IJMSO

International Journal of Mathematical Sciences and Optimization: Theory and Applications
11(1), 2025, Pages 89 - 95
https://doi.org/10.5281/zenodo.15175819

Convergence of Implicit Noor Iteration in Convex b-Metric Space

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Article Info
Received: 02 August 2024 Revised: 12 September 2024
Accepted: 17 September 2024 Available online: 20 March 2025

Abstract

The convergence to a fixed point and stability of the implicit Noor iteration in a convex b-metric space is established in this work. The class of mappings considered here is an extension of a class of weak contractions which has been used by several authors to obtain quite interesting results on the existence of unique fixed points as well as convergence and stability of iterative schemes in the literature.

Keywords: Implicit Noor Iteration, Convex b-Metric Space, Convergence, Convexity.
MSC2010: 47H10, 54H25.

1 Introduction and Preliminaries

Let (X, d) be a metric space, and T a selfmap of X. There exist a constant $a \in [0, 1)$ and monotone increasing function $\varphi : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ with $\varphi(0) = 0$, such that

$$d(Tx, Ty) \leq \varphi(d(x, Tx)) + ad(x, y), \quad x, y \in X.$$

With respect to this class of mappings, Imoru and Olatinwo [12] proved that the Picard and Mann iterations are T-stable in a Banach space. There are other classes of mappings for which the Picard, Mann and other iterative processes have been shown to be T-stable. See [1–3, 7, 10, 11]. The convergence of implicit iterations to fixed points of several classes of mappings in convex metric spaces have been established by many authors. In 2007, Song [19] proved that a Mann type implicit iteration converges to a fixed point of $T : K \longrightarrow K$, where K is a nonempty closed convex subset of a Banach space and T is a continuous pseudo-contractive mapping. In 2015, Chugh et al. [7] showed that, for the class of weakly contractive mappings T above, the implicit Noor iterative sequence converges to a fixed point of T, and is T-stable, in a convex metric space.

The mapping $W : X \times X \times [0, 1] \longrightarrow X$ is a convex structure on X if for each triple $(x, y, \mu) \in X \times X \times [0, 1]$ and $u \in X$, we have

$$d(u, W(x, y, \mu)) \leq \mu d(u, x) + (1 - \mu)d(u, y).$$

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The metric space X together with W is a convex metric space. A nonempty subset C of X is said to be convex if $W(x, y, \lambda) \in C$ for all $(x, y, \mu) \in C \times C \times [0, 1]$. All normed spaces and their convex subsets are convex metric spaces, but not all convex metric spaces are embedded in a normed space (See [20]).

Given any initial approximation $x_0 \in C$, and sequences $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ of real numbers in $[0, 1]$, the implicit Noor iteration is defined in a convex metric space as:

$$\begin{aligned} x_n &= W(y_{n-1}, Tx_n, a_n) \\ y_{n-1} &= W(z_{n-1}, Ty_{n-1}, b_n) \\ z_{n-1} &= W(x_{n-1}, Tz_{n-1}, c_n), \quad n = 0, 1, 2, ... \end{aligned} \tag{1}$$

The implicit Ishikawa can be obtained by putting $c_n = 1$ and implicit Mann is obtained by putting $b_n = c_n = 1$. The implicit Noor iterative scheme was found to converge faster than Mann, Ishikawa, Noor, implicit Mann and implicit Ishikawa iterations [7].

In the last three decades, investigation of existence and approximation of fixed points in generalized metric spaces, such as G-metric space, fuzzy metric space, 2-metric space, b-metric space and others, was given a considerable attention. Interested readers may see [5, 6, 20]. Czerwik [6] introduced the concept of b-metric by multiplying the right hand side of the triangle inequality by some real number. The b-metric space thereafter became an interesting tool for the improvement of Banach's contraction principle.

Definition. Let X be a nonempty set and $S \geq 1$ a given real number. A function $X \times X \longrightarrow \mathbf{R}_+$ is a b-metric if for each $x, y, z \in X$, the following conditions are satisfied:
(b1) $d(x, y) = 0$ iff $x = y$;
(b2) $d(x, y) = d(y, x)$;
(b3) $d(x, y) = S[d(x, z) + d(z, y)]$.
The pair (X, d) is called a b-metric space.

The following are some of the common examples of b-metric in the literature. See [4, 6, 19].

Example 1. Let X be the set of all sequences $\{x_n\}$ in $\mathbf{R}$ such that $\sum_{k=1}^{\infty} |x_k|^p$ is finite. Define $d(x, y) = \left(\sum_{k=1}^{\infty} |x_k - y_k|^p\right)^{1/p}$, $0 < p < 1$, where $x = \{x_n\}$ and $y = \{y_n\}$. Then $d(x, y)$ is a b-metric with constant $S = 2^{1/p}$.

Example 2. Let X be the space $L_p[0, 1] (0 < p < 1)$ of measurable real functions $f(t), t \in [0, 1]$, such that $\int_0^1 |f(t)|^p dt < \infty$. Then, $d(x, y) = \left(\int_0^1 |x(t) - y(t)|^p dt\right)^{1/p}$ is a b-metric.

Example 3. Let X be the set $\mathbf{R}$ of all real numbers with metric $d(x, y) = |x - y|^2 = (x - y)^2$, $x, y \in \mathbf{R}$. Then $d(x, y)$ is a b-metric with constant $S = 2$.

In [5], Chen et al introduced the concept of convex b-metric space by means of the convex structure w. They gave the following definition.

Definition. Let (H, d_b) be a b-metric space and let $I = [0, 1]$. Define the mapping $d_b : H \times H \longrightarrow [0, \infty]$ and a continuous function $w : H \times H \times I \longrightarrow$. Then, w is said to be the convex structure on H if the following holds:

$$d_b(o, w(u, v; \lambda)) \leq \lambda d_b(o, u) + (1 - \lambda) d_b(o, v) \tag{C}$$

for each $o \in H$ and $(u, v, \lambda) \in H \times H \times I$.

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In this work we shall define our convex structure $W : X \times X \times [0,1] \longrightarrow X$ on the b-metric space (X, d) by

$$d(u, W(x, y, \mu)) \leq S[\mu d(u, x) + (1-\mu)d(u, y)] \tag{2}$$

for each $u \in X$, $(x, y, \mu) \in X \times X \times [0,1]$ and some constant $S \geq 1$.
The inequality (C) is a special case of (2) with $S = 1$.

In the sequel, we shall consider a map $T : X \longrightarrow X$ on a convex b-metric space satisfying the following: there exists a constant $\lambda \in [0,1)$ and function $\phi : [0,\infty) \times [0,\infty) \longrightarrow [0,\infty)$, with $\phi(0,t) = \phi(t,0) = 0$, such that

$$d(Tx, Ty) \leq \phi(d(x, Tx), d(y, Ty)) + \lambda d(x, y), \;\; x, y \in X. \tag{3}$$

Example 4. (i) $\phi(u, v) = uv$ and (ii) $\phi(u, v) = \min\{u, v\}$ are simple examples of the function ϕ.

Example 5. Let $X = [0,1]$ be endowed with b-metric $d(x, y) = |x-y|^2 = (x-y)^2$. Define $T : X \longrightarrow X$ by $Tx = \frac{x}{2}$ and $\phi(x, y) = \min\{x, y\}$. Then,
$d(x, Tx) = (x - \frac{x}{2})^2 = x^2/4$, so that $\phi(d(x, Tx), d(y, Ty)) = \frac{1}{4}\min\{x^2, y^2\}$. Thence, there exists some constant $\lambda \in [0,1)$ such that
$\phi(d(x, Tx), d(y, Ty)) + \lambda d(x, y) = \frac{1}{4}\min\{x^2, y^2\} + \lambda(x-y)^2$. Whereas, $d(Tx, Ty) = \frac{1}{4}(x-y)^2$. Therefore, choosing $\frac{1}{4} \leq \lambda \leq 1$, the mapping T satisfies (3).Moreover, T has a unique fixed point 0.

In 2006, Rafiq [20] showed that the Mann-type implicit iteration ($x_n = \alpha_n x_{n-1} + (1-\alpha_n)Tx_n$) associated with hemicontractive mappings converges strongly to a fixed point in Hilbert spaces. The condition $\alpha_n \in [\delta, 1-\delta]$ for some $\delta \in (0,1)$ was imposed to obtain his result. However, Song [19] observed that the control condition did exclude the choice of $\alpha_n = \frac{1}{n}$ and, consequently, the existence of $x_i, i = 1, 2, 3, \cdots$, satisfying $x_i = \alpha_i x_{i-1} + (1-\alpha_i)Tx_i$ was not guaranteed. In other words, it is not known whether, or not, the mapping $S_i = \alpha_i x_{i-1} + (1-\alpha_i)T$ has a fixed point. Song filled the gap left by Rafiq by imposing the control condition $\alpha_n \in (0, b] \subset (0,1)$.
In this work, we shall impose the condition that for any $\alpha \in [0,1], y \in C$, there is some $t \in C$ such that the map $t \mapsto W(y, Tt, \alpha)$ has a fixed point.

2 Main Results

In this section we first establish the convergence of the implicit Noor iteration to a fixed point of the class of mappings T defined by (3) in b-metric spaces, under certain conditions. Our results are obtained under the assumption that for any $\alpha \in [0,1], y \in C$, there is some $t \in C$ such that the map $t \mapsto W(y, Tt, \alpha)$ has a fixed point.

Theorem 1. *Let C be a nonempty, closed convex subset of a convex b-metric space X with constant $S \geq 1$. If T is a selfmap of C satisfying (3) and the set of fixed points F_T is nonempty, then the sequence $\{x_n\}_{n=0}^{\infty}$ generated by (1) converges to a fixed point of T, provided $S\lambda < 1$ and $S^3 a_n b_n c_n < (1-S\lambda)^3 k$ for all n large enough, where $k \in [0,1)$.*

Proof. Let $p \in F_T = \{x \in C : x = Tx\}$. There exists an initial approximation $x_0 \in C$ such that, for all $n = 1, 2, 3, ...$, we have

$$\begin{aligned} d(x_n, p) &= d(W(y_{n-1}, Tx_n, a_n), p) \\ &\leq S[a_n d(y_{n-1}, p) + (1-a_n)d(Tx_n, p)] \\ &\leq S a_n d(y_{n-1}, p) + S(1-a_n)[\phi(0, d(x_n, Tx_n)) + \lambda d(x_n, p)]. \end{aligned}$$

This yields

$$d(x_n, p) \leq \frac{S a_n}{1 - S\lambda(1-a_n)} d(y_{n-1}, p).$$

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Similarly,

$$d(y_{n-1},p) \leq \frac{Sb_n}{1-S\lambda(1-b_n)} d(z_{n-1},p) \text{ and } d(z_{n-1},p) \leq \frac{Sc_n}{1-S\lambda(1-c_n)} d(x_{n-1},p).$$

Therefore,

$$d(x_n,p) \leq \left(\frac{Sa_n}{1-S\lambda(1-a_n)}\right)\left(\frac{Sb_n}{1-S\lambda(1-b_n)}\right)\left(\frac{Sc_n}{1-S\lambda(1-c_n)}\right) d(x_{n-1},p). \quad (4)$$

That is,

$$d(x_n,p) \leq \frac{S^3 a_n b_n c_n}{(1-S\lambda)^3} d(x_{n-1},p).$$

Using the condition: $S\lambda < 1$; $S^3 a_n b_n c_n < (1-S\lambda)^3 k$ (for all n large enough), we have

$$d(x_n,p) \leq kd(x_{n-1},p) \leq k^2 d(x_{n-2},p) \leq \cdots \leq k^n d(x_0,p),$$

where $0 \leq k < 1$.
This shows that $\lim_{n\to\infty} d(x_n,p) \leq 0$, meaning that the sequence $\{x_n\}$ converges to p.

Example 6. Let $C = [0,1]$ with the $d(x,y) = (x-y)^2$ and define $T : C \longrightarrow C$ by $Tx = \frac{x}{4}$.
Let $\varphi(x,y) = \min\{x,y\}$ and put $\lambda = \frac{1}{10}$, $S = \frac{10}{9}$, $a_n = b_n = c_n = \frac{1}{n}$.
Then, $S\lambda = \frac{1}{9} < 1$, $S^3 a_n b_n c_n = \frac{1000}{729n^3}$ and $(1-S\lambda)^3 k = \frac{512}{729}k$, $k \in [0,1)$.
So, $S^3 a_n b_n c_n < (1-S\lambda)^3 k$ for all $n > \frac{5}{4k}$.

Now, $d(Tx,Ty) = (\frac{x}{4} - \frac{y}{4})^2 = \frac{1}{16}(x-y)^2$ and
$\varphi(d(x,Tx), d(y,Ty)) + \lambda d(x,y) = \frac{9}{16}\min\{x^2,y^2\} + \frac{1}{10}(x-y)^2$.
Thus,
$d(Tx,Ty) - \varphi(d(x,Tx), d(y,Ty)) + \lambda d(x,y) = -\frac{3}{16}\left[\frac{(x-y)^2}{5} + 3\min\{x^2,y^2\}\right]$. Therefore the inequality (3) is satisfied.
Furthermore, the inequality $d(x_n,p) \leq \frac{S^3 a_n b_n c_n}{(1-S\lambda)^3} d(x_{n-1},p)$ in the proof of Theorem 1 yields $(x_n - p)^2 \leq \frac{125}{64n^3}(x_{n-1}-p)^2$. That is,

$$\begin{aligned}
&(x_1-p)^2 \leq \tfrac{125}{64}(x_0-p)^2;\\
&(x_2-p)^2 \leq \tfrac{125}{64(2^3)}(x_1-p)^2 \leq \left(\tfrac{125}{64}\right)^2 \tfrac{1}{2^3}(x_0-p)^2;\\
&(x_3-p)^2 \leq \tfrac{125}{64(3^3)}(x_2-p)^2 \leq \left(\tfrac{125}{64}\right)^3 \tfrac{1}{2^3.3^3}(x_0-p)^2;\\
&.\\
&.\\
&.\\
&(x_n-p)^2 \leq \left(\tfrac{125}{64}\right)^n \tfrac{1}{(n!)^3}(x_0-p)^2
\end{aligned}$$

Since $\left(\frac{125}{64}\right)^n \frac{1}{(n!)^3} < 1$ for $n \geq 2$, then, $\lim_{n\to\infty}(x_n - p) = 0$.
Finally, using the implicit Noor iteration (1), we obtain the following.
$z_{n-1} = \frac{x_{n-1}}{n} + (1-1/n)\frac{z_{n-1}}{4} \Rightarrow z_{n-1} = \frac{4}{3n+1}x_{n-1}$;
$y_{n-1} = \frac{z_{n-1}}{n} + (1-1/n)\frac{y_{n-1}}{4} \Rightarrow y_{n-1} = \left(\frac{4}{3n+1}\right)^2 x_{n-1}$;
$x_n = \frac{y_{n-1}}{n} + (1-1/n)\frac{x_n}{4} \Rightarrow x_n = \left(\frac{4}{3n+1}\right)^3 x_{n-1}$.
Thus, $x_n = \left(\frac{4}{3n+1}\right)^{3n} x_0$. The sequence $\{x_n\}$ converges to 0, the fixed point of T.

In what follows, we use the following modification of (3) involving two mappings $F, G : X \longrightarrow X$ satisfying $FX \subseteq GX$, and

$$d(Fx,Fy) \leq \phi(d(Gx,Fx), d(Gy,Fy)) + \lambda d(Gx,Gy), \; x,y \in X. \quad (5)$$

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Given an initial point $x_0 \in X$, we can generate a sequence $\{x_n\}_{n=1}^{\infty}$, where

$$\begin{aligned} Gx_n &= W(Gy_{n-1}, Fx_n, a_n) \\ Gy_{n-1} &= W(Gz_{n-1}, Fy_{n-1}, b_n) \\ Gz_{n-1} &= W(Gx_{n-1}, Fz_{n-1}, c_n), \quad n = 0, 1, 2, ... \end{aligned} \tag{6}$$

Theorem 2. *Let C be a nonempty, compact convex subset of a convex b-metric space X with constant $S \geq 1$. Let F and G be two weakly compatible selfmappings of C satisfying $FC \subseteq GC$ and (5). If GC is closed in C, and the set $\{p \in C : Gp = Fp\}$ of coincidence points of F and G is nonempty, then the sequence $\{Gx_n\}_{n=0}^{\infty}$ generated by (6) converges to Gq, where $q = Gp = Fp$, provided $S\lambda < 1$ and $S^3 a_n b_n c_n < (1 - S\lambda)^3 k$ for all n large enough, where $k \in [0, 1)$. Furthermore, if F is injective, $\{Gx_n\}_{n=0}^{\infty}$ converges to a common fixed point of F and G.*

Proof. Let $q = Fp = Gp$. By the weak compatibility of F and G, $Gq = GFp = FGp$ and $Fq = FGp = GFp$. Therefore, $Fq = Gq$.
Then, for $n = 1, 2, 3, ...$, we have

$$\begin{aligned} d(Gx_n, Gq) &= d(W(Gy_{n-1}, Fx_n, a_n), Gq) \\ &\leq S[a_n d(Gy_{n-1}, Gq) + (1 - a_n) d(Fx_n, Fq)] \\ &= S a_n d(Gy_{n-1}, Gq) + S(1 - a_n)[\phi(d(Gx_n, Fx_n), d(Gq, Fq)) + \lambda d(Gx_n, Gq)] \\ &= S a_n d(Gy_{n-1}, Gq) + S(1 - a_n)\lambda d(Gx_n, Gq). \end{aligned}$$

This yields

$$d(Gx_n, Gq) \leq \frac{S a_n}{1 - S\lambda(1 - a_n)} d(Gy_{n-1}, Gq).$$

Following the proof of Theorem 1, we obtain

$$d(Gx_n, Gq) \leq k d(Gx_{n-1}, Gq) \leq k^2 d(x_{n-2}, Gq) \leq \cdots \leq k^n d(Gx_0, Gq).$$

This shows that $\lim_{n\to\infty} d(Gx_n, Gq) \leq 0$, meaning that the sequence $\{Gx_n\}$ converges to $Gq(= GFp = FGp = Fq)$.
Since GC is closed in C there is a point $v \in C$ such that $v = Gq = Fq$. Then, $Gv = GFq = FGq = Fv$.
From (5), $d(Fv, Fp) \leq \phi(d(Gv, Fv), d(Gp, Fp)) + \lambda d(Gv, Gp) = \lambda d(Fv, Fp)$, which means $Fv = Fp$, and by the injectivity of F we have $v = p$. Thus, $\{Gx_n\}$ converges to p.
Finally,
$d(Fp, Fq) \leq \phi(d(Gp, Fp), d(Gq, Fq)) + \lambda d(Gp, Gq) = \lambda d(Fp, Fq)$. So $p = q$, meaning p is a common fixed point of F and G.

Corollary 1. *Let C be a nonempty, compact convex subset of a convex b-metric space X with constant $S \geq 1$. Let F and G be two weakly compatible selfmappings of C satisfying $FC \subseteq GC$ and (5). If G is continuous in C, and the set $\{p \in C : Gp = Fp\}$ of coincidence points of F and G is nonempty, then the sequence $\{x_n\}_{n=0}^{\infty}$ generated by (6) converges to the common fixed point of F and G, provided $S\lambda < 1$ and $S^3 a_n b_n c_n < (1 - S\lambda)^3 k$ for all n large enough, where $k \in [0, 1)$.*

Proof. It has been shown above that $Fq = Gq$, where $q = Fp = Gp$, and that $\{Gx_n\}$ converges to Gq (before using the injective property of F). The continuity of G then implies that $\{x_n\}$ converges to q. Moreover,

$$d(q, Fq) = d(Fp, Fq) \leq \phi(d(Gp, Fp), d(Gq, Fq)) + \lambda d(Gq, Gp) = \lambda d(Fq, q),$$

meaning $q = Fq$. So, q is a common fixed point of F and G.

Corollary 2. *Let C be a nonempty, compact convex subset of a convex b-metric space X with constant $S \geq 1$. Let F and G be two weakly compatible selfmappings of C satisfying $FC \subseteq GC$ and*

(5). If GC is closed in C, and the set $\{p \in C : Gp = Fp\}$ of coincidence points of F and G is nonempty, then the sequence $\{Gx_n\}_{n=0}^{\infty}$ of the modified implicit Ishikawa iteration

$$Gx_n = W(Gy_{n-1}, Fx_n, a_n);\ Gy_{n-1} = W(Gx_{n-1}, Fy_{n-1}, b_n), \quad n = 1, 2, \cdots, \tag{7}$$

converges to Gq, where $q = Gp = Fp$, provided $S\lambda < 1$ and $S^2 a_n b_n < (1 - S\lambda)^2 k$ for aal n large enough, where $k \in [0, 1)$. Furthermore, if either F is injective or G is continuous, then $\{Gx_n\}_{n=0}^{\infty}$ converges to a common fixed point of F and G.

Proof. Put $c_n = 1$ in the foregoing.

Remarks. The following were emphasized in [7]:
(i) The speed of implicit iterations depends the parameters a_n, b_n and c_n.
(ii) The implicit Noor iteration converges faster than implicit Ishikawa and implicit Mann iterations. It is therefore more preferable for applications in various disciplines of science.

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