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MODELLING THE EFFECTS OF HEAT AND MASS TRANSFER ON STEADY TWO-DIMENSIONAL HYDROMAGNETIC FLOW OVER AN IMPERMEABLE SURFACE

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Abstract

The role of heat and mass transfer in moving fluid is important in view of several physical problems such as those dealing with chemical reactions and those encountered with dissociating fluids. A lot of interest has been built in the study of the flow of heat and mass transfer. The aim of present investigation was to study the effects of heat and mass transfer on steady hydromagnetic boundary layer flow over an impermeable horizontal surface with ohmic and viscous heat dissipation. The model formulated taking into consideration the viscous energy dissipation. The surface is assumed to be impermeable. The governing equations formulated based on the conservation of momentum, species and energy were considered in steady state form and solved analytically using direct integration. The results obtained are presented graphically and discussed. The results revealed the effects of operating parameters on the flow and heat transfer over an impermeable surface. Our findings showed that there is a continuous increase in fluid velocity and a decrease in medium temperature along the distance while fluid velocity decreases and medium temperature increases as values of magnetic parameter and Reynolds number increases. These results might be used for interpretation or experiments planning of the more complex flow and heat transfer processes.

Keywords: Analytical solution, hydromagnetic flow, impermeable surface, steady, viscous fluid.
MSC2010: 35Q20.

1 Introduction

Heat and mass transfer from a heated moving surface to a quiescent ambient medium occurred in many manufacturing processes such as hot rolling, wire drawing and crystal growing [1]. Hydromagnetic flow is one of the fundamental problems in heat and mass transfer [2]. Investigations on

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hydrodynamic boundary layer flow and heat transfer over a stretching surface have gained appreciable attention due to its extensive applications in industry and its importance to several technological processes which include the aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, and crystal growing. Crane [3] investigated the steady boundary layer flow due to stretching surface with linear velocity. Many researchers such as Gupta and Gupta [4], Vleggaar [5], and Chen and Char [6] extended the work of Crane [3] by considering the effects of heat and mass transfer analysis under different physical situations.

Simultaneous heat and mass transfer from different geometries may arise during industrial operations where the surface is sometimes stretched because of the process of drawing, for example, the process of cooling continuous strips or filament by drawing them through quiescent fluid where simultaneous heat and mass transfer may occur during the cooling. Hence, it can be deduced that the combined heat and mass transfer can play a vital role in the problems of hydromagnetic flow over an impermeable surface. A new facet of approaching such problems can be given by considering the effect of thermal radiation. Thermal radiation effect might play a significant role in controlling heat process in polymer processing industry. The quality of the final product depends greatly on the heat controlling factors and the knowledge of radiative heat transfer can perhaps lead to a desired product with a sought characteristic.

Many works have been reported on flow and heat transfer over a stretched surface in the presence of radiation. An analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium was given by Anjali Devi and Kayalvizhi [7]. Makinde and Sibanda [8] investigated the chemical reaction effects over the stretching surface in the presence of internal heat generation. Seini and Makinde [9] studied the radiation and chemical reaction effects on MHD boundary layer flow over a stretching surface. Abdul Hakeem et al. [10] investigated the thermal radiation effects on hydromagnetic flow over a stretching surface. Influence of thermal radiation on MHD flow over a stretching surface was studied by Jonnadula et al. [11].

Taking consideration of dissipation effects in the study of heat and mass transfer boundary layer problems adds new dimension to it. Gebhart [12] was the first who studied the problem taking into account the viscous dissipation. Kayalvizhi et al. [13] and Dessie and Kishan [14] examined the effects of viscous dissipation and ohmic dissipation on MHD flow over a stretching surface with thermal radiation effects.

All the above-mentioned studies are confined to the steady state flow problems. But, in certain practical problems, the motion of the stretched surface may start impulsively from rest. In such cases, the transient or unsteady aspects become more interesting. Effects of radiation and heat transfer over an unsteady stretching surface in the presence of heat source or sink were studied by Elbashbeshy and Emam [15]. Makinde ([16], [17]) analyzed the chemically reacting hydromagnetic unsteady flow of a radiating fluid. Yusof et al. [18] analyzed the radiation effect on unsteady MHD flow over a stretching surface. Mass transfer and MHD effect on an unsteady stretching surface were investigated by Ramana Reddy and Bhaskar Reddy [19]. Recently, unsteady MHD flow and heat transfer over a stretching permeable surface were investigated by Choudhary et al. [20]. Reddy et al. [21] considered the thermal radiation and viscous dissipation effects on unsteady MHD flow over a stretching surface. Durojaye and Agee [22] investigated the one-dimensional, positive temperature coefficient (PTC) thermistor equation, using the hyperbolic-tangent function as an approximation to the electrical conductivity of the device. They observed that the steady state solution using the new approximation yielded a distribution of device temperature over the spatial dimension and all the phases of the temperature distribution of the device without having to look for a moving boundary. They analysed the steady state solution and the numerical solution of the unsteady state.

Durojaje et al. [23] also presented a mathematical model for free racial polymerization in the presence of material diffusion. They proved the existence and uniqueness of solution of the model. They used parameter expanding method and seek direct eigenfunctions expansion to obtain analytical solution of the model. The results were presented graphically and discussed. It was discovered that the mixture temperature and monometer concentration were significantly influenced by Kamenetskii number and thermal diffusivity of the mixture. Durojaye and Ayeni [24] consider a steady

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state solution reaction kinetics model of polymerization in the presence of material diffusion. They obtained steady state equations for the resulting partial differential equations. Criteria for existence and Uniqueness of solutions of the equations and numerical results were also provided. They concluded that steady state equation is bounded and has solution under reasonable physical conditions. Adeniyan and Adigun [25] conducted numerically analysis on forced-convective heat and reactive solute mass transfer of a steady incompressible, electrically conducting, chemically reacting and Joule dissipating viscous fluid streaming towards a stationary porous planar surface embedded in a saturated non-Darcian porous medium in the presence of surface mass flux, pressure stress-work and velocity slip. Fatunmbi et al. [26] investigated stagnation-point flow in magneto-Williamson nanofluid along a convectively heated nonlinear stretchable material in a porous medium. The impacts of Joule heating, thermophoresis together with Brownian motion are also checked in this investigation.
However, to the best of author's knowledge, no attempt has been made to investigate the effects of thermal radiation, viscous dissipation, ohmic dissipation and heat and mass transfer effects on steady hydromagnetic flows over an impermeable surface. Being motivated by the extensive applications, this paper seeks to investigate the heat and mass transfer effects on steady two-dimensional hydromagnetic flow over an impermeable surface.
The objective of this paper is to provide an analytical solutions capable of describing steady two-dimensional hydromagnetic flow over an impermeable surface with ohmic and viscous heat dissipation.

2 Model Formulation

The two-dimensional, transient hydromagnetic flow of a viscous, incompressible, electrically conducting, and radiating fluid along with heat and mass transfer over an impermeable surface with ohmic and viscous dissipation is considered. A constant magnetic field B is applied in the direction perpendicular to that of the fluid flow.

The formulation of our model is being guided by the following assumptions:

(i) The fluid is considered to be grey

(ii) The radiative heat flux in the x-direction is negligible in comparison with that in the y-direction

Under these assumptions, the governing equations that are based on the laws balancing mass, linear momentum, energy, and concentration for the present investigation are given as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2 u}{\rho}=0 \tag{2.2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_0\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_P}\frac{\partial q_r}{\partial y}+\frac{v}{\rho c_P}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2 u^2}{\rho c_P} \tag{2.3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{2.4}$$

Rosseland approximation is used to simplify the radiative heat flux term in the energy equation which has the form

$$q_r=-\frac{4\sigma^*}{3k_1}\frac{\partial T^4}{\partial y} \tag{2.5}$$

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The temperature difference within the flow is assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. On expanding T^4 in a Taylor series about T_∞ and thereby neglecting the higher order terms, it is obtained as follows:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{2.6}$$

The above governing equations are associated with the following initial and boundary conditions:

$$\left.\begin{array}{r} u(x,y,t)=0, \quad T(x,y,t)=T_0, \quad C(x,y,t)=C_0, \quad x\geq 0, y\geq 0, \quad t=0. \\ \left.\begin{array}{c} \dfrac{\partial u}{\partial x}=0, \quad \dfrac{\partial u}{\partial y}=0, \\ \dfrac{\partial T}{\partial x}=0, \quad \dfrac{\partial T}{\partial y}=0, \\ \dfrac{\partial C}{\partial x}=0, \quad \dfrac{\partial C}{\partial y}=0, \end{array}\right\}. \quad x=0, y=0, \quad t>0. \\ u(x,y,t)=U_\infty, \quad T(x,y,t)=T_\infty, \quad C(x,y,t)=C_\infty, \quad x=L, y=H, \quad t\geq 0. \end{array}\right\}. \tag{2.7}$$

Where u is the velocity component along the x-axis, v is the velocity component along the y-axis, ν is the kinematic coefficient of viscosity, σ is the electrical conductivity of the fluid, B is the strength of the applied variable magnetic field, ρ is the fluid density, T is the temperature of the fluid, $\alpha_0 = \frac{K}{\rho c_\rho}$ is the thermal diffusivity with K as the thermal conductivity of the fluid, c_ρ is the specific heat capacity at constant pressure, q_r is the radiative heat flux, C is the concentration, D is the coefficient of mass diffusivity, q_r is the radiative heat flux, σ^* is the Stefan-Boltzman constant, k_1 is the mean absorption coefficient.

3 Method of Solution

3.1 Transformation

Introducing the following new space variable [27]

$$z = x + y \tag{3.1}$$

The equations (1) – (6) together with initial and boundary conditions (7) become

$$\frac{\partial U}{\partial z} = 0 \tag{3.2}$$

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B^2 u}{\rho} \tag{3.3}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial z} = \alpha_0\frac{\partial^2 T}{\partial z^2} = \frac{16\sigma^* T_\infty}{3k_1\rho c_p}\frac{\partial^2 T}{\partial z^2} + \frac{V}{c_p}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{\sigma B^2 u^2}{\rho c_p} \tag{3.4}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} \tag{3.5}$$

$$\left.\begin{array}{ll} u(z,t)=0, \quad T(z,t)=T_0, \quad C(z,t)=C_0 & z\geq 0, t=0. \\ \dfrac{\partial u}{\partial z}=0, \quad \dfrac{\partial T}{\partial z}=0, \quad \dfrac{\partial C}{\partial z}=0 & z=0, t>0. \\ u(z,t)=U_\infty, \quad T(z,t)=T_\infty, \quad C(z,t)=C_\infty & z=h, t\geq 0. \end{array}\right\}. \tag{3.6}$$

Where $U = u + v$

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3.2 Dimensional Analysis

Introducing the following non-dimensional variables:

$$t' = \frac{Ut}{h}, \quad z' = \frac{z}{h}, \quad u' = \frac{u}{U}, \quad \theta = \frac{T - T_0}{T_\infty - T_0}, \quad \phi = \frac{C - C_0}{C_\infty - C_0} \tag{3.7}$$

Then, equations (9) to (13) become:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial z} = \frac{1}{R_e}\frac{\partial^2 u}{\partial z^2} - Mu \tag{3.8}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial z} = \frac{1}{p_e}\frac{\partial^2 \theta}{\partial z^2} - R\frac{\partial^2 \theta}{\partial z^2} + \frac{E_c}{R_e}\left(\frac{\partial u}{\partial z}\right)^2 + E_c M u^2 \tag{3.9}$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial z} = \frac{1}{p_{em}}\frac{\partial^2 \phi}{\partial z^2} \tag{3.10}$$

$$\left.\begin{aligned} u(z,0) = 0, \quad & \left.\frac{\partial u}{\partial z}\right|_{z=0} = 0, \quad u(1,t) = \alpha \\ \theta(z,0) = 0, \quad & \left.\frac{\partial \theta}{\partial z}\right|_{z=0} = 0, \quad \theta(1,t) = 1 \\ \phi(z,0) = 0, \quad & \left.\frac{\partial \phi}{\partial z}\right|_{z=0} = 0, \quad \phi(1,t) = 1 \end{aligned}\right\} \tag{3.11}$$

Where

$R_e = \frac{hv}{\nu}$ =Reynolds Number, $M = \frac{\sigma B^2 h}{\rho v}$=Magnetic Parameter, $p_e = \frac{hv}{\alpha_0}$= Peclet Number,

$E_c = \frac{v^2}{c_p(T_\infty - T_0)}$= Eckert Number, $p_{em} = \frac{hv}{D}$=Peclet Mass Number,

$R = \frac{16\sigma^* T_\infty}{3k_1\rho c_p h v}$=Radiation Number, $\alpha = \frac{U_\infty}{U}$

For steady state $\frac{\partial \star}{\partial t} = 0 \quad \star = \{u, \phi, \theta\}$, equations (15) - (18) reduce to

$$\left.\begin{aligned} \frac{d^2 u}{dz^2} - R_e\frac{du}{dz} - R_e M u = 0 \\ \left.\frac{du}{dz}\right|_{z=0} = 0, \quad u(1) = \alpha \end{aligned}\right\} \tag{3.12}$$

$$\left.\begin{aligned} \frac{d^2\theta}{dz^2} - \left(\frac{P_e}{1 - RP_e}\right)\frac{d\theta}{dz} + \frac{Ec}{R_e}\left(\frac{P_e}{1 - RP_e}\right)\left(\frac{du}{dz}\right)^2 + EcM\left(\frac{P_e}{1 - RP_e}\right)u^2 = 0 \\ \left.\frac{d\theta}{dz}\right|_{z=0} = 0, \quad \theta(1) = 1 \end{aligned}\right\} \tag{3.13}$$

$$\left.\begin{aligned} \frac{d^2\phi}{dz^2} - P_{em}\frac{d\phi}{dz} = 0 \\ \left.\frac{d\phi}{dz}\right|_{z=0} = 0, \quad \phi(1) = 1 \end{aligned}\right\} \tag{3.14}$$

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3.3 Analytical Solution

By direct integration, we solving equations (19)- (21) and obtain the following solutions:

$$u(z) = a_1 e^{m_1 z} + b_1 e^{m_2 z} \tag{3.15}$$

$$\phi(z) = 1 \tag{3.16}$$

$$\theta(z) = a_2 + b_2 e^{pz} + A e^{2m_1 z} + B e^{(m_1+m_2)z} + C e^{2m_2 z}. \tag{3.17}$$

Where

$p = \left(\frac{P_e}{1-RP_e}\right)$, $r = \frac{E_c}{R_e}p$, $s = E_c M p$, $m_{1,2} = \frac{R_e + \sqrt{R_e^2 + 4R_e M}}{2}$ $A = \frac{a_1^2(s+rm_1^2)}{2m_1(p-2m_1)}$,
$B = \frac{2a_1b_1(s+rm_1m_2)}{(m_1+m_2)(p-(m_1+m_2))}$, $C = \frac{b_1^2(s+rm_2^2)}{2m_2(p-2m_2)}$, $a_2 = (1-(b_2e^p + Ae^2m_1 + Be^{(m_1+m_2)} + Ce^{2m_2}))$,
$a_1 = -\left(\frac{m_2}{m_1e^{m_2} - m_2e^{m_1}}\right)$, $b_1 = \left(\frac{m_2}{m_1e^{m_2} - m_2e^{m_1}}\right)$, $b_2 = -\frac{1}{p}(2((Am_1 + Cm_2)) + B(m_1+m_2))$

The computations were done on equations (22) to (24) using computer symbolic algebraic package MAPLE 2021 Version.

4 Results and Discussion

4.1 Analysis of Results

Steady flow and heat transfer processes over an impermeable surface are simulated analytically using direct integration. Analytical solutions given by equations (22) - (24) are computed using computer symbolic algebraic package MAPLE 2021. and graphical simulation are shown in Figures 2 to 8.

4.2 State Variables Dynamics

We performed the analytical simulations of the system of differential equations of the state variables to determine the changes in the various state variables with space. There seems to be a continuous increase in the fluid velocity and a decrease in medium temperature along the distance.

Figure 2 depicts the graph of fluid velocity $u(z)$ against distance z for different values of Magnetic parameter M. It is observed that the velocity of the fluid increases along the distance and the maximum fluid velocity decreases as values of Magnetic parameter increases. As noticed M in this figure, a boost in creates a drag in the fluid motion due to the action of the Lorentz force which is responsible for the resistance in the fluid motion and at such, a reduction in the velocity.

Figure 3 displays the graph of medium temperature $\theta(z)$ against distance z for different values of Magnetic parameter M. It is observed that the temperature of the medium decreases along the distance and this medium temperature increases as values of Magnetic parameter increases. Higher values of M improves heat transfer such that the impermeable surface takes less time to cool.

Figure 4 shows the graph of fluid velocity $u(z)$ against distance z for different values of Reynolds number R_e. It is observed that the velocity of the fluid increases along the distance and the maximum fluid velocity decreases as Reynolds number increases.

Figure 5 depicts the graph of medium temperature $\theta(z)$ against distance z for different values of Reynolds number R_e . It is observed that the temperature of the medium decreases along the distance and this medium temperature decreases as Reynolds number increases.

Figure 6 displays the graph of medium temperature $\theta(z)$ against distance z for different values of Peclet energy number P_e. It is observed that the temperature of the medium decreases along the distance and this medium temperature increases as Peclet energy number increases.

Figure 7 shows the graph of medium temperature $\theta(z)$ against distance z for different values of Radiation number R. It is observed that the temperature of the medium decreases along the distance and this medium temperature increases as Radiation number increases.

Figure 8 depicts the graph of medium temperature $\theta(z)$ against distance z for different values of Eckert number E_c. It is observed that the temperature of the medium decreases along the distance and this medium temperature increases as Eckert number increases. An increase in the Eckert number E_c causes thickness of the thermal boundary layer to swell up due to the friction between the fluid particles and consequently, the temperature distribution is increased

It is worth pointing out that the effects observed in Figures 2 to 8, are important for studying the effect of heat generation and absorption on the moving fluids.

5 Conclusion

The effects of mass transfer and radiation heat transfer on steady two-dimensional hydromagnetic flow over an impermeable surface with ohmic and viscous heat dissipation, are analyzed in this work. Analytical solutions are obtained for the governing equations and the effects of the concerned physical parameters over the dimensionless velocity, temperature and concentration distribution are presented graphically. The study revealed the following:
1. Eckert number, Peclet energy number, Radiation number and Magnetic parameter enhanced the medium temperature.
2. Magnetic parameter reduced the fluid velocity.
3. Reynolds number reduced both the fluid velocity and medium temperature.
The results of this study may be of importance to engineers and scholars attempting to develop programming standards and to researchers interested in the theoretical aspects of computer programming

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