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A Deterministic Model Analysis for Youth Involvement in *Yahoo-Yahoo*(Cybercrime) and *Yahoo+*(Ritualism)

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Abstract

The rising involvement of young people in *yahoo-yahoo* (cybercrime) and *yahoo+* (ritualism) has become a major societal concern, particularly in Nigeria, where economic instability and peer pressure drive vulnerable individuals into these illicit activities. This study develops a deterministic compartmental model to analyze the transition of youths through different stages: vulnerability, engagement in *yahoo-yahoo*, progression into *yahoo+* (ritualism), and recovery through rehabilitation. The model accounts for key factors such as recruitment rates, peer influence, the probability of transitioning to ritualistic practices, rehabilitation effectiveness, and relapse risks. A stability analysis of the equilibrium points, which are the yahoo-free and endemic, provides insight into long-term trends based on the basic reproduction number, R_0, determining whether *yahoo-yahoo* (cybercrime) and *yahoo+* (ritualism) persist or decline. Numerical simulations reveal potential strategies to reduce youth participation in these activities. Based on the model's findings, the study offers policy recommendations to support sustainable youth development and mitigate this growing societal threat.

Keywords: Modeling, Positvity and Boundedness, Simulations.
MSC2010: 92B05.

1 Introduction

Cybercrime, widely known as *yahoo-yahoo* (cybercrime) in Nigeria, encompasses various forms of internet fraud involving deception, impersonation, and fraudulent schemes to exploit unsuspecting victims. Emerging in the early 2000s, *yahoo-yahoo* (cybercrime) initially involved email scams, phishing, and identity theft but has since evolved into more sophisticated financial fraud, including romance scams, investment fraud, and business email compromise. The rapid advancement of digital technology, coupled with economic hardship and high unemployment, has contributed significantly to the proliferation of this illicit activity, attracting many young Nigerians who view cyber fraud as a means of financial survival [1, 2].

Over time, some *yahoo-yahoo* (cybercrime) practitioners have incorporated ritualistic practices, a phenomenon referred to as *yahoo+* (ritualism). This involves the use of human sacrifices, charms, and occult rituals to enhance the success of fraudulent activities. Many perpetrators believe that engaging in ritual sacrifices grants them supernatural powers that improve their ability to deceive victims and acquire wealth effortlessly. The gruesome nature of *yahoo+* (ritualism) has led to heinous crimes such as kidnappings, body mutilations, and ritual killings, which have been widely documented across Nigeria [3]. The transition from *yahoo-yahoo* (cybercrime) to *yahoo+* (ritualism) is often driven by desperation, greed, and peer influence, as cybercriminals seek alternative ways to ensure their scams remain effective. The perceived power of spiritual fortification and sacrifices has contributed to a disturbing rise in ritual killings, particularly among young individuals in urban and semi-urban areas [5].

In response, the Nigerian government has attempted to curb *yahoo-yahoo* (cybercrime) and *yahoo+* (ritualism) through law enforcement agencies such as the Economic and Financial Crimes Commission (EFCC) and the police. However, these efforts have been hindered by corruption, inadequate surveillance, and the absence of effective rehabilitation programs for offenders. Although arrests and convictions have increased in recent years, cybercrime remains deeply ingrained in the country's socio-economic fabric. Addressing this growing menace requires a multifaceted approach, including stricter law enforcement, youth empowerment programs, digital literacy initiatives, and psychological rehabilitation. Understanding the intricate dynamics of cybercrime and ritualism is essential for designing effective policies that can curb their spread and protect Nigeria's youth from further involvement in these illegal activities [4].

Several studies have examined *yahoo-yahoo* (cybercrime) and *yahoo+* (ritualism) from different perspectives, including sociology, criminology, culture, and mathematics. Scholars such as [6–8] emphasize the need for government intervention and adult education to combat cyber fraud, arguing that economic instability and lack of formal employment contribute to youth involvement in these crimes. Criminological perspectives, such as those of [9–11], frame *yahoo-yahoo* (cybercrime) as a moral and religious crisis requiring ethical reorientation and community-driven reforms. Other researchers have explored the role of spiritual beliefs, governance challenges, and public perception in sustaining cybercrime. Mathematical modeling approaches have also been used to analyze crime dynamics [22, 23], with studies examining amnesty and rehabilitation strategies, crime propagation, and financial fraud population dynamics [12–18]. However, there remains a significant gap in the literature regarding a structured mathematical model approach to analyzing *yahoo-yahoo* (cybercrime) and *yahoo+* (ritualism). This study aims to fill this gap by developing a compartmental model that captures the complexities of youth transitions between vulnerability, cybercrime involvement, ritualistic practices, rehabilitation, and relapse. Sections 2 and 3 present the model formulation, analysis and numerical simulations, while Section 4 concludes with key recommendations.

2 Model Formulation and Analysis

We define the following compartments:

- $V(t)$ - Vulnerable youths engaged in *yahoo-yahoo*(cybercrime) and *yahoo+*(ritualist).
- $Y(t)$ - Youths engaged in *yahoo-yahoo*(cybercrime).
- $R(t)$ - Youths engaged in *yahoo+*(ritualist).
- $Q(t)$ - Youths who have recovered (rehabilitated) from *yahoo-yahoo*(cybercrime) and *yahoo+*(ritualist).

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In view of the meanings above,

$$\left.\begin{aligned}\frac{dV}{dt} &= \alpha - \beta VY - \mu V,\\ \frac{dY}{dt} &= \beta VY - \gamma Y - \theta Y - \mu_Y Y,\\ \frac{dR}{dt} &= \gamma Y - (\zeta + \mu + \delta_o)R + \eta R,\\ \frac{dQ}{dt} &= \theta Y + \zeta R - (\mu + \eta)Q.\end{aligned}\right\} \tag{2.1}$$

The model (2.1) parameter descriptions are:

- α - Recruitment rate of youths into society.
- β - Influence/contact rate between vulnerable youths and those engaged in Yahoo-Yahoo.
- μ - Natural death rate.
- γ - Progression rate from *yahoo-yahoo*(cybercrime) to *yahoo+*(ritualist).
- θ - Progression rate of *yahoo-yahoo*(cybercrime) individuals into rehabilitation.
- ζ - Progression rate of *yahoo+*(ritualist) individuals into rehabilitation.
- δ_o - Death rate due to ritual killings.
- η - Relapse rate of rehabilitated youths.
- μ - Natural death rate of vulnerable youths .
- μ_Y - Arrest rate of youths into yahoo-yahoo crimes.

2.1 Existence and Uniqueness of Solutions

To establish that the system of differential equations has a unique solution, we invoke the Picard-Lindelöf theorem (also known as the Cauchy-Lipschitz theorem) [21]. This theorem states that if a system of ordinary differential equations satisfies the conditions of continuity and Lipschitz continuity in a domain, then a unique solution exists.

Theorem 2.1. *Consider the general system of first-order ordinary differential equations*

$$\frac{dx}{dt} = f(t,x), \quad x(t_0) = x_0. \tag{2.2}$$

If $f(t,x)$ is continuous in t and Lipschitz continuous in x in a domain D, then there exists a unique solution $x(t)$ for all t in some interval around t_0.

Proof. In view of (2.1), we define the vector function, given by

$$F(V,Y,R,Q) = \begin{bmatrix} \alpha - \beta VY - \mu V \\ \beta VY - \gamma Y - \theta Y - \mu_Y Y \\ \gamma Y - (\zeta + \mu + \delta_o)R + \eta R \\ \theta Y + \zeta R - (\mu + \eta)Q \end{bmatrix}. \tag{2.3}$$

The function $F(V,Y,R,Q)$ consists of linear and bilinear terms. Since they are continuous and differentiable everywhere in $\mathbb{R}^4$, the function is continuous in its domain.

To verify the Lipschitz condition, we check whether there exists a constant $L > 0$ such that for all (V_1,Y_1,R_1,Q_1) and (V_2,Y_2,R_2,Q_2), we have,

$$\|F(V_1,Y_1,R_1,Q_1) - F(V_2,Y_2,R_2,Q_2)\| \leq L\|(V_1,Y_1,R_1,Q_1) - (V_2,Y_2,R_2,Q_2)\|. \tag{2.4}$$

The function $F(V,Y,R,Q)$ consists of linear and bilinear terms. To establish Lipschitz continuity, we need to show that there exists a constant $L>0$ such that

$$\|F(V_1,Y_1,R_1,Q_1)-F(V_2,Y_2,R_2,Q_2)\| \le L\|(V_1,Y_1,R_1,Q_1)-(V_2,Y_2,R_2,Q_2)\|. \tag{2.5}$$

For the second model compartment V,

$$F_V = \alpha - \beta VY - \mu V. \tag{2.6}$$

The difference for two states (V_1,Y_1) and (V_2,Y_2) is

$$|F_V(V_1,Y_1)-F_V(V_2,Y_2)| = |-\beta(V_1Y_1-V_2Y_2)-\mu(V_1-V_2)|. \tag{2.7}$$

Using the mean-value theorem on the bilinear term, that is,

$$|V_1Y_1-V_2Y_2| = |V_1(Y_1-Y_2)+Y_2(V_1-V_2)|. \tag{2.8}$$

Thus, we can bound it as

$$|F_V(V_1,Y_1)-F_V(V_2,Y_2)| \le (\beta\max\{Y_1,Y_2\}+\mu_v)|V_1-V_2|+\beta\max\{V_1,V_2\}|Y_1-Y_2|. \tag{2.9}$$

So the function is Lipschitz with constant

$$L_V = \max\{\beta Y_{\max}+\mu, \beta V_{\max}\}. \tag{2.10}$$

For the second model compartment Y,

$$F_Y = \beta VY - \gamma Y - \theta Y - \mu_Y Y. \tag{2.11}$$

The difference is

$$|F_Y(V_1,Y_1)-F_Y(V_2,Y_2)| = |\beta(V_1Y_1-V_2Y_2)-(\gamma+\theta+\mu_Y)(Y_1-Y_2)|. \tag{2.12}$$

For the third compartment R

$$F_R = \gamma Y - (\zeta+\mu+\delta_o)R + \eta R. \tag{2.13}$$

Taking the difference between two states (Y_1,R_1) and (Y_2,R_2), becomes

$$|F_R(Y_1,R_1)-F_R(Y_2,R_2)| = |\gamma(Y_1-Y_2)-(\zeta+\mu+\delta_o+\eta)(R_1-R_2)|. \tag{2.14}$$

Since $\gamma,\zeta,\mu,\delta_o$, and η are constants, we can bound this by

$$|F_R(Y_1,R_1)-F_R(Y_2,R_2)| \le \gamma|Y_1-Y_2|+|\zeta+\mu+\delta_o+\eta||R_1-R_2|. \tag{2.15}$$

Thus, the Lipschitz constant for this compartment is

$$L_R = \max\{\gamma, |\zeta+\mu+\delta_o+\eta|\}. \tag{2.16}$$

For the fourth Compartment Q,

$$F_Q = \theta Y + \zeta R - (\mu+\eta)Q. \tag{2.17}$$

Taking the difference for two states (Y_1,R_1,Q_1) and (Y_2,R_2,Q_2),

$$|F_Q(Y_1,R_1,Q_1)-F_Q(Y_2,R_2,Q_2)| = |\theta(Y_1-Y_2)+\zeta(R_1-R_2)-(\mu+\eta)(Q_1-Q_2)|. \tag{2.18}$$

Bounding each term, one obtains

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$$|F_Q(Y_1, R_1, Q_1) - F_Q(Y_2, R_2, Q_2)| \leq \theta|Y_1 - Y_2| + \zeta|R_1 - R_2| + (\mu + \eta)|Q_1 - Q_2|. \tag{2.19}$$

Thus, the Lipschitz constant for this compartment is

$$L_Q = \max\{\theta, \zeta, \mu + \eta\}. \tag{2.20}$$

We compute the partial derivatives (Jacobian matrix) of F,

$$J = \begin{pmatrix} -\beta Y - \mu & -\beta V & 0 & 0 \\ \beta Y & \beta V - \gamma - \theta - \mu_Y & 0 & 0 \\ 0 & \gamma & -(\zeta + \mu + \delta_o) + \eta & 0 \\ 0 & \theta & \zeta & -(\mu + \eta) \end{pmatrix}. \tag{2.21}$$

Each term in J is either a constant or a function of the variables V, Y, R, Q. For the social relevant population (non-negative and bounded) considered in his work, we find

$$\left.\begin{aligned} |-\beta Y - \mu| &\leq \beta Y_{\max} + \mu, \\ |\beta Y| &\leq \beta Y_{\max}, \\ |\beta V - \gamma - \theta - \mu_Y| &\leq \beta V_{\max} + \gamma + \theta + \mu_Y, \\ |\gamma| &\text{ is constant}, \\ |-(\zeta + \mu + \delta_o) + \eta| &= |\zeta + \mu + \delta_o + \eta|, \\ |\theta| &\text{ is constant}, \\ |\zeta| &\text{ is constant}. \end{aligned}\right\} \tag{2.22}$$

Thus, the function $F(V, Y, R, Q)$ satisfies a Lipschitz condition with some constant L, ensuring that the solutions do not diverge. By the Picard-Lindelöf theorem, since $F(V, Y, R, Q)$ is continuous and satisfies a Lipschitz condition, the system has a unique solution for given initial conditions. Hence, the model is mathematically well-posed. □

Boundedness of the Model

To ensure that the system remains meaningful in a social sense, we analyze the boundedness of the model to confirm that all state variables remain non-negative and do not grow indefinitely. We define the total population of individuals in the system as

$$N(t) = V(t) + Y(t) + R(t) + Q(t). \tag{2.23}$$

Summing the equations, we obtain

$$\begin{aligned} \frac{dN}{dt} &= \frac{dV}{dt} + \frac{dY}{dt} + \frac{dR}{dt} + \frac{dQ}{dt} \\ &= \alpha - \mu V - \mu_Y Y - \mu R - \delta_o R - (\mu + \eta)Q. \end{aligned} \tag{2.24}$$

Since all parameters $\mu_Y, \mu, \delta_o, \eta$ are positive, it follows that

$$\frac{dN}{dt} \leq \alpha - \min(\mu_Y, \mu, \delta_o, \eta)N. \tag{2.25}$$

Applying Gronwall's inequality, we obtain

$$N(t) \leq \frac{\alpha}{\min(\mu_Y, \mu, \delta_o, \eta)}. \tag{2.26}$$

Thus, as $t \to \infty$, the total population remains bounded, that is,

$$N(t) \leq N_{\max} = \frac{\alpha}{\min(\mu_Y, \mu, \delta_o, \eta)}. \tag{2.27}$$

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To ensure the social relevance of the model, we must show that all state variables remain non-negative for all $t \geq 0$. This guarantees that the system does not produce negative populations, which would be meaningless in real-world scenarios.

Theorem 2.2. *Let* $(V(0), Y(0), R(0), Q(0))$ *be non-negative initial conditions, i.e.,*

$$V(0) \geq 0, \quad Y(0) \geq 0, \quad R(0) \geq 0, \quad Q(0) \geq 0.$$

Then, for all $t \geq 0$*, the solutions* $V(t), Y(t), R(t), Q(t)$ *remain non-negative.*

Proof. We apply the method of proof by contradiction and differential inequality techniques. Rewriting the equation for V:

$$\frac{dV}{dt} = \alpha - \beta VY - \mu_v V. \tag{2.28}$$

We rearrange it as

$$\frac{dV}{dt} + \mu V = \alpha - \beta VY. \tag{2.29}$$

Applying Gronwall's inequality, since $\alpha > 0$, $\beta \geq 0$, and $Y \geq 0$, we conclude that $V(t)$ remains non-negative for all $t \geq 0$, provided $V(0) \geq 0$.

Also, consider the equation for Y,

$$\frac{dY}{dt} = \beta VY - (\gamma + \theta + \mu_Y)Y. \tag{2.30}$$

Factor out Y so that

$$\frac{dY}{dt} = Y(\beta V - \gamma - \theta - \mu_Y). \tag{2.31}$$

If $Y(0) \geq 0$, then for all $t \geq 0$, the right-hand side remains non-negative or zero, ensuring that $Y(t) \geq 0$.

Also, consider the equation for R,

$$\frac{dR}{dt} = \gamma Y - (\zeta + \mu + \delta_o)R + \eta R. \tag{2.32}$$

Since $\gamma, \zeta, \mu, \delta_o, \eta$ are positive parameters and $Y(t) \geq 0$, we conclude that $R(t)$ remains non-negative for all $t \geq 0$ if $R(0) \geq 0$.

We consider the equation for Q,

$$\frac{dQ}{dt} = \theta Y + \zeta R - (\mu + \eta)Q. \tag{2.33}$$

Since $\theta Y \geq 0$, $\zeta R \geq 0$, and $(\mu + \eta)Q$ is a removal term, it follows that $Q(t)$ remains non-negative for all $t \geq 0$, provided $Q(0) \geq 0$.

Since all compartments remain non-negative for $t \geq 0$, we conclude that the model is well-posed. Hence, the theorem is proved. □

Yahoo-Yahoo-Free Equilibrium (YYFE) and Basic Reproduction Number R_0

The crime compartments are Y (Youths who engage in *yahoo-yahoo*(cybercrime)) and R and youths who engage in *yahoo+*(ritualism). The Yahoo-Yahoo-free equilibrium (YYFE) is given by

$$E_0 = \left(\frac{\alpha}{\mu}, 0, 0, 0\right). \tag{2.34}$$

The basic reproduction number R_0 is computed using the next-generation matrix method. The new affected ones into Y and R are given by,

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$$\mathcal{F} = \begin{pmatrix} \beta V Y \\ \gamma Y \end{pmatrix}. \tag{2.35}$$

The transition terms are

$$\mathcal{V} = \begin{pmatrix} (\gamma + \theta + \mu_Y)Y \\ (\zeta + \mu + \delta_o + \eta)R \end{pmatrix}. \tag{2.36}$$

The affected Matrix F yeilds

$$F = \begin{pmatrix} \beta V^* & 0 \\ \gamma & 0 \end{pmatrix} = \begin{pmatrix} \frac{\beta\alpha}{\mu} & 0 \\ \gamma & 0 \end{pmatrix}. \tag{2.37}$$

The transition Matrix V becomes

$$V = \begin{pmatrix} \gamma + \theta + \mu_Y & 0 \\ 0 & (\zeta + \mu + \delta_o) + \eta \end{pmatrix}. \tag{2.38}$$

Since V is diagonal, its inverse is becomes

$$V^{-1} = \begin{pmatrix} \frac{1}{\gamma+\theta+\mu_Y} & 0 \\ 0 & \frac{1}{\zeta+\mu+\delta_o+\eta} \end{pmatrix}. \tag{2.39}$$

Multiplying F by V^{-1} yields,

$$FV^{-1} = \begin{pmatrix} \frac{\beta\alpha}{\mu} & 0 \\ \gamma & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\gamma+\theta+\mu_Y} & 0 \\ 0 & \frac{1}{\zeta+\mu+\delta_o+\eta} \end{pmatrix}. \tag{2.40}$$

Performing the multiplication,

$$FV^{-1} = \begin{pmatrix} \frac{\beta\alpha}{\mu_v(\gamma+\theta+\mu_Y)} & 0 \\ \frac{\gamma}{\zeta+\mu+\delta_o+\eta} & 0 \end{pmatrix}. \tag{2.41}$$

The dominant eigenvalue of FV^{-1} gives R_0, such that

$$R_0 = \max\left(\frac{\beta\alpha}{\mu(\gamma + \theta + \mu_Y)}, \frac{\gamma}{\zeta + \mu + \delta_o + \eta} \right). \tag{2.42}$$

- If $R_0 < 1$, Yahoo-Yahoo and ritualism will decline over time.
- If $R_0 > 1$, Yahoo-Yahoo and ritualism will persist and spread.

Theorem 2.3. *The Yahoo-Yahoo-free equilibrium* $E_0 = (V^*, Y^*, R^*, Q^*) = \left(\frac{\alpha}{\mu}, 0, 0, 0\right)$ *is locally asymptotically stable if the basic reproduction number satisfies*

$$R_0 = \max\left(\frac{\beta\alpha}{\mu(\gamma + \theta + \mu_Y)}, \frac{\gamma}{\zeta + \mu + \delta_o + \eta} \right) < 1. \tag{2.43}$$

Proof. To analyze the local stability of E_0, we perform the Jacobian matrix analysis, given by

$$J = \begin{pmatrix} -\mu & -\beta V & 0 & 0 \\ 0 & \beta V - (\gamma + \theta + \mu_Y) & 0 & 0 \\ 0 & \gamma & -(\zeta + \mu + \delta_o + \eta) & 0 \\ 0 & \theta & \zeta & -(\mu + \eta) \end{pmatrix}. \tag{2.44}$$

Evaluating at E_0, where $V^* = \frac{\alpha}{\mu}$ and $Y^* = 0$, we obtain

$$J(E_0) = \begin{pmatrix} -\mu & -\beta\frac{\alpha}{\mu} & 0 & 0 \\ 0 & \beta\frac{\alpha}{\mu} - (\gamma+\theta+\mu_Y) & 0 & 0 \\ 0 & \gamma & -(\zeta+\mu+\delta_o+\eta) & 0 \\ 0 & \theta & \zeta & -(\mu+\eta) \end{pmatrix}. \tag{2.45}$$

The characteristic equation is given by

$$\det(J - \lambda I) = 0. \tag{2.46}$$

Observing the block diagonal structure, we get the eigenvalues,

- First eigenvalue: $-\mu$ (always negative, implying stability in that direction).
- Second eigenvalue: $\beta\frac{\alpha}{\mu} - (\gamma+\theta+\mu_Y)$.

This second eigenvalue is negative if,

$$\frac{\beta\alpha}{\mu(\gamma+\theta+\mu_Y)} < 1. \tag{2.47}$$

The last two eigenvalues correspond to the lower 2×2 block given by

$$\begin{pmatrix} -(\zeta+\mu+\delta_o+\eta) & 0 \\ \gamma & -(\mu+\eta) \end{pmatrix}. \tag{2.48}$$

The corresponding eigenvalues are the roots of

$$\begin{vmatrix} -(\zeta+\mu+\delta_o+\eta) - \lambda & 0 \\ \gamma & -(\mu+\eta) - \lambda \end{vmatrix} = 0. \tag{2.49}$$

Solving for λ, we obtain the stability condition:

$$\frac{\gamma}{\zeta+\mu+\delta_o+\eta} < 1. \tag{2.50}$$

Thus, for E_0 to be locally asymptotically stable, we require:

$$R_0 = \max\left(\frac{\beta\alpha}{\mu(\gamma+\theta+\mu_Y)}, \frac{\gamma}{\zeta+\mu+\delta_o+\eta}\right) < 1. \tag{2.51}$$

This ensures that all eigenvalues have negative real parts, completing the proof. □

Theorem 2.4. *The Yahoo-Yahoo-free equilibrium* $E_0 = (V^*, Y^*, R^*, Q^*) = \left(\frac{\alpha}{\mu_v}, 0, 0, 0\right)$ *is globally asymptotically stable if the basic reproduction number* $R_0 < 1$.

Proof. To establish the global asymptotic stability, we construct a suitable Lyapunov function. Consider the function:

$$L(Y, R, Q) = Y + R + Q. \tag{2.52}$$

This function is non-negative and equals zero only at $(Y, R, Q) = (0, 0, 0)$, which corresponds to the Yahoo-Yahoo-free equilibrium E_0. Differentiating L along the system's trajectories,

$$\frac{dL}{dt} = \frac{dY}{dt} + \frac{dR}{dt} + \frac{dQ}{dt}. \tag{2.53}$$

Substituting from the model,

$$\frac{dL}{dt} = (\beta VY - (\gamma+\theta+\mu_Y)Y) + (\gamma Y - (\zeta+\mu+\delta_o+\eta)R) + (\theta Y + \zeta R - (\mu+\eta)Q). \tag{2.54}$$

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Rearranging terms,

$$\frac{dL}{dt} = \beta VY - (\gamma + \theta + \mu_Y - \gamma - \theta)Y - (\zeta + \mu + \delta_o + \eta - \zeta)R - (\mu + \eta)Q. \tag{2.55}$$

Since $V^* = \frac{\alpha}{\mu}$, we substitute

$$\beta V = \beta \frac{\alpha}{\mu}. \tag{2.56}$$

Thus, we obtain

$$\frac{dL}{dt} = \left(\beta \frac{\alpha}{\mu} - (\gamma + \theta + \mu_Y) \right) Y + (\gamma - (\zeta + \mu + \delta_o + \eta))\, R - (\mu + \eta)Q. \tag{2.57}$$

Since the basic reproduction number is defined as,

$$R_0 = \max \left(\frac{\beta \alpha}{\mu(\gamma + \theta + \mu_Y)}, \frac{\gamma}{\zeta + \mu + \delta_o + \eta} \right), \tag{2.58}$$

it follows that if $R_0 < 1$, then

$$\beta \frac{\alpha}{\mu} < (\gamma + \theta + \mu_Y) \quad \text{and} \quad \gamma < (\zeta + \mu + \delta_o + \eta). \tag{2.59}$$

Thus,

$$\frac{dL}{dt} \leq -(\mu + \eta)Q \leq 0. \tag{2.60}$$

Equality holds only if $Q = 0$, and from the system equations, this further implies $Y = 0$ and $R = 0$, meaning the system converges to E_0. Since $L(Y, R, Q)$ is a Lyapunov function and $\frac{dL}{dt} \leq 0$, with equality only at E_0, the Yahoo-Yahoo-free equilibrium is globally asymptotically stable whenever $R_0 < 1$.

□

Theorem 2.5. *The Yahoo-present equilibrium $E_1 = (V^{**}, Y^{**}, R^{**}, Q^{**})$ is locally asymptotically stable whenever the basic reproduction number $R_0 \geq 1$.*

Proof. To analyze the local stability of E_1, we perform a Jacobian matrix analysis. The Yahoo-present equilibrium E_1 satisfies:

$$\left. \begin{aligned} V^* &= \frac{\alpha}{\mu_v + \beta Y^*}, \\ Y^* &= \frac{\alpha}{\mu_v} \cdot \frac{R_0 - 1}{\beta}, \quad \text{(since } R_0 > 1\text{)}, \\ R^* &= \frac{\gamma Y^*}{\zeta + \mu + \delta_o + \eta}, \\ Q^* &= \frac{\theta Y^* + \zeta R^*}{\mu + \eta}. \end{aligned} \right\} \tag{2.61}$$

Substituting $V^{**}, Y^{**}, R^{**}, Q^{**}$ into J, we obtain,

$$J(E_1) = \begin{pmatrix} -\mu_v - \beta Y^{**} & -\beta V^{**} & 0 & 0 \\ \beta Y^{**} & \beta V^{**} - (\gamma + \theta + \mu_Y) & 0 & 0 \\ 0 & \gamma & -(\zeta + \mu + \delta_o + \eta) & 0 \\ 0 & \theta & \zeta & -(\mu + \eta) \end{pmatrix}. \tag{2.62}$$

The characteristic equation is given by $\det(J - \lambda I) = 0$. Observing the block diagonal structure, we obtain the following eigenvalues

- First eigenvalue: $-\mu_v - \beta Y^{**}$, which is always negative.
- Second eigenvalue: $\beta V^{**} - (\gamma + \theta + \mu_Y)$.

Since $R_0 > 1$, we conclude that,

$$\beta V^{**} - (\gamma + \theta + \mu_Y) < 0. \tag{2.63}$$

Thus, the second eigenvalue is also negative. Since all eigenvalues of $J(E_1)$ have negative real parts, it follows that the Yahoo-present equilibrium E_1 is locally asymptotically stable whenever $R_0 > 1$. □

□

Theorem 2.6. *The Yahoo-present equilibrium $E_1 = (V^{**}, Y^{**}, R^{**}, Q^{**})$ is globally asymptotically stable whenever the basic reproduction number $R_0 > 1$.*

Proof. To establish the global asymptotic stability, we construct a suitable Lyapunov function. Consider the function,

$$L(Y, R, Q) = (Y - Y^{**})^2 + (R - R^{**})^2 + (Q - Q^{**})^2. \tag{2.64}$$

This function is non-negative and equals zero only at $(Y, R, Q) = (Y^{**}, R^{**}, Q^{**})$. Differentiating L along the system's trajectories,

$$\frac{dL}{dt} = 2(Y - Y^{**})\frac{dY}{dt} + 2(R - R^{**})\frac{dR}{dt} + 2(Q - Q^{**})\frac{dQ}{dt}. \tag{2.65}$$

Substituting from the system equations

$$\left.\begin{aligned} \frac{dY}{dt} &= \beta VY - (\gamma + \theta + \mu_Y)Y, \\ \frac{dR}{dt} &= \gamma Y - (\zeta + \mu + \delta_o + \eta)R, \\ \frac{dQ}{dt} &= \theta Y + \zeta R - (\mu + \eta)Q, \end{aligned}\right\} \tag{2.66}$$

we obtain

$$\left.\begin{aligned} \frac{dL}{dt} &= 2(Y - Y^{**})\left[\beta VY - (\gamma + \theta + \mu_Y)Y\right] \\ &\quad + 2(R - R^{**})\left[\gamma Y - (\zeta + \mu + \delta_o + \eta)R\right] \\ &\quad + 2(Q - Q^{**})\left[\theta Y + \zeta R - (\mu + \eta)Q\right]. \end{aligned}\right\} \tag{2.67}$$

Expanding and rearranging terms yields

$$\left.\begin{aligned} \frac{dL}{dt} &= 2(Y - Y^{**})\beta VY - 2(Y - Y^{**})(\gamma + \theta + \mu_Y)Y \\ &\quad + 2(R - R^{**})\gamma Y - 2(R - R^{**})(\zeta + \mu + \delta_o + \eta)R \\ &\quad + 2(Q - Q^{**})\theta Y + 2(Q - Q^{**})\zeta R - 2(Q - Q^{**})(\mu + \eta)Q. \end{aligned}\right\} \tag{2.68}$$

Grouping similar terms becomes

$$\left.\begin{aligned} \frac{dL}{dt} &= -2(\gamma + \theta + \mu_Y)(Y - Y^{**})^2 \\ &\quad - 2(\zeta + \mu + \delta_o + \eta)(R - R^{**})^2 \\ &\quad - 2(\mu + \eta)(Q - Q^{**})^2. \end{aligned}\right\} \tag{2.69}$$

Since all parameters are positive, $\frac{dL}{dt} \leq 0$, with equality only when $Y = Y^{**}, R = R^{**}, Q = Q^{**}$. By Lyapunov's LaSalle Invariance Principle, the system converges to E_1, proving global asymptotic stability. □

□

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3 Discussion of Results

We employ a numerical integration solver in Python to simulate the dynamics of youth involvement in Yahoo-Yahoo and Yahoo+ (ritualist) in Nigeria. This approach allows us to analyze the model's behavior over time, given the complexity of the system. Since daily incidence data on this phenomenon is scarce, we rely on estimates from existing literature to provide a reasonable approximation for the model parameters and initial conditions in Table 1.

Table 1: Model parameters, meanings, estimated values, and sources

Symbol	Meaning	Value/Unit	Source
α	Recruitment rate of youths into society	$\approx$ 0.52 million per year	[1–6]
β	Influence/contact rate between vulnerable and cybercrime youths	0.74/year	[1–6]
μ	Natural death rate	0.012/year	[1–6]
γ	Progression rate from cybercrime to ritualism	0.63/year	[1–6]
θ	Progression rate to quitting stage (rehabilitation)	0.16/year	[1–6]
ζ	Progression rate to quitting stage (rehabilitation)	0.13/year	[1–6]
δ_o	Death rate due to ritualism	0.3/year	Assumed
η	Relapse rate into cybercrime/ritualism after quitting	0.77/year	[1–18]

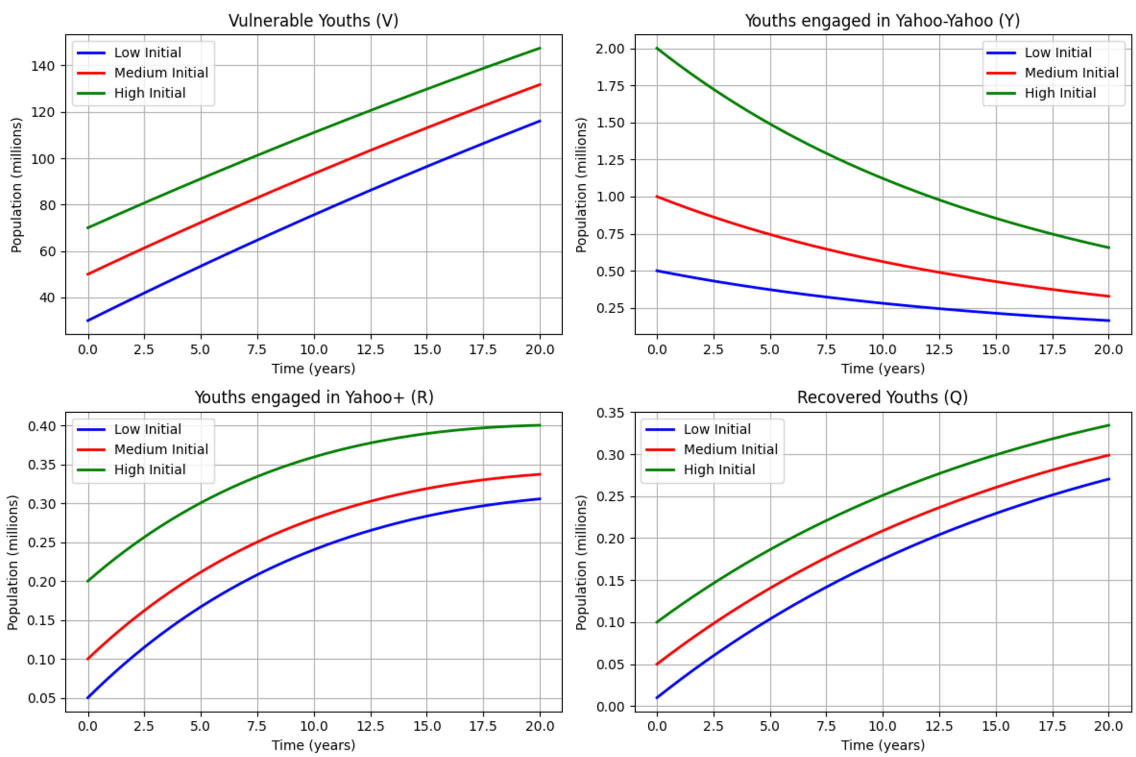

Figure 1: The behavior of the model variables under different initial rates

Figure 1 shows how the model variables behave under three different initial conditions: low (blue), medium (red), and high (green). The low initial condition represents a scenario where fewer individuals start in a given category, while the medium condition serves as the default scenario. The high initial condition depicts a situation where a significant number of individuals begin in a particular category. From the simulations, it is evident that a higher initial population of vulnerable youths leads to a greater number of individuals engaging in yahoo-yahoo (cybercrime) over time.

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Also, if the initial population is small, yahoo-yahoo(cybercrime) takes longer to gain momentum. For those already involved in yahoo-yahoo, a high initial value in Y results in a rapid increase in ritualism by youths into yahoo+ (R). However, when the initial value of Y is low, the rise in yahoo+ is slower but still persistent. If the initial number of youths into yahoo+ (R) is high, ritualism remain a significant issue over time. However, even if R_0 starts small, it can still grow as more individuals transition from yahoo-yahoo to yahoo+(ritualism). Regarding recovery via rehabilitation, a higher initial number of recovered youths (Q) accelerates the decline of crime rates, leading to faster recovery and social stability. However, if the initial value of Q is low, relapse effects (η) can sustain high crime rates for a prolonged period.

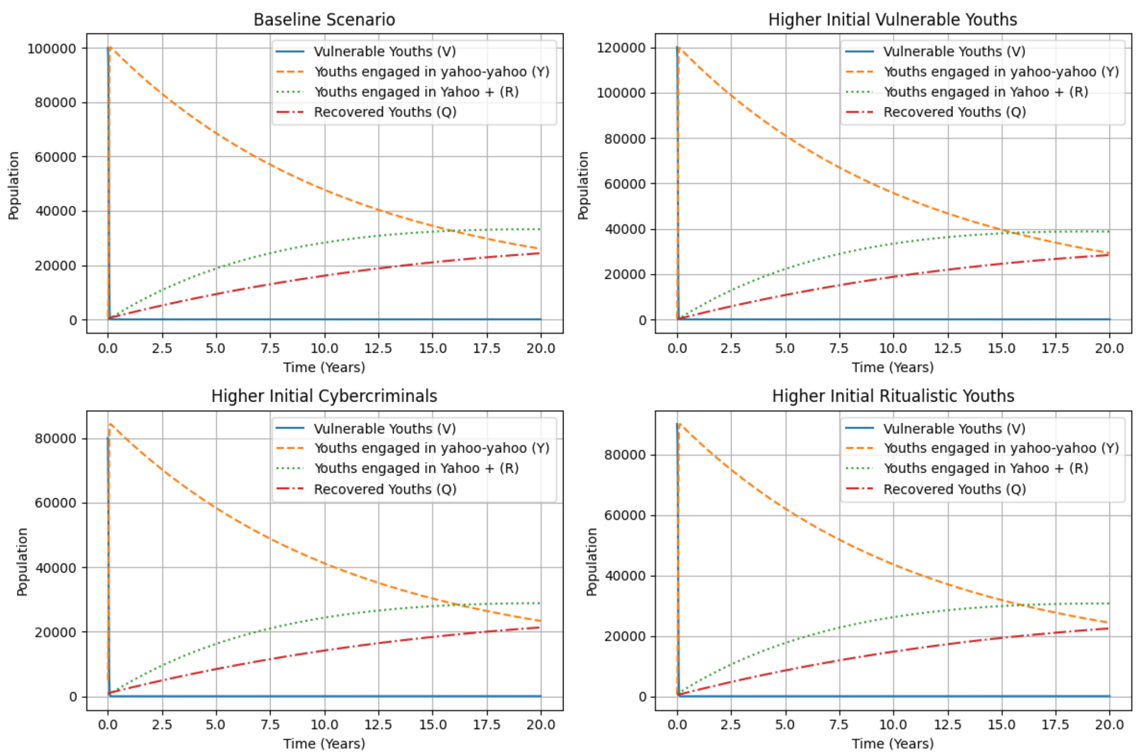

Figure 2: The behavior of the model variables under different initial rates

Figure 2 shows how different initial conditions affect the evolution of vulnerable youths (V), youths engaged in yahoo-yahoo (cybercrime) (Y), those engaged in both yahoo+ (ritualist) (R), and recovered youths via rehabilitation (Q). The results show that a rapid increase in yahoo-yahoo(cybercriminals) (Y) can lead to a surge in youths engaged in yahoo+(ritualist) (R), unless rehabilitation efforts (θ, ζ) are strong enough to counteract the trend. When the initial number of yahoo-yahoo youths is high, cybercrime spreads rapidly, making rehabilitation crucial in preventing further escalation. Additionally, when a large number of youths are vulnerable, contact rate (β) can cause a gradual rise in crime over time. Furthermore, if yotuhs into yahoo+(ritualist) start with a high initial population (R_0), merely controlling cybercrime may not be sufficient. In such cases, direct interventions targeting ritualism itself become necessary. These emphasize the need for early rehabilitation programs to aid recovery in order to prevent an uncontrollable increase in both yahoo-yahoo(cybercrime) and yahoo+(ritualist).

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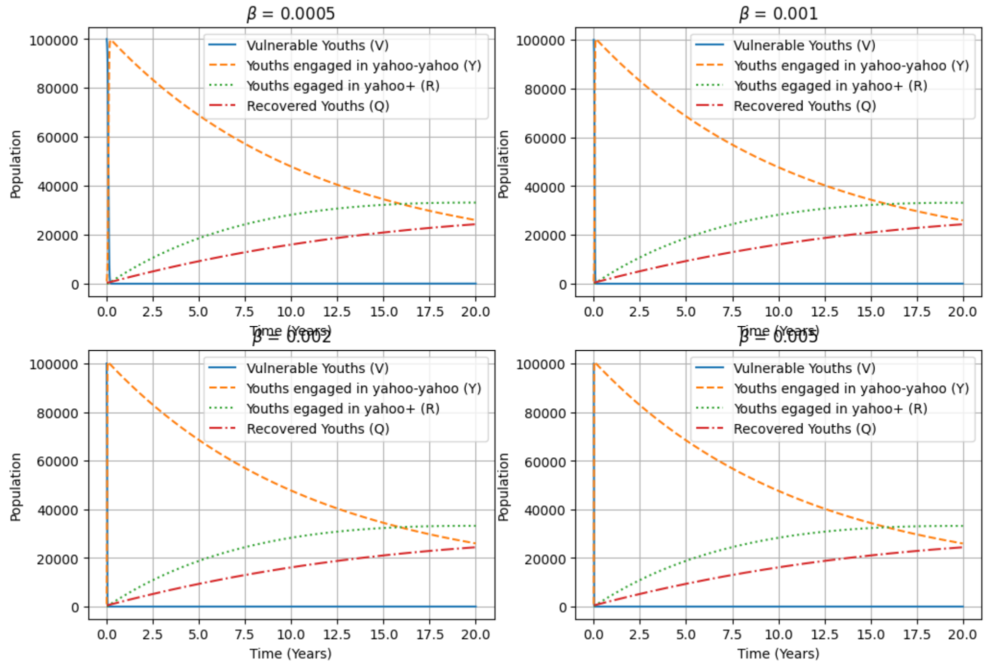

Figure 3: The behavior of the model variables under the variation of different contact (β) rates

In Figure 3, we vary the contact rate (β) to examine increasing exposure to yahoo-yahoo(cybercrime). We test different values of β. When β is low ($\beta = 0.0005 - 0.001$), yahoo-yahoo remains at a low, with many youths remaining in the vulnerable category. As β increases to a moderate range ($\beta = 0.002$), more youths transition into yahoo-yahoo, and if rehabilitation efforts are weak, the number of yahoo+(ritualists) (R) also begins to rise. When β is high ($\beta = 0.01$), cybercrime becomes endemic, driving the system towards instability as recruitment into crime accelerates. These findings provides insights for policymakers, the need for interventions to regulate youth exposure to cybercrime.

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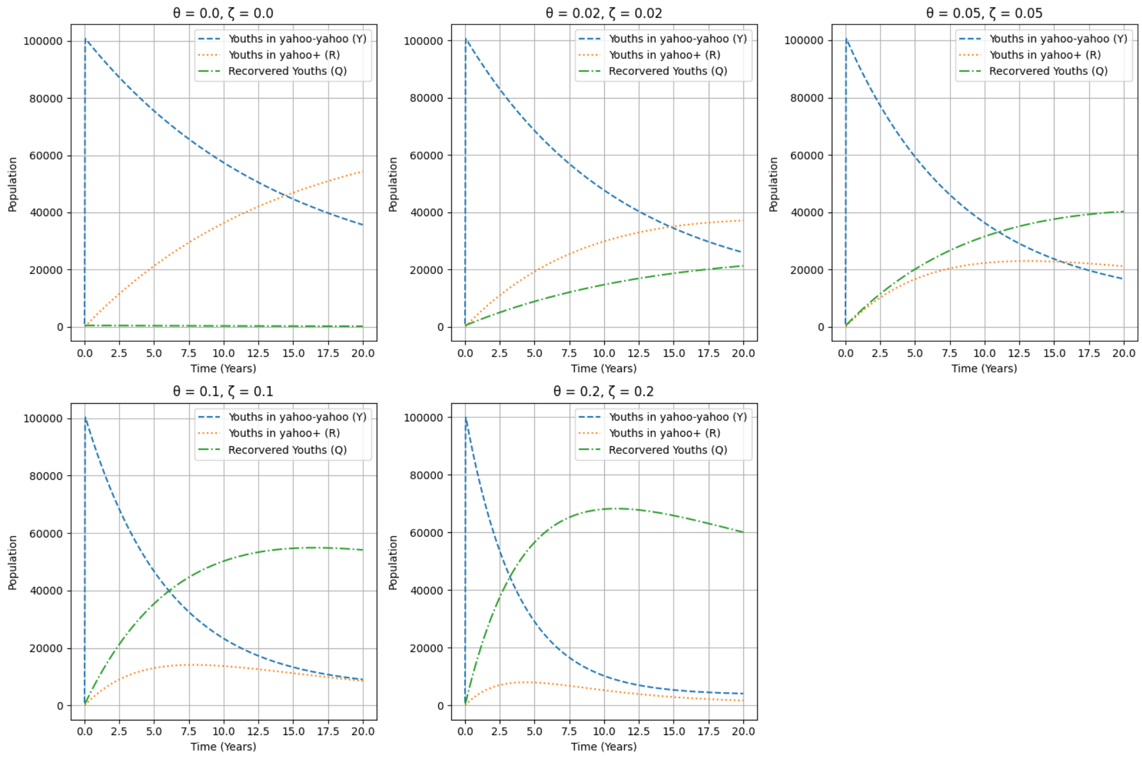

Figure 4: The effect of rehabilitation effects on the behavior of the model variables

Figure 4 evaluates the impact of different rehabilitation strategies leading to recovery. The five scenarios tested includes:

1. No rehabilitation ($\zeta = 0, \theta = 0$) - the worst-case scenario.
2. Low rehabilitation ($\theta = 0.02, \zeta = 0.02$).
3. Moderate rehabilitation ($\theta = 0.05, \zeta = 0.05$).
4. High rehabilitation ($\theta = 0.1, \zeta = 0.1$).
5. Very high rehabilitation ($\theta = 0.2, \zeta = 0.2$) - the best-case scenario.

When no rehabilitation is implemented, both yahoo-yahoo and yahoo+(ritualist) grow unchecked, with yahoo+(ritualist) surpassing yahoo-yahoo(cybercrime) over time. Low rehabilitation slightly slows the rise of yahoo+(ritualist) but fails to significantly reduce yahoo-yahoo(cybercrime). Moderate rehabilitation leads to a visible decline in yahoo-yahoo and containment of yahoo+(ritualist). With high rehabilitation, yahoo-yahoo(cybercrime) stabilizes at a low level, and in the case of very high rehabilitation, yahoo-yahoo(cybercrime) nearly disappears, and yahoo+(ritualist) becomes insignificant. These results offer some policy insights:

- At least 10% of cybercriminals and ritualists ($\theta, \zeta \geq 0.1$) must undergo rehabilitation to achieve a significant reduction in crime.
- A 20% rehabilitation rate can almost eliminate yahoo-yahoo(cybercrime).
- Higher rehabilitation rates reduce relapse effects (η), preventing individuals from returning to the two crimes.

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- If rehabilitation is not enforced, yahoo+(ritualism) will surpass yahoo-yahoo(cybercrime) within six years, highlighting the urgency of early interventions.

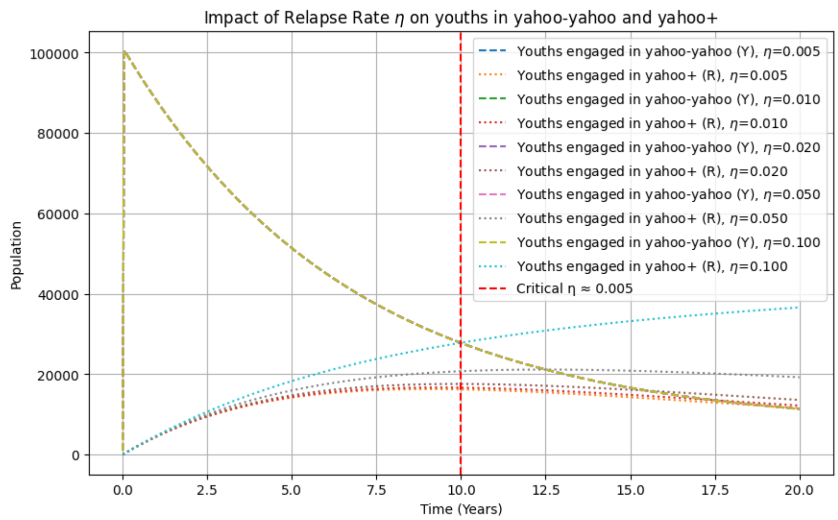

Figure 5: The effect of relapse (η) rates on youths engaged in yahoo-yahoo (cybercrime) and yahoo+(ritualist)

Figure 5 shows the impact of varying the relapse rate from 0.005 to 0.1 on yahoo-yahoo activities and yahoo+ (ritualist) . Here, a critical threshold is identified where the number of (yahoo+(ritualists) exceeds 2000 and remains persistently high, to show the failure of society to eradicate yahoo-yahoo crime. This critical relapse rate is marked on the graph with a red dashed line at year 10. When, at lower relapse rates, both yahoo-yahoo and yahoo+ (ritualist) decline over time. With moderate relapse rates, yahoo-yahoo activity persists but remains manageable. However, at higher relapse rates, yahoo-yahoo and yahoo+ (ritualist) become entrenched problems, as a significant number of rehabilitated individuals revert to the crimes. Policy recommendations suggest that reducing relapse below 0.02 through strict monitoring and reintegration programs can help sustain crime-free interventions. This simulation shows the importance of maintaining low relapse rates through effective policy interventions.

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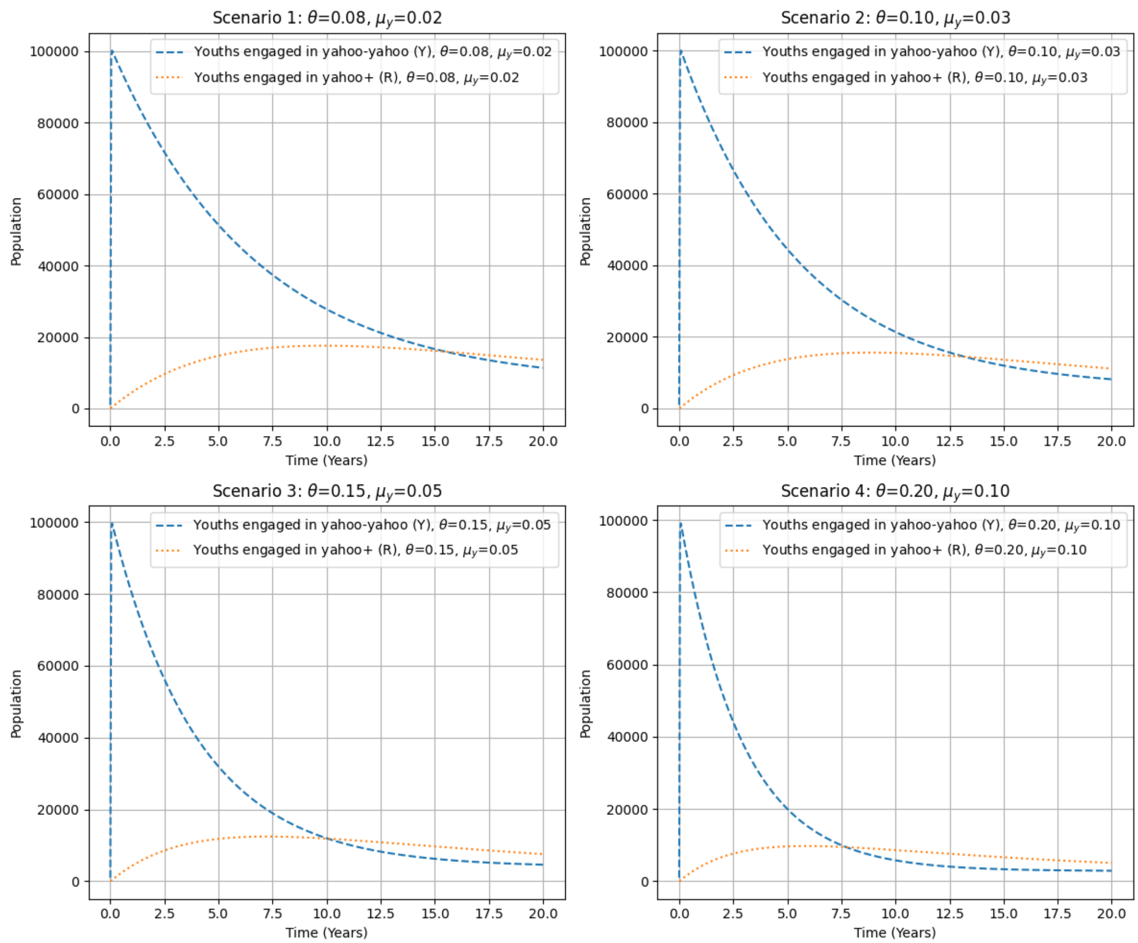

Figure 6: The long-term trends of yahoo-yahoo crimes and yahoo+ (ritualist) over a 20-year period

Figure 6 presents the long-term trends of yahoo-yahoo crimes and yahoo+ (ritualist) over a 20-year period. The effects of varying rehabilitation θ on cybercrime reduction is analyzed. Four different scenarios are displayed in the subplots, comparing the effect of $(\theta,\ \mu_y)$. Under current conditions, yahoo-yahoo(cybercrime) persists for years before gradually declining by increasing rehabilitation accelerates the reduction. Enhancing law enforcement delays cybercrime but does not eliminate it completely without concurrent rehabilitation efforts. On the other hand, solely increasing arrest of youths engaged in yahoo-yahoo is a weaker strategy, as it merely delays cybercrime without resolving it. Without any intervention, yahoo-yahoo(cybercrime) and yahoo+(ritualist) will persist indefinitely.

4 Conclusion

The findings of this study show the alarming persistence of youth involvement in yahoo-yahoo(cybercrime) and its ritualistic extension (Yahoo+), probably driven by economic instability, social influence, and weak rehabilitation structures. The developed compartmental model effectively captures the dynamics of youth transitioning between different states, described with variables and parameters. The model solutions are found to exist, positive and bounded, while stability analysis in view of the R_0 reveals that without targeted and sustained interventions, yahoo-yahoo(cybercrime) and yahoo+(ritualist) may persist at endemic levels. Numerical simulations indicate that reducing relapse rates and increasing rehabilitation success are critical to curbing these activities. Importantly, a critical threshold exists where high relapse rates render rehabilitation efforts ineffective, leading to sustained or increasing yahoo-yahoo levels. These findings show the need for a holistic approach that combines preventive and corrective measures to ensure long-term eradication of cybercrime among youths. Policymakers should therefore develop intervention frameworks that incorporate these mathematical insights to formulate policies that are both theoretically sound and practically effective. Continuous monitoring of these parameter values through real-time data collection and mathematical modeling should guide policymakers in dynamically adjusting intervention strategies for maximum impact. By implementing these policies and mathematical recommendations, stakeholders can collaboratively dismantle the structures sustaining yahoo-yahoo(cybercrime) and yahoo+(ritualist), to yield a safer and more productive society for future generations.

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