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Another New Two Parameter Estimator in Dealing with Multicollinearity in the Logistic Regression Model

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Abstract

In logistic regression models, the maximum likelihood method is always one of the commonly used to estimate the model parameters. However, unstable parameter estimates are obtained due to the problem of multicollinearity. In this article, a new two parameter biased estimator is proposed to combat the issue of multicollinearity in the binary logistic regression models. The proposed estimator is a general estimator which includes other biased estimators, such as the Logistic Ridge, Logistic Liu and the estimators with two biasing parameters as special cases. The properties of the proposed estimator were derived, and six (6) forms of biasing parameter k (generalized, maximum, median, mid-range, arithmetic and harmonic means) were used in this study. Necessary and sufficient conditions for the superiority of the new two parameter biased estimator over the existing estimators are obtained. Also, Monte Carlo simulation studies are executed to compare the performance of the proposed biased estimator. Finally, a numerical example is given to illustrate some of the theoretical results. The proposed estimator outperforms all the other estimators in the various design of experiment used in this study.

Keywords: Multicollinearity; Logistics; Estimators; Simulations; Parameter.
MSC2010: 626M10, 68U20, 91G70, 65C20, 65C05.

1 Introduction

A binary logistic regression model has been one of the commonly used to describe the relationship between a binary response variable and one or more predictor variables. This model is often used in the areas of applied sciences such as bio-statistics, criminology, business and finance, engineering, biology and medical research. One among the methods of estimating the parameters in a logistic model is the maximum likelihood estimator (MLE). The predictor variables may be correlated when they are continuous, giving rise to the problem that is termed multicollinearity. When multicollinearity is present, and there is adoption of MLE, the conclusion about the estimated model

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might not be reliable [1]. An alternative method of estimation is the ridge regression (RR) estimator attributed to Hoerl and Kennard [2]. They introduced a biasing parameter k to the ML estimator to reduce the effect of multicollinearity. Several authors have worked on the improvement in the RR estimator in the linear regression model and also bringing this idea to the binary logistic. Authors like Schaeffer et al [3] adjusted the ridge estimator to the logistic regression model to handle the problem of multicollinearity. Also, Kibria et al. [4] evaluate the performances of some biasing parameters in logistic ridge regression. The Liu-type estimator was introduced to the logistic regression model by Inan and Erdogan [5]. The restricted ridge estimator was harmonized to the logistic regression model by Asar et al. [6]. Asar and Genc [7]. adjusted the two-parameter ridge estimator in the binary logistic regression. First-order approximated ridge estimator is proposed for use in logistic regression also by Ozkale [8]. New two-parameter estimators for the logistic regression model by Awwad et al. [9] was also introduced. Ertan and Akay [10], proposed a general estimator which includes other biased estimators, such as the Logistic Ridge, Logistic Liu and the estimators with two biasing parameters as special cases too. In addition the Logistic Kibria-Lukman estimator (LKLE) and (LMRT) were also proposed to handle multicollinearity for the logistic regression model by Lukman et al [1, 11]. Other researchers such as Huang [12] also work on handling multicollinearity for the logistic regression model.

Ahmad and Aslam [13] introduced the modified ridge-type estimator in a linear regression model. The focus of this paper is to harmonize this estimator into the binary logistic regression model. The property of this new estimator was derived and compared with the existing ones using the mean squared error (MSE) criterion. The rest of the paper is as follows: In Section 2, the statistical methodology is described. The simulation results and numerical example are discussed in Section 3. Finally, a conclusion is given in section 4.

2 Statistical Methodology

The logistic regression model, where the distribution of the response (y) is Bernoulli: such that

$$y_i \sim Ber(v_i)$$
$$v_i = \frac{e^{x_i\beta}}{1+e^{x_i\beta}} \tag{2.1}$$

where x_i is given as the i^{th} row of matrix X with the dimension of $n \times p$ and β is unknown coefficients vector with the dimension of $p \times 1$. The transformation of logit is

$$f(x_i) = \ln\left(\frac{v_i}{1-v_i}\right) = x_i\beta. \tag{2.2}$$

The MLE is used widely in parameter estimation of this logistic model. The function of log likelihood is

$$L = \sum_{i=1}^{n} y_i \log(v_i) + \sum_{i=1}^{n}(1-y_i)\log(1-v_i), \tag{2.3}$$

$$\frac{\partial L}{\partial \beta} = \sum_{i=1}^{n}(y_i - v_i)x_i = 0. \tag{2.4}$$

With the iteratively reweighted least squares (IRLS), equation (2.4) is solved. Since it is nonlinear in parameter. So the MLE for the logistic model is defined as

$$\hat{\beta}_{MLE} = J^{-1}X'\hat{Q}\hat{z}, \tag{2.5}$$

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where $J = X'\hat{G}X$, and $\hat{G} = diag\Big(\hat{v}_i(1-\hat{v}_i)\Big)$ and $\hat{z}_i = \log(v_i) + \dfrac{y-\hat{v}_i}{\hat{v}_i(1-\hat{v}_i)}$.

In the presence of multicollinearity, an alternative to the MLE was introduced to handle the problem of multicollinearity called ridge regression for logistic (LRR) [3, 14]. The LRR is defined as:

$$\hat{\beta}_{LRR} = (J + kI_p)^{-1} J \hat{\beta}_{MLE}, \tag{2.6}$$

where I is the identity matrix, k $(k > 0)$ is the ridge parameter.

Also, Lukman et al. [15] proposed the modified ridge-type (MRT) estimator in linear regression model. This work is also extended to the logistic regression model too. The Logistic modified ridge-type (LMRT) [15] estimator is defined as:

$$\hat{\beta}_{LMRT} = (J + k(I+d)I_p)^{-1} J \hat{\beta}_{MLE}, \tag{2.7}$$

where k $(k > 0)$ and d $(0 < d < 1)$.

Ahmad and Aslam [13] introduced the modified new two parameter estimator in a linear regression model, this estimator is define as:

$$\hat{\beta}_{MNTP} = (X'X + I)^{-1}(X'X + dI)(X'X + kd)^{-1} X'y. \tag{2.8}$$

Therefore, in this study, we developed the logistic version of this estimator. The logistic new two parameter estimator is

$$\hat{\beta}_{LNTP} = ((J + I_p)^{-1}(J + dI_p)(J + kdI_p)^{-1} J \hat{\beta}_{MLE}, \tag{2.9}$$

where k $(k > 0)$ and d $(0 < d < 1)$.

2.1 MSEM and MSE Properties of the Estimators

The MSEM and MSE estimators of $\hat{\beta}$ are defined as

$$MSEM(\hat{\beta}) = \text{Cov}(\hat{\beta}) + \text{bias}(\hat{\beta})(\text{bias}(\hat{\beta}))^T. \tag{2.10}$$

$$MSE(\hat{\beta}) = \text{trace}\Big(\text{Cov}(\hat{\beta})\Big) + (\text{bias}(\hat{\beta}))^T \text{bias}(\hat{\beta}). \tag{2.11}$$

By the use of spectral decomposition of the matrix, $X^T\hat{G}X = T\Lambda T^T$, where T is the matrix whose columns are the eigenvectors of $X^T\hat{G}X$ and Λ is the matrix of eigenvalues of $X^T\hat{G}X$. The MSEM of the estimators is

$$MSEM(\hat{\alpha}_{MLE}) = T\Lambda^{-1}T^T \tag{2.12}$$

$$MSEM(\hat{\alpha}_{LRR}) = T\Lambda_k \Lambda \Lambda_k T^T + k^2 \Lambda_k \alpha\alpha^T \Lambda_k, \tag{2.13}$$

$\Lambda_k = (\Lambda + kI)^{-1}$.

$$MSEM(\hat{\alpha}_{LMRT}) = T\widetilde{\Lambda}_k \Lambda^{-1} \widetilde{\Lambda}_k T^T + (\widetilde{\Lambda}_k - 1)\alpha\alpha^T(\widetilde{\Lambda}_k - 1), \tag{2.14}$$

$\widetilde{\Lambda}_k = \Lambda(\Lambda + k(1+d)I)^{-1}$.

$$MSEM(\hat{\alpha}_{LNTP}) = T\Lambda_c\Lambda\Lambda_c^T + T(\Lambda_c - 1)\alpha\alpha^T(\Lambda_c - 1)^T T^T, \tag{2.15}$$

$\Lambda_c = (\Lambda + 1)^{-1}(\Lambda + d)(\Lambda + kd)^{-1}$.

where $\alpha = T^T\beta$, then the MSE of the estimators are

$$MSE(\hat{\alpha}_{MLE}) = \sum_{j=1}^{p} \frac{1}{\lambda_j}. \tag{2.16}$$

$$MSE(\hat{\alpha}_{LRE}) = \sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + k)^2} + k^2 \sum_{j=1}^{p} \frac{\alpha_j^2}{(\lambda_j + k)^2} \tag{2.17}$$

$$MSE(\alpha_{LMRT}) = \sum_{j=1}^{p} \frac{\lambda_j + [k(1+d)]^2]\hat{\alpha}_j^2}{(\lambda_j + k(1+d))^2} \tag{2.18}$$

$$MSE(\hat{\alpha}_{LNTP}) = \sum_{j=1}^{p} \frac{\lambda_j(\lambda_j + d)^2}{(\lambda_j + 1)^2(\lambda_j + kd)^2} + \sum_{j=1}^{p} \frac{[(1 - d + kd)\lambda_j + kd]^2\hat{\alpha}_j^2}{(\lambda_j + 1)^2(\lambda_j + kd)^2}. \tag{2.19}$$

The following lemmas are needful to prove the statistical property of the proposed estimator.

Lemma 2.1. *Let M be a positive definite matrix, that is, $M > 0$, and α be some vector, then $M - \alpha\alpha^T \geq 0$ if and only id $\alpha^T M^{-1}\alpha \leq 1$ [16].*

Lemma 2.2. *Let $\hat{\beta}_j = A_{jy}, j = 1, 2$ be two linear estimators of β [17]. Suppose that $D = Cov(\hat{\beta}_1) - Cov(\hat{\beta}_2) > 0$, where $Cov(\hat{\beta}_j), j = 1, 2$ denotes the covariance matrix of $\hat{\beta}_j$ and $b = bias(\hat{\beta}_j) = (A_jX - I)\beta, j = 1, 2$. Consequently,*

$$\begin{aligned} \Delta(\hat{\beta}_1 - \hat{\beta}_2) &= MSEM(\hat{\beta}_1) - MSEM(\hat{\beta}_2) \\ &= \sigma^2 D + b_1b_1^T - b_2b_2^T > 0 \end{aligned} \tag{2.20}$$

if and only if $b_2^T[\sigma^2 D + b_1b_1^T]^{-1}b_2 < 1$, where $MSEM(\hat{\beta}_j) = Cov(\hat{\beta}_j) + b_j^T b_j$.

Comparison between $\hat{\alpha}_{LNTP}$ and $\hat{\alpha}_{MLE}$.

Theorem 2.3. *If $k > 0$ the estimator $\hat{\alpha}_{LNTP}$ is preferable to the estimator $\hat{\alpha}_{MLE}$ in the MMSE sense where $MSEM(\hat{\alpha}_{MLE}) - MSEM(\hat{\alpha}_{LNTP}) > 0$ if and only if*

$$(\Lambda_c - 1)\alpha T^T[\Lambda^{-1} - \Lambda_c\Lambda\Lambda_c^T](\Lambda_c - I)^T T^T\alpha^T < 1.$$

Proof

$$\begin{aligned} &\text{Cov}\Big(\alpha_{MLE}\Big) - \text{Cov}\Big(\alpha_{LNTP}\Big) \\ &= \Lambda^{-1} - (\Lambda + 1)^{-1}(\Lambda + dI)(\Lambda + kdI)^{-1}\Lambda(\Lambda + kd)^{-1}(\Lambda + dI)^T(\Lambda + I)^{-1} \\ &= T\text{diag}\left[\frac{1}{\lambda_i} - \frac{(\lambda_i + d)^2\lambda_i}{(\lambda_i + 1)^2(\lambda_i + kd)^2}\right]_{i=1}^{p} T^T. \end{aligned}$$

Now, $\left[\Lambda^{-1} - \Lambda_c\Lambda\Lambda_c^T\right]$ will be pd if and only if

$$(\lambda_i + 1)^2(\lambda_i + kd)^2 - \lambda_i^2(\Lambda_i + d)^2 > 0.$$

The result is true for $k > 0$ and $0 < d < 1$ and hence, the prove is completed by Lemma 2.2.

Comparison between $\hat{\alpha}_{LNTP}$ and $\hat{\alpha}_{LRE}$.

Theorem 2.4. *If $k > 0$ the estimator $\hat{\alpha}_{LNTP}$ is preferable to the estimator $\hat{\alpha}_{LRE}$ in the MMSE sense where $MSEM(\hat{\alpha}_{LRE}) - MSEM(\hat{\alpha}_{LNTP}) > 0$ if and only if*

$$(\Lambda_c - 1)\alpha T^T \left[\left(\Lambda_k \Lambda \Lambda_k^T - \Lambda_c \Lambda \Lambda_c^T\right) + k^2 \Lambda_k \alpha \alpha^T \Lambda_k^T\right] (\Lambda_c - 1)^T T^T \alpha^T < 1.$$

Proof

$$\begin{aligned}
&\mathrm{Cov}\Big(\alpha_{LRE}\Big) - \mathrm{Cov}\Big(\alpha_{LNTP}\Big) \\
&= T\Big((\Lambda + kI)^{-1}\Lambda(\Lambda + kI)^{-1} \\
&\quad - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + kdI)^{-1}\Lambda(\Lambda + kd)^{-1}(\Lambda + dI)^T(\Lambda + I)^{-1}\Big)T^T \\
&= T\mathrm{diag}\left[\frac{\lambda_i}{(\lambda_i + k)^2} - \frac{(\lambda_i + d)^2\lambda_i}{(\lambda_i + 1)^2(\lambda_i + kd)^2}\right]_{i=1}^p T^T \\
&= T\mathrm{diag}\left[\frac{\lambda_i(\lambda_i + 1)^2(\lambda_i + kd)^2 - (\lambda_i + k)^2(\lambda_i + d)^2\lambda_i}{(\lambda_i + k)^2(\lambda_i + 1)^2(\lambda_i + kd)^2}\right]_{i=1}^p T^T
\end{aligned}$$

Now, $\left[\Lambda_k \Lambda \Lambda_k^T - \Lambda_c \Lambda \Lambda_c^T\right]$ will be pd if and only if

$$\lambda_i(\lambda_i + 1)^2(\lambda_i + kd)^2 - (\lambda_i + k)^2(\lambda_i + d)^2\lambda_i > 0.$$

The result is true for $k > 0$ and $0 < d < 1$ and hence, the prove is completed by Lemma 2.2.

Comparison between $\hat{\alpha}_{LNTP}$ and $\hat{\alpha}_{LMRT}$.

Theorem 2.5. *If $k > 0$ and $0 < d < 1$ the estimator $\hat{\alpha}_{LNTP}$ is preferable to the estimator $\hat{\alpha}_{LMRT}$ in the MMSE sense where $MSEM(\hat{\alpha}_{LMRT}) - MSEM(\hat{\alpha}_{LNTP}) > 0$ if and only if*

$$(\Lambda_c - 1)\alpha T^T \left[\left(\widetilde{\Lambda}_k \Lambda \widetilde{\Lambda}_k^T - \Lambda_c \Lambda \Lambda_c^T\right) + (\widetilde{\Lambda}_k - 1)\alpha \alpha^T (\widetilde{\Lambda}_k - 1)\right] (\Lambda_c - 1)^T T^T \alpha^T < 1.$$

Proof

$$\begin{aligned}
&\mathrm{Cov}\Big(\alpha_{LMRT}\Big) - \mathrm{Cov}\Big(\alpha_{LNTP}\Big) \\
&= T((\Lambda + k(1+d))^{-1}\Lambda(\Lambda + k(1+d))^{-1} - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + kdI)^{-1}\Lambda(\Lambda + kd)^{-1}(\Lambda \\
&+ dI)^T(\Lambda + I)^{-1})T^T \\
&= T\mathrm{diag}\left[\frac{\lambda_i}{(\lambda_i + k(1+d))^2} - \frac{(\lambda_i + d)^2\lambda_i}{(\lambda_i + 1)^2(\lambda_i + kd)^2}\right]_{i=1}^p T^T \\
&= T\mathrm{diag}\left[\frac{\lambda_i(\lambda_i + 1)^2(\lambda_i + kd)^2 - (\lambda_i + k(1+d))^2(\lambda_i + d)^2\lambda_i}{(\lambda_i + k(1+d))^2(\lambda_i + 1)^2(\lambda_i + kd)^2}\right]_{i=1}^p T^T
\end{aligned}$$

Now, $\left[\widetilde{\Lambda}_k \Lambda \widetilde{\Lambda}^{T - \Lambda_c \Lambda \Lambda_c^T}_k\right]$ will be pd if and only if

$$\lambda_i(\lambda_i + 1)^2(\lambda_i + kd)^2 - (\lambda_i + k(1+d))^2(\Lambda_i + d)^2\lambda_i > 0.$$

The result is true for $k > 0$ and $0 < d < 1$ and hence, the prove is completed by Lemma 2.2.

2.2 Selection of Parameters k and d for LNTP

In this section, we develop some iterative methods to estimate k and d such that the mean squared error is minimized. Following Ahmad and Aslam [13] their k and d in logistic form is defined as follows:

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$$\hat{k}_{opt} = \frac{(\lambda_i + d) - (1-d)\lambda_i \hat{\alpha}_i^2}{(\lambda_i + I) d\hat{\alpha}_i^2}. \tag{2.21}$$

$$\hat{d}_{opt} = \sum_{i=1}^{p} \left[\frac{\lambda_i(\hat{\alpha}_i^2 - 1)}{(\lambda_i \hat{\alpha}_i^2 + 1 - \hat{k}_{opt} \lambda_i \hat{\alpha}_i^2)} \right]. \tag{2.22}$$

While the iterative steps are as follows:

Step 1: Obtain an initial estimate of d using $\hat{d} = \left(\frac{(\alpha_i^2 - 1)\lambda_i}{(1 + \lambda \alpha_i^2)} \right)$ to be substituted into k_{opt}.

Step 2: Obtain five different estimates of k_{opt} using $\hat{d}$ obtain in step 1

$$\hat{k}_{AM} = \frac{1}{p} \sum_{i=1}^{p} \frac{(\lambda_i + d) - \lambda_i(1-d)\hat{\alpha}_i^2}{d(\lambda_i + 1)\hat{\alpha}_i^2} = LNTP1 \tag{2.23}$$

$$\hat{k}_{HM} = p \sum_{i=1}^{p} \frac{(\lambda_i + d) - \lambda_i(1-d)\hat{\alpha}_i^2}{d(\lambda_i + 1)\hat{\alpha}_i^2} = LNTP2 \tag{2.24}$$

$$\hat{k}_{MAX} = \text{Maximum} \left(\sum_{i=1}^{p} \frac{(\lambda_i + d) - \lambda_i(1-d)\hat{\alpha}_i^2}{d(\lambda_i + 1)\hat{\alpha}_i^2} \right) = LNTP3 \tag{2.25}$$

$$\hat{k}_{MED} = \text{Median} \left(\sum_{i=1}^{p} \frac{(\lambda_i + d) - \lambda_i(1-d)\hat{\alpha}_i^2}{d(\lambda_i + 1)\hat{\alpha}_i^2} \right) = LNTP4 \tag{2.26}$$

$$\hat{k}_{MR} = \frac{1}{2} \left(\hat{k}_{MAX} + \hat{k}_{MIN} \right) = LNTP5. \tag{2.27}$$

Step 3: Estimates $\hat{d}_{opt}$ using $\hat{k}_{opt}$ in Step 2 as:

$$\hat{d}_{opt} = \sum_{i=1}^{p} \left[\frac{\lambda_i(\hat{\alpha}_i^2 - 1)}{(\lambda_i \hat{\alpha}_i^2 + 1 - \hat{k}_{opt} \lambda_i \hat{\alpha}_i^2)} \right] \tag{2.28}$$

Step 4: Finally, if $\hat{d}_{opt} < 0$, consider $\hat{d}_{opt} = \hat{d}$.

2.3 Monte Carlo simulation study

2.3.1 Design of the simulation

The effective factors in the simulation are chosen to be the degree of correlation among the explanatory variables, the sample size n and the number of explanatory variables p. Following McDonald and Galarneau [18], Kibria [19] , Owolabi et al. [20],Oladapo [21] and Idowu et al. [22].

$$x_{ij} = \left(1 - \rho^2\right)^{1/2} z_{ij} + \rho z_{ip}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, p \tag{2.29}$$

where z_{ij} is an independent standard normal distribution with mean zero and unit variance, ρ is the correlation between any two explanatory variables and p is the number of explanatory variables. The

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values of p will be taken as 0.8, 0.9, 0.95 and 0.99 respectively. Thus, the correlations between the variable are the same. Moreover, sample sizes equal to 10, 20, and 30,100,350 and 500 are considered. Finally the number of explanatory variables and logit model consisting of $p = 2, 3, 4, 5, 6, 7, 8, 9$ and 10 are considered in the design of the experiment. The response variable is generated from the Bernoulli distribution such that
$y_i \sim Ber(v_i)$ and $v_i = \frac{e^{x_i\beta}}{1+e^{x_i\beta}}$. The parameter values were chosen such that $\beta'\beta = 1$, which is a common restriction in simulation studies by Newhouse and Oman [23]. The estimated MSE is calculated as

$$MSE(\hat{\beta}) = \frac{1}{1000}\sum_{i=1}^{1000}\sum_{j=1}^{p}\left(\hat{\beta}_{ij} - \beta_j\right). \tag{2.30}$$

3 Results and Discussion

3.1 Results of the simulation

The experiment is repeated 1000 times. We provide the simulated SMSE values of the estimators in Tables 1–4. We summarize the results in the followings:

- According to the simulated SMSE values given in Table 1, the proposed estimator LNTP outperforms MLE and thereby the proposed LNTP3 out performs all other estimators compared with in this study.
- According to the simulated SMSE values given in Table 2, the proposed estimator LNTP outperforms MLE and thereby the proposed LNTP3 out performs all other estimators compared with in this study except at few instances when the degree of correlation is at 0.8 and 0.9 LNTP4 shows superiority.
- According to the simulated SMSE values given in Table 3 and 4, the proposed estimator LNTP outperforms MLE and thereby the proposed LNTP4, LNTP3 and LNTP2 competes favorably dominating one another at one point in this study.
- It's also observe that If the correlation among the explanatory variables is high as 0.99, LNTP3 dominates all other estimators in this entire study.
- It is also observe from the table that an increase in the sample size n makes a decrease in the SMSE of the estimators with a few exceptions. If the correlation among the explanatory variables is high, increasing the sample size affects the estimators positively.
- Moreover, if we consider the performances of the estimators according to the degree of correlation ρ, we conclude that SMSE values of the estimators increase as ρ increases in general.
- Also, if we consider the performances of the estimators according to the degree of correlation ρ, we conclude that SMSE values of the estimators increases as the number of explanatory variable increases at a ρ level.

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Table 1. Estimated MSE for different estimator when p=2, 3 and 4

n	ρ	p	ML	LRIDGE	LMRT	LNTP	LNTP1	LNTP2	LNTP3	LNTP4	LNTP5
10	0.8	2	125332.5	9.4384	10.1243	17.8234	19.4968	19.4968	**5.1612**	10.9460	19.4968
		3	3758011	9.7023	13.2901	24.1114	25.5716	8.3202	**1.1537**	3.3747	6.7171
		4	7561030	9.5897	13.3318	25.7473	8.1381	2.3506	**0.8973**	1.3001	1.9441
	0.9	2	310417.7	16.9045	17.6019	32.0419	34.3837	34.3837	**8.3433**	18.6688	34.3837
		3	6687670	19.1049	24.9820	47.2948	111.0005	31.2331	**1.9280**	10.1149	23.7311
		4	15764720	17.8263	24.4980	45.9435	17.7334	4.0246	**0.8992**	1.8506	3.3380
	0.95	2	720321.8	31.7220	32.4327	60.3225	63.5864	63.5864	**14.7627**	33.8344	63.5864
		3	12034180	37.6203	47.3791	83.9736	78.4938	48.5756	**2.6957**	20.9630	49.5468
		4	29660120	35.1410	48.5813	88.7416	39.2246	8.1954	**0.9602**	3.2421	6.8126
	0.99	2	1173183	155.5364	156.2107	293.2940	300.4217	300.4217	**69.0065**	159.3312	300.4217
		3	62495120	194.2840	243.6252	404.1797	366.8548	260.8227	**12.5436**	116.1595	263.4664
		4	196472800	209.1604	288.9481	432.0622	226.6464	54.2938	**2.0963**	18.2079	42.5936
20	0.8	2	189.32921	4.6703	5.3648	7.1627	6.3874	6.3874	**2.8333**	4.4375	6.3874
		3	139349.9	15.2629	17.9498	27.9140	25.8748	15.8314	**1.1943**	6.2211	14.7220
		4	565495.3	13.8110	19.6941	26.1922	12.4062	6.0103	**0.7202**	2.4553	4.9388
	0.9	2	276.7577	8.4388	9.1409	13.4906	12.7815	12.7815	**4.6582**	8.3283	12.7815
		3	26289810	93.2708	119.2709	5279.9110	104.6964	73.0860	**11.8314**	39.5941	67.2955
		4	1134631	25.4636	35.3503	47.9507	23.1545	10.6581	**0.7517**	4.1558	8.8647
	0.95	2	411.6574	15.5304	16.2611	25.4410	24.6415	24.6415	**7.9818**	15.3791	24.6415
		3	48259210	179.0751	216.7037	20349.65	195.2208	139.2025	**21.6984**	76.2253	128.7320
		4	2167485	49.7345	67.6214	92.9252	45.5175	20.2171	**0.8701**	7.4547	16.7832
	0.99	2	1348.375	72.6507	73.3035	117.6530	116.8536	116.8536	**36.4798**	72.1961	116.8536
		3	216790400	868.8194	975.3795	483101.7	929.0802	677.2772	**96.5388**	377.8956	639.7939
		4	11412940	239.2485	322.1041	450.6092	232.3002	93.9258	**1.8482**	32.7368	77.8528
30	0.8	2	12.869457	4.1231	4.6625	6.0161	5.4412	5.4412	**2.5506**	3.8952	5.4412
		3	675.524	8.0841	10.7050	13.6872	9.9673	4.5176	**0.8078**	2.3758	4.0825
		4	345459.8	13.5498	19.3615	23.4093	9.8578	4.7142	**0.6384**	2.1098	3.8947
	0.9	2	23.420274	7.2198	7.7659	10.9704	10.4213	10.4213	**4.1230**	7.0569	10.4213
		3	969.9359	14.3313	17.9712	24.3284	18.2109	8.1816	**1.0538**	4.0875	7.4247
		4	543617.9	24.5602	34.4456	43.2918	18.3892	8.0605	**0.6395**	3.3472	6.5916
	0.95	2	43.75956	13.2478	13.7448	20.5732	19.9833	19.9833	**7.1032**	13.0283	19.9833
		3	109.4695	26.2225	31.7192	44.0400	33.7218	15.5177	**1.7637**	7.8604	14.3417
		4	936710.6	44.4375	60.6009	78.8472	39.0094	16.5044	**0.9343**	7.0075	13.8007
	0.99	2	198.4947	58.4734	58.9086	92.3797	91.5101	91.5101	**30.6587**	58.2872	91.5101
		3	433.8411	106.9732	128.1819	181.2852	142.5104	70.2667	**6.2014**	35.0098	64.2789
		4	3973275	185.3305	245.0112	319.7612	186.8946	74.3418	**1.6648**	26.9463	59.5066

Bold values show the smallest MSE

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Table 2: Estimated MSE for different estimators when p=2, 3, 4, 5,6,7,8,9,10 and n=100

n	ρ	p	ML	LRIDGE	LMRT	LNTP	LNTP1	LNTP2	LNTP3	LNTP4	LNTP5
		2	2.61323	1.0176	1.4087	1.2011	0.7694	0.7694	**0.6383**	0.6765	0.7694
		3	5.38281	1.7406	2.6245	2.4642	1.3364	0.9139	**0.4792**	0.6872	0.8942
		4	8.589513	2.5045	3.9059	3.9020	1.6651	0.7958	**0.4856**	0.5172	0.7064
		5	12.95187	3.4837	5.5375	5.7087	2.4939	0.8516	0.5581	**0.5483**	0.7201
	0.8	6	19.49529	4.9836	8.0464	8.8392	2.6811	0.8534	0.5992	**0.4729**	0.6806
		7	26.32997	6.3827	10.4110	11.1575	3.8668	0.8862	0.6721	**0.5092**	0.6794
		8	31.77028	7.3261	12.4032	13.2366	3.7347	0.8658	0.7311	**0.5262**	0.6409
		9	52.6131	10.4395	17.6243	19.3029	5.7760	0.9698	0.7608	**0.5306**	0.6915
		10	18391.27	12.7146	21.5546	23.3231	6.3589	0.9845	0.7929	**0.4928**	0.5959
		2	4.7064133	1.6624	2.0884	2.1471	1.7091	1.7091	**1.1959**	1.4272	1.7091
		3	9.915866	3.0898	4.2205	4.5042	2.8917	1.6210	**0.5301**	1.1050	1.5425
		4	15.70225	4.4106	6.4195	7.1820	3.0739	1.4374	**0.4094**	0.7622	1.2404
		5	23.5194	6.1850	9.3129	10.3060	4.6336	1.5073	**0.4707**	0.7278	1.1774
	0.9	6	34.37297	8.6601	13.5619	15.2461	4.7095	1.3928	**0.5218**	0.6263	1.0522
		7	44.96551	10.8797	17.1971	19.1038	6.6992	1.5551	**0.5841**	0.6398	1.0499
		8	55.23405	12.7916	20.9538	22.6592	6.8440	1.3747	0.6563	**0.5117**	0.8079
		9	86.82022	17.5850	29.2195	32.5356	10.0271	1.6341	0.6809	**0.5381**	0.8851
100		10	18490.55	21.9897	36.6934	40.5930	11.3127	1.8981	0.7267	**0.5878**	0.9434
		2	9.230376	3.1921	3.6110	4.4532	4.0643	4.0643	**2.2108**	3.1107	4.0643
		3	18.85266	5.7179	7.2582	8.6769	6.0703	2.9518	**0.6385**	1.7886	2.7859
		4	29.28337	8.0735	11.2167	13.5030	5.6163	2.6189	**0.3804**	1.2173	2.2280
		5	44.01192	11.5372	16.8125	19.8633	8.6747	2.6513	**0.3895**	1.0745	2.0339
	0.95	6	62.64786	15.6795	23.8461	27.8667	9.0930	2.7482	**0.4221**	0.9238	1.8764
		7	79.45901	19.2477	30.0358	34.3997	12.0511	2.6451	**0.5023**	0.9043	1.7274
		8	98.38952	22.9338	37.0477	41.2078	12.8405	2.1632	**0.5894**	0.6865	1.2740
		9	150.935	30.8671	50.4685	57.3672	18.0521	2.7012	**0.6150**	0.6799	1.3503
		10	187.3834	37.8023	63.0590	69.1266	18.7344	2.5099	**0.6704**	0.6402	1.1889
		2	43.6017	14.3313	14.7172	21.4573	21.1319	21.1319	**8.0571**	14.2102	21.1319
		3	88.76302	25.6902	30.6579	41.7968	32.0685	15.6224	**2.0149**	8.6433	14.4278
		4	135.5694	37.1699	48.9015	64.1985	27.9879	13.3246	**0.4793**	5.4801	11.1854
		5	200.082	52.3964	73.4540	92.0879	44.1492	14.1556	**0.3342**	5.0819	10.8784
	0.99	6	279.3253	70.2817	104.0271	126.1366	45.1155	13.4643	**0.2971**	3.9974	9.2754
		7	345.7138	84.5829	129.8054	152.9925	59.7759	11.5142	**0.3303**	2.6433	6.6740
		8	427.4579	101.9472	159.2161	183.2628	60.6668	11.2916	**0.3785**	2.1404	5.5461
		9	600.0443	136.5561	217.0108	245.9353	82.1386	11.7087	**0.4171**	2.0230	5.2950
		10	728.0066	154.8061	253.8591	283.6435	83.9024	12.3542	**0.4809**	2.0398	5.4081

Bold values show the smallest MSE

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Table 3. Estimated MSE for different estimator when p=2, 3, 4, 5, 6,7,8,9 and 10

n	ρ	p	ML	LRIDGE	LMRT	LNTP	LNTP1	LNTP2	LNTP3	LNTP4	LNTP5
		2	0.5638962	0.2694	0.5674	0.4040	**0.1271**	**0.1271**	0.1746	0.1425	**0.1271**
		3	1.109471	0.4566	0.9542	0.7165	0.2541	0.2767	0.3358	**0.2609**	0.2896
		4	1.831482	0.6575	1.3414	1.0480	0.3609	0.3159	0.4513	**0.2914**	0.3211
		5	2.731089	0.9111	1.7916	1.4958	0.5502	**0.2775**	0.5159	0.2810	0.2813
	0.8	6	3.954384	1.2454	2.3690	2.0795	0.6397	**0.2906**	0.5995	0.3168	0.3022
		7	5.649396	1.6602	3.0486	2.7817	0.9199	**0.3370**	0.6493	0.3456	0.3370
		8	6.946472	1.9599	3.6342	3.3866	0.9244	**0.3124**	0.7172	0.3651	0.3373
		9	8.714361	2.3968	4.4075	4.2128	1.1662	**0.3205**	0.7605	0.3727	0.3336
		10	10.01353	2.6929	4.9177	4.7554	1.2304	**0.3148**	0.7972	0.3879	0.3404
		2	1.0379335	0.4164	0.7827	0.5215	0.2222	0.2222	0.2143	**0.2066**	0.2222
		3	2.0392811	0.7317	1.3773	1.0540	0.4385	0.4373	0.3449	**0.3533**	0.4391
		4	3.4056051	1.1189	2.0201	1.6895	0.6965	0.3713	0.3851	**0.2808**	0.3441
		5	4.922689	1.4905	2.6697	2.3701	0.9732	0.3983	0.4524	**0.3058**	0.3599
	0.9	6	7.07393	2.0789	3.6318	3.3776	1.0959	0.3789	0.5381	**0.3019**	0.3339
		7	10.12599	2.8185	4.8773	4.7379	1.5894	0.4090	0.5669	**0.2963**	0.3371
		8	12.25591	3.3155	5.8333	5.6878	1.5932	0.3837	0.6387	**0.3058**	0.3151
		9	15.22944	4.0319	7.0631	7.0729	2.0836	0.4039	0.6771	**0.3207**	0.3184
350		10	17.25434	4.5030	7.8941	7.9285	2.1365	0.4079	0.7368	**0.3604**	0.3555
		2	1.988132	0.7047	1.1207	0.8241	0.4697	0.4697	**0.3326**	0.3897	0.4697
		3	3.916875	1.2871	2.0764	1.7745	0.9563	0.6952	**0.3182**	0.4741	0.6618
		4	6.367515	1.9327	3.1577	2.9084	1.3003	0.5527	0.3259	**0.3159**	0.4644
		5	9.201272	2.6531	4.3228	4.2177	1.8594	0.5590	0.3693	**0.3161**	0.4613
	0.95	6	13.28881	3.7763	6.1256	6.1080	1.9619	0.5427	0.4355	**0.2896**	0.3987
		7	18.70282	5.0725	8.3210	8.4831	2.9221	0.5704	0.4879	**0.2980**	0.3979
		8	22.23708	5.7951	9.7282	10.0439	2.9095	0.6036	0.5394	**0.2895**	0.3861
		9	27.9824	7.2985	12.2652	12.8115	3.8197	0.5861	0.5898	**0.2878**	0.3527
		10	31.22896	8.0850	13.6545	14.2664	4.0791	0.6606	0.6504	**0.3339**	0.4169
		2	9.4559717	2.8955	3.3268	4.1958	3.8830	3.8830	**1.8173**	2.7925	3.8830
		3	18.8773	5.7175	7.1546	8.6375	6.2765	3.1495	**0.4986**	1.6862	2.8617
		4	29.60242	8.2925	11.3898	13.3771	6.0502	2.6771	**0.2533**	1.1904	2.1799
		5	42.0759	11.5232	16.6057	19.1673	8.8317	2.4024	**0.2489**	0.8221	1.6738
	0.99	6	60.73508	16.7192	24.9837	28.4840	10.5687	2.4042	**0.2713**	0.6834	1.4702
		7	83.55982	21.9398	34.0622	38.5102	14.0478	2.5483	**0.3038**	0.6554	1.4653
		8	99.7769	25.7998	40.6039	45.6965	14.6094	2.8355	**0.3449**	0.7398	1.6371
		9	121.1319	30.5985	49.2469	55.1348	18.5933	2.3883	**0.3844**	0.5542	1.1728
		10	134.2877	34.0159	55.7709	61.0410	17.3755	2.2879	0.4370	**0.4324**	0.9282

Bold values show the smallest MSE

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Table 4. Estimated MSE for different estimator when p=2, 3, 4, 5, 6,7,8,9 and 10

n	ρ	p	ML	LRIDGE	LMRT	LNTP	LNTP1	LNTP2	LNTP3	LNTP4	LNTP5
500	0.8	2	0.3528762	0.1891	0.4150	0.3170	**0.0918**	**0.0918**	0.1217	0.1078	**0.0918**
		3	0.8087279	0.3674	0.7980	0.6129	0.1968	**0.1809**	0.2853	0.2003	0.1921
		4	1.387308	0.5523	1.1213	0.8823	0.2870	**0.2563**	0.4064	0.2691	0.2706
		5	2.0229	0.7066	1.4602	1.1809	0.3918	**0.2480**	0.4832	0.2894	0.2630
		6	2.728624	0.8911	1.7860	1.4836	0.4699	**0.2435**	0.5755	0.3141	0.2665
		7	3.325482	1.0478	2.0822	1.7831	0.5675	**0.2470**	0.6583	0.3325	0.2820
		8	4.586665	1.3876	2.6665	2.3779	0.6811	**0.2593**	0.7019	0.3443	0.2914
		9	5.722459	1.6820	3.1711	2.9016	0.8602	**0.2603**	0.7567	0.3818	0.3139
		10	6.93031	1.9922	3.6885	3.4470	0.9207	**0.2610**	0.7905	0.3790	0.3163
	0.9	2	0.6637589	0.2945	0.6133	0.4137	0.1403	**0.1403**	0.1611	0.1456	0.1403
		3	1.476618	0.5710	1.1468	0.8514	0.3065	**0.2913**	0.3056	0.2644	0.3007
		4	2.558362	0.9011	1.6670	1.3487	0.5259	**0.3334**	0.3586	0.2633	0.3240
		5	3.660321	1.1538	2.1822	1.8625	0.7381	**0.2896**	0.4107	0.2476	0.2758
		6	4.938111	1.4874	2.7141	2.3815	0.7800	**0.3142**	0.5037	0.2782	0.2963
		7	6.024327	1.7783	3.2115	2.9181	1.0437	**0.3420**	0.5933	0.3046	0.3154
		8	8.097346	2.3031	4.1375	3.8917	1.1679	**0.3106**	0.6296	0.2946	0.2872
		9	9.925171	2.7539	4.9405	4.7504	1.4162	**0.3003**	0.6918	0.3213	0.2940
		10	11.89501	3.2506	5.7504	5.6343	1.5464	**0.2897**	0.7188	0.3183	0.2922
	0.95	2	1.2932657	0.4900	0.8820	0.5893	0.2639	0.2639	**0.2159**	0.2345	0.2639
		3	2.744971	0.9456	1.6407	1.2930	0.6158	0.4821	**0.2845**	0.3556	0.4749
		4	4.728357	1.5123	2.5364	2.2133	0.9382	0.4417	**0.3024**	0.2791	0.3885
		5	6.94674	2.0482	3.4984	3.2424	1.3799	0.4180	**0.3513**	0.2626	0.3524
		6	9.101306	2.5863	4.3879	4.1893	1.3363	0.4162	0.4435	**0.2701**	0.3410
		7	11.01472	3.0865	5.2085	5.0714	1.8550	0.4352	0.5199	**0.3020**	0.3511
		8	14.68191	4.0047	6.7804	6.7886	2.1648	0.4267	0.5401	**0.2648**	0.3084
		9	17.97793	4.8149	8.2962	8.3705	2.5484	0.4589	0.6021	**0.2975**	0.3407
		10	21.23405	5.6108	9.6616	9.8253	2.7241	0.4642	0.6341	**0.2932**	0.3239
	0.99	2	6.1061619	1.9005	2.3355	2.6536	2.3155	2.3155	**1.3073**	1.7878	2.3155
		3	12.92049	3.8956	5.1952	5.8059	4.1667	1.7241	**0.3596**	1.0023	1.5921
		4	21.79521	6.3662	8.8859	9.9502	4.1998	2.0324	**0.2182**	0.9442	1.6935
		5	32.048	8.8702	12.9594	14.6899	6.3704	1.7772	**0.2122**	0.6488	1.2732
		6	42.47068	11.6666	17.4835	19.5732	6.9873	1.8361	**0.2442**	0.5657	1.2028
		7	49.25341	13.2600	20.4810	22.8040	8.2785	1.6679	**0.3092**	0.4600	0.9491
		8	64.87688	16.9304	26.8172	29.5350	9.9125	1.6963	**0.3357**	0.3837	0.8171
		9	78.71493	20.4638	33.1393	36.3955	11.6855	1.6110	**0.3757**	0.4033	0.8349
		10	91.55276	23.3955	38.4855	41.8101	11.9766	1.8040	**0.4225**	0.4561	0.8268

Bold values show the smallest MSE

3.2 Application

The dataset used in this study was originally adopted by Pena et al. [24] and later used by Özkale [8]. Pena et al. employed logistic model to examine the regressors of temperature effect, pH, as well soluble solids concentration with the nisin concentration on the response of Alicyclobacillus growth probability for apple Juice. The eigenvalues of the matrix are 13464.7990, 1715.9257, 56.5515 and 3.5445. Consequently, the condition index (C.I) is 61.6342, which revealed presence of multicollinearity in the model. The estimated values of regression coefficient from each of the estimators and their corresponding mean squared error are available in Table 5. From Table 5, the ML estimator provides the least performance as expected when there is multicollinearity. The efficiency of biased estimators depends on the choice of the biasing parameter k and d. The proposed estimators all performed very well and one of them provides the smallest mean square error which also align with the simulation result too.

Table 5: Regression coefficients and MSE

	ML	LRIDGE	LMRT	LNTP	LNTP1	LNTP2	LNTP3	LNTP4	LNTP5
$\hat{\beta}_1$	1.0834	0.0010	0.8922	0.0029	0.0110	0.0054	0.0029	0.0008	0.0108
$\hat{\beta}_2$	-0.0543	-0.0163	-0.0515	-0.0277	-0.0345	-0.0317	-0.0277	-0.0165	-0.0345
$\hat{\beta}_3$	0.0529	0.0055	0.0543	0.0155	0.0278	0.0211	0.0155	0.0056	0.0277
$\hat{\beta}_4$	-0.4255	-0.0010	-0.3710	-0.0060	-0.0292	-0.0140	-0.0060	-0.0009	-0.0289
MSE	0.3005	0.2196	0.2168	0.1851	0.1788	0.1819	0.1855	0.2188	0.1788

4 Conclusion

In this article, we proposed a logistic new two parameter estimator to mitigate the multicollinearity problem in a logistic regression model. Also, we established the superiority of this new estimator over the existing ones in terms of their corresponding MSE. The performance of the estimators were evaluated using both the theoretical approach and the Monte Carlo simulation study. In the design of the Monte Carlo experiment, factors such as the degree of correlation, the sample size and the number of explanatory variables were varied. The results showed that the performance of the estimators was highly dependent on these factors. Finally, to illustrate the efficiency of the proposed estimator, we applied a pena dataset and observed that the results agreed with those of the simulation study to some extent. The findings of this study is hereby suggested to be helpful and useful for practitioners and applied researchers who use a logistic regression model with correlated explanatory variables in their various fields and research works.

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