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Markov Switching Autoregressive Model: A Volatility Model With Application to All Share Index Returns

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Abstract

This study explored the Autoregressive models incorporating Regime switching or Markov switching non-linear predictive models. This stemmed from the complexities observed in economic phenomena the understanding of which will help in recommending better model fits. The study collected data on all share index returns (Jan, 1985 - December 2019) from the Nigeria Stock Exchange, fit an appropriate MS-AR model and estimate its parameters. The parameters of the model were obtained while their properties like the expected duration, autocorrelation measure and the goodness of fit were equally computed in testing the applicability of the model. The result shows that the MS (3)-AR (3) as a predictive model was appropriate, efficient and robust enough for forecasting the returns of the all-share index of the Nigerian stock exchange over the sampled period. The study is therefore relevant for modelling all share index returns by investors, policy makers, researchers and the general public.

Keywords: ASIR, AutoRegressive, Markov Switching, Regimes, Transition probability, Volatility.
MSC2010: 13P25.

1 Introduction

The complexities of events observed in financial statistics show that the traditional modelling techniques which assumed linearity and a Gaussian process no longer hold [1]. In all fields of human endeavour, non-linear models are becoming conventional for the complexity and interrelatedness of phenomena. [2] observed that the dynamism of time events makes modelling a thought-provoking task and that the dynamics of systems are no longer quite explained by linear models. Hence, non-linear models of which Markov switching is classified have gained prominence in capturing time-varying events and modelling them with better precision. Markov switching (MS) or regime-switching (RS) models according to [3], characterise time variant event properties at different regimes because the conditional homoscedastic assumptions of the autoregressive moving average models have failed. Hence, the need for a model that takes into account the heteroscedastic properties of the dynamic model under consideration and a Markov or regime switching becomes appropriate. However, the development and fitting of these models are quite tasking but with the aid of available computing power, they are better coped with.

The application of Markov Switching models to economic, financial, environmental, epidemiological, physical and other areas has been explored by different authors, stock market performance and economic regimes in Nigeria [4] modelling stock market returns in Nigeria [3], Exchange rate in Nigeria [5], Implied volatility of market indices with S & P 500 and DAX [6], Oil shocks in G-7 countries [7], heteroscedasticity in Bitcoin [8], US GNP growth [9], performance in controlling the Covid-19 pandemic [10], and to infer dry and rainy periods from telecommunication microwave link signals [11]. Markov switching models are a family of models that introduce time variation in the parameters of a time series in the form of their state- or regime-specific values. These models can capture the dynamics of complex processes that switch between different modes of behaviour, such as economic cycles, financial crises, or mood disorders. The switching between states is governed by a latent discrete-valued stochastic process, usually assumed to be a Markov chain, which means that the current state depends only on the previous state and not on the entire history of the process. Markov switching models can be applied to various types of time series models, such as autoregressive, dynamic regression, or state-space models.

Markov switching models are applicable in modelling the interest rate as a two-stage process that switches between high and low-interest rate regimes, modelling the GDP growth rate as a three-state process that switches between expansion, recession, and crisis regimes, modelling the mood states of bipolar patients as a two-stage process that switches between manic and depressive episodes [12]. Markov switching models can be estimated using various methods, such as maximum likelihood, Bayesian inference, or filtering and smoothing algorithms. The estimation results can provide information about the state-dependent parameters, the transition probabilities between states, the expected duration of each state, and the probabilities of being in each state at any given time. [13] noted that in MS models, the long-term behaviour of a time series is disintegrated by recognising the states or regimes that it alternates, and the average time it remains in each state or regime. The disintegration of the series' complete behaviour is then designated by the probability distribution called the transition probability that rules the shifts among the states or regimes. Markov Switching models introduce time variation in the parameters as states or regime-specific values which are characterised by detailed patterns of volatility, skewness, and kurtosis that arise in these models with different forms of the transition matrix and analyses of the implied properties of the model [14, 15].

The Markov Switching model has key assumptions that the switching mechanism is controlled by an unobservable state variable that follows a first-order Markov chain, the current value (X_n) of the regime's variable is contingent on its immediate past value (X_{n-1}), with a prevalent structure for a random period which may change when it undergoes a switch, the model emphasises

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on the expected behaviour of the variables, the transition probabilities designate the prospect that the current regime remains unchanged, the unidentified error distribution is approximately normal [12, 16]. Markov Switching models as regime-switching models differ from other threshold models because the threshold models switch deterministically within a threshold switch stochastically. It differs from the hidden Markov model because Markov Switching models are a first-order Markov chain that depends only on the current state and the state before while hidden Markov models are doubly stochastic processes involving an underlying unobservable stochastic process which results observable. Also, the Markov switching model assumes that the error distribution is unknown but is approximated by a blend of normal distributions while other regime-switching models make different assumptions about the distribution of the error ([14, 16, 17, 18]. With the varied applications in different fields, the present study explores the volatility model incorporating regime or Markov switching (RSM or MSM) model to fit all share index returns of the Nigeria stock exchange (NGX).

2 Methods and Material

This section specifies the appropriate model of interest - the Markov Switching Autoregressive model and estimates its parameters, transition probabilities, expected durations and other characteristics with applications.

2.1 Model Specification for MS-AR

The regimes of the Markov Switching Autoregressive (MS-AR) model with state-dependent mean and variance for the returns, Z_t (differenced log all share index) at time t was determined using the Akaike information criteria using the equation stated by [3]) as:

$$r_t = c_{S_t} + \varphi_1 \left(r_{t-1} - c_{s_{t-1}}\right) + \varphi_2 \left(r_{t-2} - c_{s_{t-2}}\right) + \ldots + \varphi_p \left(r_{t-p} - c_{s_{t-p}}\right) + \varepsilon_t \tag{2.1}$$

$$\varepsilon_t \sim i.i.d\; N(0,\; {\sigma_{S_t}}^2) \tag{2.2}$$

where

$$c_{s_t} = c_0 S_{0t} + c_1 S_{1t} + c_2 S_{2t} + \ldots + c_p S_{pt} + {\sigma_{S_t}}^2$$

$$= {\sigma_1}^2 S_{1t} + {\sigma_2}^2 S_{2t} + \ldots + {\sigma_p}^2 S_{pt} \tag{2.3}$$

c_{S_t} is the state-dependent mean, ${\sigma_{S_t}}^2$ is the state-dependent variance and $\varphi_1, \varphi_2, \varphi_3$ are the autoregressive coefficients produced by different subsamples. This study modelled the regimes, $S_{it,}\quad i = 1\,,\;\ldots,\; m$ are the outcomes of the unobserved m-state Markov chain with S_t independent of the ε_t for all t.

[19] initiated the stochastic regime-switching MS-AR model for the three regimes while [20] modelled three regimes MS-AR with autoregressive AR(p) using:

$$r_t = \begin{cases} a_1 + B_{11}r_{t-1} + \cdots + B_{p1}r_{t-p} + \epsilon_t & \forall S_t = 1 \\ a_2 + B_{12}r_{t-1} + \cdots + B_{p2}r_{t-p} + \epsilon_t & \forall S_t = 2 \\ a_1 + B_{13}r_{t-1} + \cdots + B_{p3}r_{t-p} + \epsilon_t & \forall S_t = 3 \end{cases} \tag{2.4}$$

with the regimes indexed by S_t.
MS-AR models have coefficients of its autoregressive component contingent on S_t at time t where S_t are separate unobservable variables. These separate variables were termed as accumulation or distribution phase (S_1), big-moves phase (S_2) and excess or panic phase (S_3) of the returns of the all-share index by [3] and were assumed to be ergodic first-order Markov chains implying that the recent S_t regimes observations of the returns captures the impacts of prior observations of the returns [21, 22, 23, 24].

2.2 Transition Probabilities

The transition probability of the regimes is given as:

$$p_{ij} = p\left(S_t = \frac{j}{S_{t-1}}\right) = i \quad \forall i, j = 1, \ldots, m \tag{2.5}$$

$$\sum_{i=1}^{m} p_{ij} = 1 \tag{2.6}$$

The transition matrix of switching from phase (regime) i to phase (regime) j is denoted as P where

$$P = \begin{bmatrix} p_{11} & \ldots & p_{1m} \\ \vdots & \vdots & \vdots \\ p_{m1} & \ldots & p_{mm} \end{bmatrix} \tag{2.7}$$

And $p_{11} + \ldots + p_{1m} = 1, p_{m1} + \ldots + p_{mm} = 1$ and higher probabilities denoting a longer time to shift to another regime.

2.3 Expected Duration

The expected duration which measures the length of stay of the system in a particular state or regime according to [25] and [3] is estimated using the function.

$$E\left(D_{S_t=i}\right) = \frac{1}{1 - \ p_{ii}} \tag{2.8}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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2.4 Model Identification

The appropriate model for this study was identified using the log-likelihood values (log L) and the Akaike information criteria (AIC) [26]. The AIC is computed as:

$$AIC \;=\; -2log\ L \;+\; 2m \tag{2.9}$$

where L is the likelihood function defined and m represents the number of parameters included in the corresponding model.

The model is applied to the returns of the All-Share Index (ASIR) of the Nigerian Stock Exchange (NGX) obtained for the period January 1985 to December 2019 and all the parameters are estimated using E-Views 9.0.

3 Results

This section presents the results of the model fitted for the Markov Switching Autoregressive (MS-AR) model, its estimates and the other characteristics estimated. The results are presented in tables, and charts and interpreted as presented. Fig. 1 shows a non stationary actual recorded all-share index of the Nigerian Stock Exchange (NGX) from January 1985 - December 2019. However, Fig. 2 presents the returns (differenced natural Log) of the all-share index for the same period which has zero mean and and approximately constant variance.

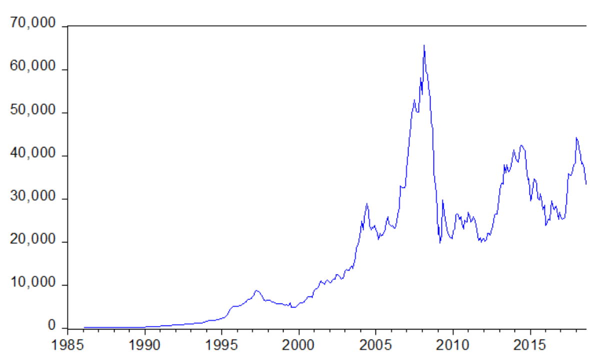

Fig. 1: Plot of the All-share index from January 1985 – December 2019

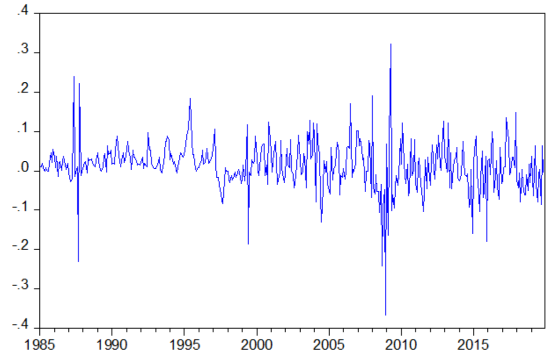

Fig. 2: Plot of the returns (Z_t = differenced natural log of All share index)

Table 1: Descriptive statistics of the all-share index (ASI)

	ASI	LN-ALSI	Z_t
Mean	16559.60	8.714979	0.013092
Median	11554.70	9.354830	0.014715
Maximum	65652.38	11.09213	0.323516
Minimum	111.3000	4.712229	-0.365883
Std.Dev.	15366.89	1.903776	0.061268
Skewness	0.671222	-0.815763	-0.432920
Kurtosis	2. 596171	2.267634	9.508078
Jarque-Bera	34.39156	55.96916	752.5373
Probability	0.000000	0.00000	0.00000
Sum	6955033	3660.291	5.485496
Sum Sq. Dev.	9.89E+10	1518.608	1.569086
Observations	420	420	419

The descriptive table shows that the All Share Index Return (ASIR or Zt) has a mean value of 1.31% with a standard deviation of 1.47%. The highest return was 32.35% while the least return was a negative or loss of 36.59% of its value. The return was shown to be slightly negatively skewed.

Table 2 reveals that the best model estimate for the MS-AR with regressors using the Akaike information Criteria (AIC) and the log-likelihood value among the three considered models above is the MS (3)–AR (3) with the least AIC (14.78152).

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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Table 2: Estimation of the MS-AR model with regressors

MS-AR Model	State	lags	Log-likelihood	AIC Values
MS(2)-AR(5)	2	5	-3140.304	15.27024
MS(3)-AR(5)	3	5	-3051.203	14.82224
MS(3)-AR(5)	3	3	-3059.557	14.78152

Table 3: Estimates of the MS (3) - AR (3)Model parameters

Variable	Coefficient	Std.Error	Z-Statistic	Prob.
		Regime 1		
μ_1	1.0141	0.0080	126.3007	0.000
σ_1^2	5.8231	0.0957	60.8668	0.0000
		Regime 2		
μ_2	0.9967	0.0171	58.2695	0.0000
σ_2^2	7.7559	0.0986	78.6928	0.0000
		Regime 3		
μ_2	1.0246	0.0061	168.8280	0.0000
σ_3^2	2.6418	0.1643	16.0773	0.0000
		Common		
AR(1)	0.1918	0.0446	4.2972	0.0000
AR(2)	0.1113	0.0323	3.4454	0.0000
AR(3)	-0.0542	0.0208	-2.6048	0.0092
Log-likelihood =	-3059	AIC	14.78152	

The results show that regime 3 (1.0246) has the highest regime-dependent intercept (expected increase in returns per period) followed by regime 1 (1.0141) while regime 2 has the least (0.9967). and all of the intercepts are significant ($p < 0.05$). This implies that returns increase the least in the accumulation period (gains or bullish) represented by Regime 1, followed by the big move represented by Regime 2 and highest in the panic phase (losses or bearish) represented by Regime 3. The variance of regime 2 was the highest (7.7559), followed by regime 1 (5.8231) with regime 3 as the least varying (2.6418). It can be seen that regime 2 has 3 times the variability in regime 3 while regime 2 was twice that of 2. Hence, $\sigma^2 st = 2 > \sigma^2 st = 1 > \sigma^2 st = 3$. The implication is that the returns fluctuate more in the big move phase (regime 2) and experience the least fluctuation in the panic phase (regime 3). The autoregressive parameters for three lags (AR 3) were homogenous for the three regimes with lags 1 and 2 positive while lag 3 is negative and lag 1 coefficient was the highest while lag 3 coefficient was the least.

The three phases identified are the big move represented by Regime 1, the accumulation period (gains or bullish) represented by Regime 2 and the panic phase (losses or bearish) represented by Regime 3. Table 4 shows that the transition probabilities which show that the probability of accumulation remaining (staying in regime 1) is 95.6% while there is only a 3.8% chance of switching

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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Table 4: Transition Probability of MS(3) - AR(3) model

	Transition Probabilities			Expected Durations		
	Regime 1	Regime 2	Regime 3	Regime 1	Regime 2	Regime 3
Regime 1	0.9558	0.0060	0.0382	22.6485	392.7099	32.5434
Regime 2	0.0001	0.9974	0.0025	Q(P)		DW stat
Regime 3	0.034	0	0.9693	0.0098(0.921)		2.236

from regime 1 to regime 3 and < 1% likelihood of moving from regime 1 to regime 2. It also revealed that the probability of returns remaining in regime 2 is 99.7% with a probability of switching from regime 2 to regimes 1 and 3 below 1%. Lastly, it can be seen that there is a 96.9% chance of regime 3 remaining while the probability of moving from regime 3 to regime 1 is 3.1% and 0 for switching from regime 3 to regime 2. The procedure also gave the smoothed probabilities which shows that the 3-state Markov model vary so much as the span of the samples transit (see Fig. 3 below). The expected duration in Regime 1 is 22.65, that of Regime 2 is 392.71 and Regime 3 is 32.54. The Durbin-Watson statistic reveals that the residual of return is not autocorrelated while the Q-statistic showed that the MS (3)-AR (3) is adequate as the goodness of fit test retained the null hypothesis of adequacy of the model (p > 0.05).

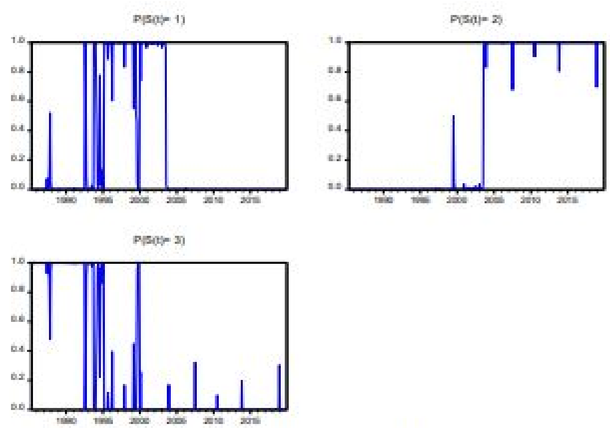

Fig. 3: The smoothed probabilities of the MS (3)-AR (3) model

4 Discussion

Hence, there is a confirmation through our study that a Markov Switching Autoregressive model (MS-AR) was appropriate for the Nigerian Stock Exchange (NGX) returns on the All-Share index (ASIR) with the linearity likelihood ratio (LR) high and so the hypothesis of switching model (MS-AR) not fitting or adequate was rejected ($p < 0.05$). This aligned with other studies which used the LR as the test statistic for the adequacy of the Markov switching model [27, 28, 29]. The study found that the appropriate model for the All-Share Index return of the Nigeria Stock Exchange is a three-regime Markov Switching 3 lagged autoregressive (MS 3-AR3) model. This was revealed by the AIC that was least for the MS (3)-AS (3) model when compared with other models (Table 2). While other studies used different criteria for their choice, Bayesian Information Criteria (BIC) [29], Schwarz Bayesian Criteria (SBC) [27], this study employed the Akaike Information Criteria (AIC) for the model selection.

The regime-dependent intercepts were estimated highest at regime 3 and least at regime 2. The regime variances were also determined with the highest variance at regime 2 and the least variance at regime 3 and the expected duration (E(D)) of stay in each regime were also computed with regime 2 showing the highest E(D) of over 200 periods and least at regime 1 showing 22 periods. The parameters (coefficients) of the lagged autoregression were determined to be homogenous for all regimes unlike in other studies where the lagged coefficients were nonhomogeneous [29] This study identified and fitted a three-regime Markov switching three-lagged autoregressive model, unlike most other studies that have obtained 3 regimes on 2 lags [3]. This difference stemmed from using data covering longer periods in this study than [3]) did.

5 Conclusion

We investigated the use of MS-AR, a non-linear model to disintegrate the All-Share Index return of the Nigerian stock exchange producing a three regime three-lagged autoregressive model. We thus concluded that the Markov Switching Autoregressive MS (3)-AR (3) model was the most appropriate for the All-Share index return (ASIR) with the least AIC value of 14.78152 and hence fitted. The transition probability of the three states was also generated and the states determined are the accumulation period with 95.6% probability of retainership, big move with 99.7% of retaining its place and the panic phase with 97.9% of remaining. Other important parameters of the model like the regime-dependent intercepts, regime variances and the autoregressive parameter were estimated. The properties of the model like the expected durations, adequacy measure, and goodness of fit test using the Q-statistic and the Durbin-Watson statistic were also measured. Hence, the MS-AR is a robust, efficient and reliable non-linear predictive model for forecasting the returns of the Nigerian stock exchange All-Share index.

Conflict of Interests

The authors declare that there is no conflict of interest regarding the publication of this paper.

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