6) = \sum_i h_{i6} = 0+1+1+1+1+0+1+1 = 6$,
$S(e_7) = \sum_i h_{i7} = 1+0+0+0+0+1+1+1 = 4$,
$S(e_8') = \sum_i h_{i8} = 1+0+1+0+1+0+1+0 = 4$,
$S(e_9) = \sum_i h_{i9} = 1+1+0+1+0+0+1+0 = 4$,
$S(e_{10}) = \sum_i h_{i10} = 0+0+1+0+1+1+0+1 = 4$,
$S(e_{11}) = \sum_i h_{i11} = 0+0+1+0+1+0+0+0 = 2$,
$S(e_{12}) = \sum_i h_{i12} = 0+0+0+1+0+1+1+1 = 4$,
$S(e_{14}) = \sum_i h_{i14} = 0+0+0+0+0+0+0+1 = 1$,
$S(e_{16}) = \sum_i h_{i16} = 0+0+1+1+1+0+0+0 = 3$,
Step 3: Obtain the subset $A \subset E$ for which S_A is a multiple of $|U| = 8$. This produces so many

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subsets:
$\{e_4, e_5, e_7\}$, $\{e_1, e_2, e_6\}$, $\{e_7, e_8, e_9, e_{10}, e_{12}\}$, $\{e_1, e_2, e_4, e_9, e_{14}, e_{16}\}$, and so which we put into candidate parameter reduction set.
Step 4: Next we filter the candidate parameter reduction set in step 3 to obtain the subset $A \subset E$ satisfying the condition $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$ and hence we delete others. Therefore, we have $\{e_1, e_2, e_4, e_9, e_{10}\}$ or $\{e_1, e_2, e_5, e_9, e_{10}\}$.
Step 5: We find the maximum cardinality of A in the candidate parameter reduction set in which case

$$E - A - C = \{e_3, e_5, e_6, e_7, e_8, e_{11}, e_{12}, e_{14}, e_{16}\}$$

or

$$E - A - C = \{e_3, e_4, e_6, e_7, e_8, e_{11}, e_{12}, e_{14}, e_{16}\},$$

is considered as the optimal normal parameter reduction.
Figure 2 above presents the new efficient normal parameter reduction algorithm which describes the steps involved.

Clearly, the above algorithms have some setback which include: (1) From the discussion above, we conclude that although the normal parameter reduction algorithm and new efficient normal parameter reduction algorithm are simple approachs toward soft set reduction, but do not consider the existence of complementary pairs in the parameter set, which if considered should reduce the computation complexity of computing the parameter importance degree and oriented parameter sum respectively (2) The algorithms did not consider using an alternative computation to the methods of parameter importance degree and oriented-parameter sum, but the proposed algorithm introduces such an alternative, the use of unit similarity matrix.

5.3 The proposed algorithm

Following from the definitions and theorems given in section 4 we present our proposed algorithm in Fig 4.
Our algorithm and the other algorithms presented above depict two different procedures to normal parameter reduction of the S-Set. There exist some differences between them as follows:
(1) We check for complementary pairs and directly put them into the reduced parameter set. This leads to the number of subsets in the candidate parameter reduction set of the proposed algorithm being lesser than that of the subsets in the other algorithm presented. Therefore a reduction in computation.
(2) Instead of using the overall sum of $A \subset E$ as presented in [34] we used an alternative computation $\sum_i^{|U|} |\bigcup_{j=1}^{|U|} s_A(h_i, h_j)|$ arising from the unit similarity matrix, which relatively is computationally similar.

(1) Input the S Set (F, E) and its parameter set E
(2) If there exists e_j^0 and e_j^1, put them into the reduced parameter set denoted by C
A new S-set (F, E') will be established without e_j^0 and e_j^1,
where $U = h_1, h_2, ..., h_n$ and $E' = e_{1'}, e_{2'}, ..., e_{r'}$
(3) For S-set (F, E') check if there exists complementary pair e_i and e_j
Put them into the reduced parameter set denoted by CP
A new S-set (F, E'') will be established without the pair e_i and e_j,
where $U = h_1, h_2, ..., h_n$ and $E'' = e_{1''}, e_{2''}, ..., e_{t''}$
(4) For S-set (F, E'') construct the unit similarity matrix
(5) Find the subset $A \subseteq E''$ in which $\sum_i^{|U|} |\bigcup_{j=1}^{|U|} s_A(h_i, h_j)|$ is a multiple of $|U|$,
Then put A into a candidate parameter reduction set
(6) Check every A in the candidate parameter reduction set,
If $|\bigcup_{j=1}^{|U|} s_A(h_j, h_1)| = |\bigcup_{j=1}^{|U|} s_A(h_j, h_2)| = \cdots = |\bigcup_{j=1}^{|U|} s_A(h_j, h_n)|$,
It will be kept; otherwise, it will be omitted
(7) Find the maximum cardinality of A in the candidate parameter reduction set
Then $E - A - C - CP$ becomes the optimal normal parameter reduction set.

Figure 3: The Proposed Algorithm

For a clearer understanding of the Proposed Algorithm, we use the same S-set over U with the tabular representation, Table 3 Given that (F, E) is the S-set with the tabular representation in Table 5. Suppose $E = \{e_1, e_2, ..., e_{16}\}$ and $U = \{h_1, h_2, ..., h_8\}$. Step 1: Pick out the parameters, e_{13}^0 and e_{15}^1 according to definitions 3.2 and 3.2 and put them into the reduced parameter set denoted by C. Therefore, we obtain a new S-set (F, E') without e_{13}^0, e_{15}^1. Where $U = \{h_1, h_2, ..., h_8\}$ and $E' = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{14}, e_{16}\}$.
Step 2: Since there exists complementary pair e_9 and e_{10} they are put into the reduced parameter set denoted by CP and a new S-set (F, E'') is obtained without the pair e_9 and e_{10}. Where $U = \{h_1, h_2, ..., h_8\}$ and $E'' = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_{11}, e_{12}, e_{14}, e_{16}\}$.
Step 3: Construct the similarity matrix for S-set (F, E''). See Table 5.
Step 4: Obtain the subset $A \subset E''$ for which $\sum_i^{|U|} |\bigcup_{j=1}^{|U|} s_A(h_i, h_j)|$ is a multiple of $|U| = 8$. This produces subsets such as:
$\{e_7, e_{12}\}$, $\{e_1, e_3, e_{14}\}$, $\{e_1, e_2, e_4\}$, $\{e_1, e_2, e_5\}$, $\{e_8, e_{14}, e_{16}\}$, $\{e_{12}, e_{14}, e_{16}\}$, $\{e_1, e_3, e_7, e_{12}, e_{14}\}$, $\{e_1, e_2, e_4, e_7, e_{12}\}$, $\{e_1, e_2, e_5, e_7, e_{12}\}$, $\{e_7, e_8, e_{12}, e_{14}, e_{16}\}$, which we put into candidate parameter reduction set.
Step 5: Next we check every subset $A \subset E''$ in the candidate parameter reduction set satisfying $|\bigcup_{j=1}^{|U|} s_A(h_j, h_1)| = |\bigcup_{j=1}^{|U|} s_A(h_j, h_2)| = \cdots = |\bigcup_{j=1}^{|U|} s_A(h_j, h_n)|$, and others are deleted. Therefore, we choose $\{e_1, e_2, e_5\}$.
Step 6: We find the maximum cardinality of the candidate parameter reduction set, thus $E - A - C - CP = \{e_3, e_4, e_6, e_7, e_8, e_{11}, e_{12}, e_{14}, e_{16}\}$ is considered as thenovel approach for normal parameter reduction, which in this case the optimal parameter reduction as given by Table 4.

U/E	e_3	e_4	e_6	e_7	e_8	e_{11}	e_{12}	e_{14}	e_{16}	$f(\cdot)$
h_1	0	1	0	1	1	0	0	0	0	3
h_2	0	1	1	0	0	0	0	0	0	2
h_3	0	0	1	0	1	1	0	0	1	4
h_4	0	1	1	0	0	0	1	0	1	4
h_5	1	1	1	0	1	1	0	0	1	6
h_6	0	1	0	1	0	0	1	0	0	3
h_7	1	0	1	1	1	0	1	0	0	5
h_8	0	1	1	1	0	0	1	1	0	5

Table 4: S-Set (F, E)

Therefore, the the original decision order is the same with the reduction soft set. Hence $E - A - C - CP = E - A - C = E - A$

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	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8
h_1	e_1, e_4, e_5, e_7, e_8	e_1, e_4, e_5	e_1, e_8	e_4, e_5	e_4, e_5, e_8	e_1, e_4, e_5, e_7	e_1, e_7, e_8	e_4, e_5, e_7
h_2		e_1, e_4, e_5, e_6	e_1, e_6	e_4, e_5, e_6	e_4, e_5, e_6	e_1, e_4, e_5	e_1, e_6	e_4, e_5, e_6
h_3			$e_1, e_2, e_6, e_8, e_{11}, e_{16}$	e_2, e_6, e_{16}	$e_2, e_6, e_8, e_{11}, e_{16}$	e_1	e_1, e_2, e_6, e_8	e_2, e_6
h_4				$e_2, e_4, e_5, e_6, e_{12}, e_{16}$	$e_2, e_4, e_5, e_6, e_{16}$	e_4, e_5, e_{12}	e_2, e_6, e_{12}	$e_2, e_4, e_5, e_6, e_{12}$
h_5					$e_2, e_3, e_4, e_5, e_6, e_8, e_{11}, e_{16}$	e_4, e_5	e_2, e_3, e_6, e_8	e_2, e_4, e_5, e_6
h_6						$e_1, e_4, e_5, e_7, e_{12}$	e_1, e_7, e_{12}	e_4, e_5, e_7, e_{12}
h_7							$e_1, e_2, e_3, e_6, e_7, e_8, e_{12}$	e_2, e_6, e_7, e_{12}
h_8								$e_2, e_4, e_5, e_6, e_7, e_{12}, e_{14}$

6 Analysis of the Proposed Method

The computational complexities of the Unit Similarity Matrix:

- Construction of the unit similarity matrix involves computing the unit similarity parameter sets for all pairs of objects.
- The number of object pairs is given by $|U|^2$, where $|U|$ is the number of objects.
- For each pair, the algorithm checks the values of the corresponding parameters and determines the similarity set.
- The time complexity for constructing the unit similarity matrix for the parameter set E is $O(|U|^2 \times |E|)$, where $|E|$ is the number of parameters.
- The space complexity is also $O(|U|^2 \times |E|)$ since the matrix needs to be stored.

If there are complementary pairs such that E'' is the set of parameters without the complementary pairs then the space and time complexity becomes $O(|U|^2 \times |E''|)$. Suppose $|U| = |E| = n$, then the space and time complexity is less or equal to $O(n^3)$ since if there are complementary pairs, $|E''| \leq n$.
Assessing the candidate parameter reduction set:

The proposed algorithm first puts parameters that form complementary pairs into the reduced parameter set CP. Assume the number of parameters in CP is denoted by t. Then, the proposed algorithm tests combinations from combination-1 to combination-t', where $t' = m - t$. In other words, the number of accessed entries in the unit similarity parameter set is assumed to be $C(t', 1) + C(t', 2) + \cdots + C(t', t')$. This implies that as the value of t increases, the number of accessed entries for the proposed algorithm decreases.

6.1 Comparison of the proposed method with the previous methods

1. Computation Complexity:

- The "Normal Parameter Reduction Algorithm" has a computation complexity of $O(|U| \times |E|^2)$, where $|U|$ is the number of objects and $|E|$ is the number of parameters. It iterates through all parameter pairs for each object, which can be computationally expensive for large datasets.
- The "New Efficient Normal Parameter Reduction Algorithm" improves the computation complexity to $O(|U| \times |E|)$. It eliminates redundant iterations and reduces the number of comparisons, resulting in faster execution.
- The proposed algorithm introduces complementary pairs and unit similarity matrices to identify indispensable and dispensable parameters. The computation complexity depends on the construction of the unit similarity matrix, which requires evaluating the Boolean functions for each parameter pair. The overall complexity is likely to be similar to or slightly higher than the "New Efficient Normal Parameter Reduction Algorithm."

2. Reduction Results:

- The "Normal Parameter Reduction Algorithm" aims to find a normal parameter reduction set that satisfies the properties of indispensability and uniformity. It may achieve good reduction results but does not consider complementary pairs explicitly.
- The "New Efficient Normal Parameter Reduction Algorithm" improves the reduction results by considering complementary pairs and removing redundant iterations. It can identify normal parameter reductions that satisfy indispensability and uniformity while reducing the computational burden.
- The proposed algorithm further enhances the reduction results by constructing unit similarity matrices. It explores the similarity patterns between parameters and captures the relationships between objects and parameters more explicitly. This approach may provide more refined reduction sets and potentially better performance in certain scenarios.

3. Scalability:

- The scalability of the algorithms depends on their computation complexity and memory requirements. As mentioned earlier, the "Normal Parameter Reduction Algorithm" has the highest complexity, making it less scalable for large datasets.
- The "New Efficient Normal Parameter Reduction Algorithm" improves scalability by reducing the number of iterations and comparisons. It can handle larger datasets more efficiently than the previous algorithm.
- The proposed algorithm introduces unit similarity matrices, which require additional memory to store the similarity information. As the dataset size increases, the memory requirements of this approach may become a scalability bottleneck. However, if memory is not a constraint, it has the potential to provide better scalability than the other algorithms due to its refined reduction results.

6.2 Analysis of the shortcomings of the proposed method

The proposed algorithm provides a new method for parameter reduction in S-Sets. While it offers a different perspective and approach, there are some possible shortcomings to consider:

1. Dependency on Unit Similarity Matrix: The algorithm heavily relies on constructing the unit similarity matrix, which involves computing and storing the unit similarity parameter sets for all pairs of objects. This matrix can be memory-intensive, especially when dealing with large universes and parameter sets. The space complexity of the algorithm can be a concern.

2. Lack of Evaluation Metrics: The algorithm does not incorporate explicit evaluation metrics or criteria to assess the quality or effectiveness of the parameter reduction. It relies on the conditions of indispensability and equal cardinality of unit similarity sets but may not capture other important aspects such as information loss, predictive accuracy, or complexity reduction.

7 Conclusion

Before now numerous algorithms have been proposed to handle problems relating to reduction of S-set, among which few are mentioned in this work and considered. Therefore, in this work we introduced the notion of unit similarity matrix with some new definitions and theorems. Base on the results we propose a novel approach for normal parameter reduction algorithm of S-set using unit similarity matrix. The proposed parameter reduction algorithm introduces a new method in handling S-set reduction problem and in some ways simplifies the reduction complexity. The example shows that the new reduction method in this work is feasible and the decision order of decision alternatives remain also invariant. Moreover, it is an alternative algorithm comparing with the algorithms mentioned. Our immediate next task is developing more general approach for parameter reduction of S-set and FS-set.

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