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Hyper-Homomorphisms in Obic Algebras

E. Ilojide [1*], O. O. George [2]

1. Department of Mathematics, Federal University of Agriculture, Abeokuta 110101, Nigeria.
2. Department of Mathematics, University of Lagos, Akoka Lagos, Nigeria.
* Corresponding author: emmailojide@yahoo.com, ilojidee@funaab.edu.ng*,
femi_george@yahoo.co.uk, oogeorge@unilag.edu.ng

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Abstract

In this paper, hyper-homomorphisms of Obic algebras which are generalizations of homomorphisms in Obic algebras are introduced. Their properties are investigated. It is shown that homomorphisms of Obic algebras preserve oscillation. Furthermore, it is established that with a suitably defined binary operation, the collection of hyper-homomorphisms in Obic algebras is also an Obic algebra. Moreover, monics, regular and preserving maps of Obic algebras are studied through their hyper-homomorphisms.

Keywords: Obic Algebras, Homomorphism, Hyper-Homomorphism.
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1 Introduction

An algebra of type (2, 0) is a non-empty set, having a constant element, on which is defined a binary operation such that certain axioms are satisfied. BCI algebras and BCK algebras, introduced in [17] and [16], are common varieties of such algebras. There are several other varieties of algebras of type (2, 0). There are also several generalizations of BCI algebras. In [5], BCH algebras were studied. In [23], D algebras were studied. In [21], the notion of BE algebras was introduced. Ideals and upper sets in BE algebras were investigated in [2] and [1]. Pre- commutative algebras were studied in [20]. Fenyves algebras were studied in [18], [15] and [19]. In [22], Q algebras were introduced. Homomorphisms of Q algebras were studied in [12].
Recently, it has been shown in [3] that algebras of type (2,0) have diverse applications in coding theory. Motivated by this, more research interest has been given to the study of algebras of type (2,0). Obic algebras were introduced in [7]. In [8], Torian algebras were studied. It was shown that the class of Torian algebras is a wider class than the class of Obic algebras. Ideals of Torian algebras were investigated in [11]. The dual and nuclei of ideals as well as congruences developed on ideals of Torian algebras were studied. In [9], right distributive Torian algebras were studied. Isomorphism Theorems of Torian algebras were studied in [10]. Kreb algebras were studied in [13]. Polian algebras were studied in [14]. In this paper, hyper-homomorphisms of Obic algebras which are generalizations of homomorphisms in Obic algebras are introduced. Their properties are investigated. It is shown

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that homomorphisms of Obic algebras preserve oscillation. Furthermore, it is established that with a suitably defined binary operation, the collection of hyper-homomorphisms in Obic algebras is also an Obic algebra. Moreover, monics, regular and preserving maps of Obic algebras are studied through their hyper-homomorphisms.

2 Preliminaries

In this section, some basic concepts necessary for proper understanding of this paper are discussed.

Definition 2.1. *[7] A triple $(X;*,0)$, where X is a non-empty set, $*$ a binary operation on X and 0 a constant element of X is called an obic algebra if the following axioms hold for all $x, y, z \in X$:*

1. $x * 0 = x$;
2. $[x * (y * z)] * x = x * [y * (z * x)]$;
3. $x * x = 0$

Example 2.2. *[7] Consider the multiplicative group $G = \{1, -1, i, -i\}$. Define a binary operation $*$ on G by $a * b = ab^{-1}$. Then $(G;*,1)$ is an obic algebra.*

Example 2.3. *[7] Let $X = \{0,1\}$. Define a binary operation $*$ on X by the following table:*

$*$	0	1
0	0	1
1	1	0

Then $(X;*,0)$ is an obic algebra.

We shall adopt the notation X for an obic algebra $(X;*,0)$ unless there is the need to emphasize the binary operation in X.

Definition 2.4. *[7] A non-empty subset S of an obic algebra X is called a sub-algebra of X if S is an obic algebra with respect to the binary operation in X.*

Proposition 2.5. *[7] A non-empty subset S of an obic algebra X is a subalgebra if and only if the following hold:*

1. $0 \in S$;
2. $x * y \in S$ *for all* $x, y \in S$.

Definition 2.6. *[7] Let $(X;*,0)$ and $(Y,\circ,0')$ be obic algebras. A function $f : X \to Y$ is called an obic homomorphism if $f(a * b) = f(a) \circ f(b)$ for all $a, b \in X$.*

Definition 2.7. *[7] Let $(X;*,0)$ and $(Y,\circ,0')$ be obic algebras, and let $f : X \to Y$ be an obic homomorphism. The set $\{x \in X : f(x) = 0'\}$ is called the kernel of f.*

Remark 2.8. *Let $(X;*,0)$ and $(Y,\circ,0')$ be obic algebras, and let $f : X \to Y$ be an obic homomorphism. The kernel of f is denoted by $ker(f)$.*

Proposition 2.9. *[7] Let $(X;*,0)$ and $(Y,\circ,0')$ be obic algebras, and let $f : X \to Y$ be an obic homomorphism. Then,*

1. $f(0) = 0'$
2. $x * y = 0 \Rightarrow f(x) \circ f(y) = 0'$ *for all* $x, y \in X$.

Definition 2.10. *[7] Let $(X;*,0)$ be an obic algebra. Define $'\wedge'$ on X by $x \wedge y = y*(y*x)$ for all $x, y \in X$*

Definition 2.11. *[7] Let X be an obic algebra. A function $\theta : X \to X$ is called a left (respectively right) monic if $\theta(x*y) = (\theta(x)*y) \wedge (x*\theta(y))$ (respectively $\theta(x*y) = (x*\theta(y)) \wedge (\theta(x)*y)$) for all $x, y \in X$.*

If θ is both a left and a right monic, then θ is called a monic.

Definition 2.12. *[7] Let X be an obic algebra. A self map θ on X is called regular if $\theta(0) = 0$*

Definition 2.13. *[7] Let X be an obic algebra. A self map θ on X is called self preserving if $\theta(x)*x = x$ for all $x \in X$.*

Definition 2.14. *[7] Let X be an obic algebra. A self map θ on X is called anti-self preserving if $x*\theta(x) = x$ for all $x \in X$.*

Definition 2.15. *[7] Let X be an obic algebra. A self map θ on X is called preserving if θ is both self preserving and anti-self preserving.*

3 Main Results

3.1 Hyper-homomorphisms

In this section, the concept of hyper-homomorphisms in obic algebras is introduced and some related properties are discussed.

Definition 3.1. *Let $(X;*,0)$ and $(Y;\cdot,0')$ be obic algebras. A map $f : X \to Y$ is called a hyper map if for any $x, y \in X$ such that $x*y \neq 0$, then either $f(x) = f(y)$ or $f(x)\cdot f(y) \neq 0'$.*

Definition 3.2. *Let $(X;*,0)$ and $(Y;\cdot,0')$ be obic algebras. A homomorphism $f : X \to Y$ is called a hyper-homomorphism if f is a hyper map.*

Definition 3.3. *Let $(X;*,0)$ and $(Y;\cdot,0')$ be obic algebras. Let X^* denote the collection of hyper-homomorphisms from X to Y. Define a binary operation $\odot$ on X^* by $(f\odot g)(x) = f(x)\cdot g(x)$ for all $x \in X$.*

Definition 3.4. *Let $(X;*,0)$ and $(Y;\cdot,0')$ be obic algebras. Then X^* is said to be a complete collection of hyper-homomorphisms if $ker(f) \cap ker(g) = ker(f\odot g)$ for any $f, g \in X^*$.*

Example 3.5. *Let $X = \{0,1,2,3\}$. Define a binary operation $*$ on X by the following table*

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $(X;*,0)$ is an obic algebra.
Now, let $f : X \to X$ be defined by $f(0) = f(1) = f(2) = f(3) = 0$;
$g : X \to X$ be defined by $g(0) = g(1) = 0, g(2) = g(3) = 1$;
$h : X \to X$ be defined by $h(0) = h(1) = 0, h(2) = h(3) = 2$;
$p : X \to X$ be defined by $p(0) = p(1) = 0, p(2) = p(3) = 3$
Then f, g, h, and p are hyper-homomorphisms. Moreover, $X^* = \{f, g, h, p\}$ is a complete collection of hyper-homomorphisms.

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Definition 3.6. *An obic algebra X is said to be oscillatory if $(a*b)*(c*d) = (a*c)*(b*d)$ for all $a, b, c, d \in X$.*

The following lemma is obvious from definition.

Lemma 3.7. *Let X be an oscillatory obic algebra. Then the following hold for all $a, b, c, d \in X$:*

1. $0*(c*d) = (0*c)*(0*d)$
2. $(a*b)*(0*d) = a*(b*d)$
3. $(a*b)*c = (a*c)*b$
4. $0*(c*d) = (a*c)*(a*d)$
5. $a*b = (a*c)*(b*c)$

The next proposition shows that homomorphisms of obic algebras preserve oscillation.

Proposition 3.8. *Let X be an oscillatory obic algebra, and let $f : X \to Y$ be a homomorphism. Then $f((a*b)*(c*d)) = f((a*c)*(b*d))$ for all $a, b, c, d \in X$*

Proof. Let $a, b, c, d \in X$. Notice that $f((a*b)*(c*d)) = f(a*b)*f(c*d) = (f(a)*f(b))*(f(c)*f(d)) = (f(a*c))*(f(b*d)) = f((a*c)*(b*d))$ as required. □

Definition 3.9. *[7] An obic algebra X is said to be implicative if $x*(y*x) = x$ for all $x, y \in X$*

Lemma 3.10. *[7] Let X be an implicative obic algebra. Then the following hold for all $x, y \in X$:*

1. $x*y = (x*y)*(0*y)$
2. $x*y = (x*(y*x))*y$
3. $x*y = x*(y*(x*y))$

The following proposition follows from Lemma 3.10.

Proposition 3.11. *Let X be an implicative obic algebra. Then the following hold for all $x, y \in X$:*

1. $(x*y)*(0*y) = (x*(y*x))*y$
2. $(x*y)*(0*y) = x*(y*(x*y))$
3. $(x*(y*x))*y = x*(y*(x*y))$

Theorem 3.12. *Let X be an oscillatory obic algebra. If X is implicative, then the following hold for all $a, b, c, d \in X$:*

1. $(0*(c*d))*(c*d) = (((0*c)*((0*d)*(0*c))))*(0*d)$
2. $0*((c*d)*(0*(c*d))) = (0*c)*((0*d)*((0*c)*(0*d)))$
3. $(a*(b*d))*(0*(b*d)) = ((a*b)*(0*d))*(0*(0*d))$
4. $(a*((b*d)*a))*(b*d) = ((a*b)*((0*d)*(a*b))*(0*d)$
5. $a*(b*d) = (a*b)*((0*d)*((a*b)*(0*d)))$
6. $((a*b)*c)*(0*c) = ((a*c)*b)*(0*b)$
7. $((a*b)*(c*(a*b)))*c = ((a*c)*(c*(a*b)))*c$
8. $(a*b)*(c*((a*b)*c)) = (a*c)*(b*((a*c)*b))$

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9. $0 * ((c * d) * (0 * (c * d))) = (a * c) * ((a * d) * ((a * c) * (a * d)))$

Proof. 1. By Lemma 3.10(2), we have

$$0 * (c * d) = (0 * ((c * d) * 0)) * (c * d) = (0 * (c * d)) * (c * d) \tag{1}$$

Also, by Lemma 3.7(1) and Lemma 3.10(2), we have,

$$0 * (c * d) = (0 * c) * (0 * d) = ((0 * c) * ((0 * d) * (0 * c))) * (0 * d) \tag{2}$$

Now, by expressions (1) and (2), we have the result.

2. By Lemma 3.10(3), we have

$$0 * (c * d) = 0 * ((c * d) * (0 * (c * d))) \tag{3}$$

Also, by Lemma 3.7(1) and Lemma 3.10(3), we have

$$0 * (c * d) = (0 * c) * (0 * d) = (0 * c) * ((0 * d) * ((0 * c) * (0 * d))) \tag{4}$$

Now, by expressions (3) and (4), we have the result.

3. By Lemma 3.10(1), we have

$$a * (b * d) = (a * (b * d)) * (0 * (b * d)) \tag{5}$$

Also, by Lemma 3.7(2) and Lemma 3.10(1), we have

$$a * (b * d) = (a * b) * (0 * d) = ((a * b) * (0 * d)) * (0 * (0 * d)) \tag{6}$$

By expressions (5) and (6), we have the result.

4. By Lemma 3.10(2), we have

$$a * (b * d) = (a * ((b * d) * a)) * (b * d) \tag{7}$$

Also, by Lemma 3.7(2) and Lemma 3.10(2), we have

$$a * (b * d) = (a * b) * (0 * d) = ((a * b) * ((0 * d) * (a * b)) * (0 * d) \tag{8}$$

By expressions (7) and (8), we have the result.

5. By Lemma 3.8(3), we have

$$a * (b * d) = a * ((b * d) * (a * (b * d))) \tag{9}$$

Also, by Lemma 3.7(2) and Lemma 3.10(3), we have

$$a * (b * d) = (a * b) * (0 * d) = (a * b) * ((0 * d) * ((a * b) * (0 * d))) \tag{10}$$

By expressions (9) and (10), we have the result.

6. By Lemma 3.10(1), we have

$$(a * b) * c = ((a * b) * c) * (0 * c) \tag{11}$$

Also, by Lemma 3.7(3) and Lemma 3.10(1), we have

$$(a * b) * c = (a * c) * b = ((a * c) * b) * (0 * b) \tag{12}$$

By expressions (11) and (12), we have the result.

7. By Lemma 3.10(2), we have

$$(a * b) * c = ((a * b) * (c * (a * b))) * c \tag{13}$$

Also, by Lemma 3.7(3) and Lemma 3.10(2), we have

$$(a * b) * c = (a * c) * b = ((a * c) * (c * (a * b))) * c \tag{14}$$

By expressions (13) and (14), we have the result.

8. By Lemma 3.10(3), we have

$$(a * b) * c = (a * b) * (c * ((a * b) * c)) \tag{15}$$

Also, by Lemma 3.7(3) and Lemma 3.10(3), we have

$$(a * b) * c = (a * c) * b = (a * c) * (b * ((a * c) * b)) \tag{16}$$

By expressions (15) and (16), we have the result.

9. By Lemma 3.10(3), we have

$$0 * (c * d) = 0 * ((c * d) * (0 * (c * d))) \tag{17}$$

Also, by Lemma 3.7(4) and Lemma 3.10(3), we have

$$0 * (c * d) = (a * c) * (a * d) = ((a * d) * ((a * c) * (a * d))) \tag{18}$$

By expressions (17) and (18), we have the result.

□

Theorem 3.13. *Let $(X; *, 0)$ be an obic algebra, and let $(Y; \cdot, 0')$ be an oscillatory obic algebra. Then $f \odot g \in X^*$ for all $f, g \in X^*$.*

Proof. Let $f, g \in X^*; x, y \in X$. Then $f \odot g(x * y) = f(x * y) \cdot g(x * y) = (f(x) \cdot f(y)) \cdot (g(x) \cdot g(y) = (f \odot g(x)) \cdot (f \odot g(y))$. So, $f \odot g$ is a homomorphism.
We now show that $f \odot g$ is a hyper map. Suppose $f \odot g$ is not a hyper map. Then there exists $x, y \in X, (x * y \neq 0)$ such that $(f \odot g)(x) \neq (f \odot g)(y)$ and $(f \odot g)(x) \cdot (f \odot g)(y) = 0'$. Notice that $0' = (f \odot g)(x) \cdot (f \odot g)(y) = (f(x) \cdot g(x)) \cdot (f(y) \cdot g(y)) = (f(x) \cdot f(y)) \cdot (g(x) \cdot g(y)) = f(x * y) \cdot g(x * y) = f \odot g(x * y)$. So, $x * y \in Ker(f \odot g) = ker(f) \cap ker(g)$. Then $0' = f(x * y) = f(x) \cdot f(y)$. Also, $0' = g(x * y) = g(x) \cdot g(y)$. Hence we have $f(x) = f(y)$ and $g(x) = g(y)$. Therefore, $f \odot g(x) = f(x) \cdot g(x) = f(y) \cdot g(y) = f \odot g(y)$; which is a contradiction to our earlier claim that $f \odot (x) \neq f \odot g(y)$. Hence $f \odot g$ is a hyper map as required. □

The following proposition is straightforward from definition.

Proposition 3.14. *Let $(X; *, 0)$ and $(Y; \cdot, 0')$ be obic algebras. The map $0_* : X \to Y$ defined by $0_*(x) = 0'$ for all $x \in X$ is a hyper-homomorphism.*

Theorem 3.15. *Let $(X; *, 0)$ be an obic algebra, and let $(Y; \cdot, 0')$ be an oscillatory obic algebra. Then $(X^*; \odot, 0_*)$ is an obic algebra.*

Proof. By Theorem 3.13, X^* is closed under the operation $\odot$. Let $f \in X^*$, and let $x \in X$. Then clearly, $f \odot 0_* = f$ and $f \odot f = 0_*$. Now, let $f, g, h \in X^*$. Notice that $((f \odot (g \odot h)) \odot f)(x) = f \odot (g \odot (h \odot f))(x)$ for all $x \in X$. So, $(f \odot (g \odot h)) \odot f = f \odot (g \odot (h \odot f))$. Hence, $(X^*; \odot, 0_*)$ is an obic algebra as required. □

Definition 3.16. *Let X be an obic algebra. Let $x, y \in X$, and let $k \in \mathbb{N}$; where $\mathbb{N}$ is the set of natural numbers. Then define $x * y^k = (((x * y) * y) * ...) * y$ (y appears k times).*

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Definition 3.17. *Let X be an obic algebra. An element $x \in X$ is called a univalent element if $0 * x^k = 0$ for some $k \in \mathbb{N}$.*

Definition 3.18. *An obic algebra X is said to be univalent if every $x \in X$ is univalent.*

Theorem 3.19. *Let $(X; *, 0)$ be an obic algebra, and let $(Y; \cdot, 0')$ be an oscillatory obic algebra. If $(Y; \cdot, 0')$ is univalent, then $(X^*; \odot, 0_*)$ is univalent.*

Proof. Let $f \in X^*; x \in X$. Then $y = f(x) \in Y$. Now, there exists $k \in \mathbb{N}$ such that $0' \cdot f(x)^k = 0'$. Notice that $0_*(x) = 0'$. So, $0_* = 0' \cdot f(x)^k = (((0' \cdot f(x)) \cdot f(x))...) \cdot f(x)$ ($f(x)$ appears k times) $= (((0_*(x) \cdot f(x)) \cdot f(x))...) \cdot f(x) = ((((0_*(x) \odot f) \odot f)...) \odot f)(x)$ (f appears k times) $= (0_* \odot f^k)(x)$. Therefore, $0_* \odot f^k = 0_*$ for all $f \in X^*$ as required. □

Definition 3.20. *Let $(X; *, 0)$ be an obic algebra, and let $(Y; \cdot, 0')$ be an oscillatory obic algebra. Let A be a subset of X and B a subset of X^*. The set $A' = \{f \in X^* : f(x) = 0'\}$ for all $x \in A$, is called the dual of A.*
The set $B' = \{x \in X : f(x) = 0'\}$ for all $f \in B$, is called the dual of B.

The following proposition is obvious from definition.

Proposition 3.21. *Let $(X; *, 0)$ be an obic algebra, and let $(Y; \cdot, 0')$ be an oscillatory obic algebra. Let A and B be subsets of X and X^* respectively. Then A' is a subalgebra of X^* and B' is a subalgebra of X.*

3.2 Monics, regular and preserving maps

In this section, we study monics, regular and preserving maps of oscillatory obic algebras through their hyper-homomorphisms.

Proposition 3.22. *Let θ be a left monic on an oscillatory obic algebra X. Then $(f(x) * \theta(g(x))) * ((f(x) * \theta(g(x))) * (\theta(f(x)) * g(x))) = \theta((f \odot g)(x))$ for all $f, g \in X^*, x \in X$.*

Proof. Let $f, g \in X^*, x \in X$. Then $\theta((f \odot g)(x)) = \theta(f(x) * g(x)) =$
$(f(x) * \theta(g(x))) * ((f(x) * \theta(g(x))) * (\theta(f(x)) * g(x)))$ as required. □

Proposition 3.23. *Let θ be a left monic on an oscillatory obic algebra X. Then the following hold for all $f \in X^*; x, y \in X$*

1. *$(f(x) * \theta(f(x))) * ((f(x) * \theta(f(x))) * (\theta(f(x)) * f(x))) = \theta(0_*(x))$;*
2. *If θ is regular, then $(f(x) * \theta(f(x))) * ((f(x) * \theta(f(x))) * (\theta(f(x)) * f(x))) =$
$(f(y) * \theta(f(y))) * ((f(y) * \theta(f(y))) * (\theta(f(y)) * f(y)))$;*
3. *If θ is self preserving, then $(f(x) * \theta(f(x))) * ((f(x) * \theta(f(x))) * f(x)) = \theta(0_*(x))$;*
4. *if θ is anti-self preserving, then $f(x) * ((f(x) * \theta(f(x))) * (\theta(f(x)) * f(x))) = \theta(0_*(x))$;*
5. *If θ is regular and self preserving, then $(f(x) * \theta(f(x))) * ((f(x) * \theta(f(x))) * f(x)) =$
$(f(y) * \theta(f(y))) * ((f(y) * \theta(f(y))) * f(y))$;*
6. *If θ is regular and anti-self preserving, then $f(x) * ((f(x) * \theta(f(x))) * (\theta(f(x)) * f(x))) =$
$f(y) * ((f(y) * \theta(f(y))) * (\theta(f(y)) * f(y)))$.*

Proof.
1. Let $x \in X$. Then $\theta(0_*(x)) = \theta((f \odot f)(x)) =$
$(f(x) * \theta(f(x))) * ((f(x) * \theta(f(x))) * (\theta(f(x)) * f(x)))$ as required.
2. Notice that $0 = 0_*(x) = \theta(0_*(x)) = \theta((f \odot f)(x)) =$
$(f(x) * \theta(f(x))) * ((f(x) * \theta(f(x))) * (\theta(f(x)) * f(x)))$ (by item (1).
Now, replacing x with y together with the fact the resulting expression gives 0, we have the result.

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3. Follows from item (1).
4. Follows from item (1).
5. Follows from items (2) and (3).
6. Follows from items (2) and (4).

□

Proposition 3.24. *Let θ be a right monic on an oscillatory obic algebra X. Then $(\theta(f(x)) * g(x)) * ((\theta(f(x) * g(x)) * (f(x) * \theta(g(x))))) = \theta((f \odot g)(x))$ for all $f, g \in X^*, x \in X$.*

Proof. Let $f, g \in X^*; x \in X$. Then $\theta((f \odot g)(x)) = \theta(f(x) * g(x)) =$
$(\theta(f(x)) * g(x)) * ((\theta(f(x) * g(x)) * (f(x) * \theta(g(x)))))$ as required. □

Proposition 3.25. *Let θ be a right monic on an oscillatory obic algebra X. Then the following hold for all $f \in X^*, x, y \in X$*

1. $(\theta(f(x)) * f(x)) * ((\theta(f(x)) * f(x)) * (f(x) * \theta(f(x)))) = \theta(0_*(x))$;
2. *If θ is regular, then* $(\theta(f(x)) * f(x)) * ((\theta(f(x)) * f(x)) * (f(x) * \theta(f(x)))) =$
$(\theta(f(y)) * f(y)) * ((\theta(f(y)) * f(y)) * (f(y) * \theta(f(y))))$;
3. *If θ is self preserving,* $f(x) * (f(x) * (f(x) * \theta(f(x)))) = \theta(0_*(x))$;
4. *If θ is anti-self preserving, then* $(\theta(f(x)) * f(x)) * ((\theta(f(x)) * f(x)) * f(x)) = \theta(0_*(x))$;
5. *If θ is regular and self preserving, then* $f(x) * (f(x) * (f(x) * \theta(f(x)))) =$
$f(y) * (f(y) * (f(y) * \theta(f(y))))$;
6. *if θ is regular and anti-self preserving, then* $(\theta(f(x)) * f(x)) * ((\theta(f(x)) * f(x)) * f(x)) =$
$(\theta(f(x)) * f(x)) * ((\theta(f(x)) * f(x)) * f(x))$

Proof.

1. Let $x \in X$. Then $\theta(0_*(x)) = \theta((f \odot f)(x)) = \theta(f(x) * f(x)) =$
$(\theta(f(x)) * f(x)) * ((\theta(f(x)) * f(x)) * (f(x) * \theta(f(x))))$ as required.
2. Notice that $0 = 0_*(x) = \theta(0_*(x)) = \theta((f \odot f)(x)) =$
$(\theta(f(x)) * f(x)) * ((\theta(f(x)) * f(x)) * (f(x) * \theta(f(x))))$.
Now, replacing x with y together with the fact that the resulting expression gives 0, we have the result.
3. Follows from item (1).
4. Follows from item (1).
5. Follows from items (2) and (3).
6. Follows from items (2) and (4).

□

Combining Proposition 3.22 and Proposition 3.24, we have the following theorem:

Theorem 3.26. *Let θ be a monic on an oscillatory obic algebra X. Then* $(f(x) * \theta(g(x))) * ((f(x) * \theta(g(x))) * (\theta(f(x)) * g(x))) =$
$(\theta(f(x)) * g(x)) * ((\theta(f(x) * g(x)) * (f(x) * \theta(g(x)))))$ *for all $f, g \in X^*, x \in X$.*

Proposition 3.27. *Let θ be a left monic on an oscillatory obic algebra X. Then* $(0 * \theta(f(x))) * ((0 * \theta(f(x))) * (\theta(0)) * f(x))) = \theta((0_* \odot f)(x))$ *for all $f \in X^*, x \in X$.*

Proof. Let $f \in X^*, x \in X$. Then $\theta((0_* \odot f)(x)) = \theta(0_*(x) * f(x)) =$
$(0 * \theta(f(x))) * ((0\theta(f(x))) * (\theta(0)) * f(x)))$ as required. □

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The following corollary is obvious.

Corollary 3.28. *Let θ be a regular left monic on an oscillatory obic algebra X. Then* $(0*\theta(f(x)))*((0*\theta(f(x)))*(0*f(x))) = \theta((0_* \odot f)(x))$ *for all* $f \in X^*, x \in X$.

Proposition 3.29. *Let θ be a right monic on an oscillatory obic algebra X. Then* $(0*f(x))*((\theta(0)*f(x))*(0*\theta(f(x)))) = \theta((0_* \odot f)(x))$ *for all* $f \in X^*, x \in X$.

Proof. Let $f \in X^*, x \in X$. Then $\theta((0_* \odot f)(x)) = \theta(0_*(x) * f(x)) =$
$(0*f(x))*((\theta(0)*f(x))*(0*\theta(f(x))))$ as required. □

The following corollary is obvious.

Corollary 3.30. *Let θ be a regular right monic on an oscillatory obic algebra X. Then* $(0*f(x))*((0*f(x))*(0*\theta(f(x)))) = \theta((0_* \odot f)(x))$ *for all* $f \in X^*, x \in X$.

Combining Proposition 3.27 and Proposition 3.29, we have the following theorem:

Theorem 3.31. *Let θ be a monic on an oscillatory obic algebra X. Then* $(0*\theta(f(x)))*((0*\theta(f(x)))*(\theta(0))*f(x))) =$
$(0*f(x))*((\theta(0)*f(x))*(0*\theta(f(x))))$ *for all* $f \in X^*, x \in X$.

Combining Corollary 3.28 and Corollary 3.30, we have the following theorem:

Theorem 3.32. *Let θ be a regular monic on an oscillatory obic algebra X. Then* $(0*\theta(f(x)))*((0*\theta(f(x)))*(0*f(x))) =$
$(0*f(x))*((0*f(x))*(0*\theta(f(x))))$ *for all* $f \in X^*, x \in X$.

4 Conclusion

In this paper, we have initiated the study of hyper-homomorphisms in obic algebras. We showed that homomorphisms of obic algebras preserve oscillation. Furthermore, it was shown that if $(X;*,0)$ is an obic algebra and $(Y;\cdot,0')$ is an oscillatory obic algebra, then the triplet $(X^*;\odot,0_*)$ is an obic algebra. Future studies can be carried out in the area of application of hyper-homomorphisms of obic algebras to binary linear codes.

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