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A Note on Transmuted Exponentiated Inverse Exponential Distribution and Application to Breast Cancer Data

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Abstract

The Transmuted Exponentiated Inverse Exponential (TEIE) Distribution has been derived using Exponentiated Inverse Exponential (EIE) distribution and the Quadratic Rank Transmutation Map (QRTM). The developed distribution is more flexible and adaptable in modeling data exhibiting different shapes of the hazard function than its sub-models. The mathematical expressions and shapes of the distribution function, probability density function, hazard rate function and reliability function are studied. The parameters of the TEIE distribution are estimated by the method of maximum likelihood. Finally, the TEIE distribution is applied to breast cancer data set and found to have a better fit than the Transmuted Inverse Exponential (EIE) distribution and the Inverse Exponential (IE) distribution.

Keywords: TEIE, QRTM, Reliability Function, Hazard Rate Function, Maximum Likelihood.
MSC2010: 13P25.

1 Introduction

Transmuted distributions have been discussed dynamically in frequently occurring large scale experimental statistical data for model selection and related issues. In applied sciences such as environmental, medicine, engineering etc. Statistical modeling and analyzing experimental data are essential. Several distributions have been developed which can be used to model such kind of experimental data. The procedures used in such a statistical analysis rely heavily on the assumed probability model or distributions. That is why the development of large classes of standard probability distributions along with relevant statistical methodologies has been expanded to cope with recent development in statistical analysis. such works includes Exponentiated Lomax Weibull distribution by [1], Gamma Generalized Power Lomax distribution by [2], Alpha Power Extended Inverted Weibull distribution by [3], Transmuted Inverted Weibull distribution by [4], Exponentiated Weibull Inverse Rayleigh distribution by [5], among many others. However, there still remain many important problems where the real data does not follow any of the standard probability or classical models.

The Inverse Exponential (IE) is a very popular statistical distribution due to its extensive applicability in several areas which includes field of flood frequency analysis, space, software reliability, structural and wind engineering among many others. [6] introduced the transmuted Inverse Exponential distribution, Mashall and Olkin Inverse exponential distribution was developed by [7]. The weighted Inverse Exponential distribution was studied by [8]. [9] developed and studied the Type II Topp-Leone Exponentiated Exponential distribution. Due to its wide applicability in different fields of science, the generalization of Inverse Exponential distribution has become important.

Nowadays transmuted distributions and their mathematical properties are widely studied for applied sciences experimental data sets. [23] developed the transmuted Weibull distribution. Transmuted Lomax distribution by [10], Transmuted exponentiated Gamma distribution was studied by [24]. [12] developed the transmuted inverse Rayleigh distribution; Transmuted Pareto distribution was studied by [13]. [14] studied the transmuted additive Weibull distribution. The Transmuted complementary Weibull Geometric distribution was developed by [15], the properties of the Transmuted Inverse Exponential distribution was investigated by [16]. The Transmuted Rayleigh distribution was developed by [17]. Transmuted Generalized Inverse Weibull distribution was studied by [11] and [18] developed the Transmuted Modified Inverse Weibull Distribution and the Transmuted Log-logistic distribution was studied [19]. Transmuted Fréchet distribution was studied by [20]. Transmuted Generalized Gamma distribution was by [12], Transmuted Weibull Lomax distribution was studied by [21] and are reported with their various structural properties including explicit expressions for the moments, quantiles, entropies, mean deviations and order statistics. All the above transmuted distributions are derived by using Quadratic Rank Transmutation Map (QRTM) studied by [22]. Report reveals that some properties of these distributions along with their parameters are estimated by using maximum likelihood and Bayesian methods. Usefulness of some of these new distributions are also illustrated with experimental data sets.

The Transmuted Inverse Exponential (TIE) distribution along with several mathematical properties was studied by [6] using Quadratic Rank Transmutation Map (QRTM) and it was reported that the TIE can be used to model reliability data. Therefore, we are motivated to developed Transmuted Exponentiated Inverse Exponential distribution using the Exponentiated Inverse Exponential distribution and the Quadratic Rank Transmutation Map (QRTM) which provides a more flexible approach in modeling lifetime data. The parameters of the TEIE distribution are estimated by the method of maximum likelihood technique, and applied to the neck cancer data set to investigate the usefulness and flexibility of the model.

2 Transmuted Exponentiated Inverted Weibull distribution

A random variable X is said to have a transmuted quadratic distribution if its cumulative distribution function (cdf) is given by

$$F(x) = (1+\lambda)G(x) + \lambda G(x)^2, \qquad |\lambda| \le 1 \tag{2.1}$$

Where F(x) is the cdf of the transmuted distribution and G(x) is the cdf of the baseline distribution. Differentiating (1) w.r.t. X, we have the probability density function (pdf) of the transmuted distribution as

$$f(x) = g(x)\left[1+\lambda - 2\lambda G(x)\right], \qquad |\lambda| \le 1 \tag{2.2}$$

Where f(x) and g(x) are represent the pdf of F(x) and G(x) respectively. It is observed that at $\lambda = 0$, (2) will revert to the base distribution of the random variable X.

International Journal of Mathematical Sciences and Optimization: Theory and Applications
11(2), 2025, Pages 45 - 54
https://doi.org/10.5281/zenodo.16740353

According to oguntunde et al. (2017) the cdf, G(x), of the Exponentiated inverse Exponential distribution is given by

$$G(x;a,v) = 1 - \left[1 - e^{-bx^{-1}}\right]^{v}, \qquad a,v \geq 0 \tag{2.3}$$

And the corresponding density function to (3) is

$$g(x;a,v) = bvx^{-2}e^{-bx^{-1}}\left[1 - e^{-bx^{-1}}\right]^{v-1}, \qquad a,v \geq 0 \tag{2.4}$$

For $a,v \geq 0$ and $|\lambda| < 1$ Using (3) as the baseline distribution and inserting it into (1), we obtain the cdf of Transmuted Exponentiated Inverse Exponential (TEIE) distribution given by

$$F(x) = (1+\lambda)\left(1 - \left[1 - e^{-bx^{-1}}\right]^{v}\right) + \lambda\left(1 - \left[1 - e^{-bx^{-1}}\right]^{v}\right), \tag{2.5}$$

The corresponding pdf of TEIE is

$$f(x) = bvx^{-2}e^{-bx^{-1}}\left[1 - e^{-bx^{-1}}\right]^{v-1}\left[(1+\lambda) - 2\lambda\left(1 - [1 - e^{-bx^{-1}}]^{v}\right)\right]. \tag{2.6}$$

Where b is a positive scale parameter and v and λ are positive shape parameters. Figure 1.0 gives the graphical representation of the cdf and the pdf of the TEIE distribution for values of b,v, and λ.

Figures 1 and 2 gives the graphical representation of the cdf and pdf of the TEIE distribution for values of a,v, and λ.

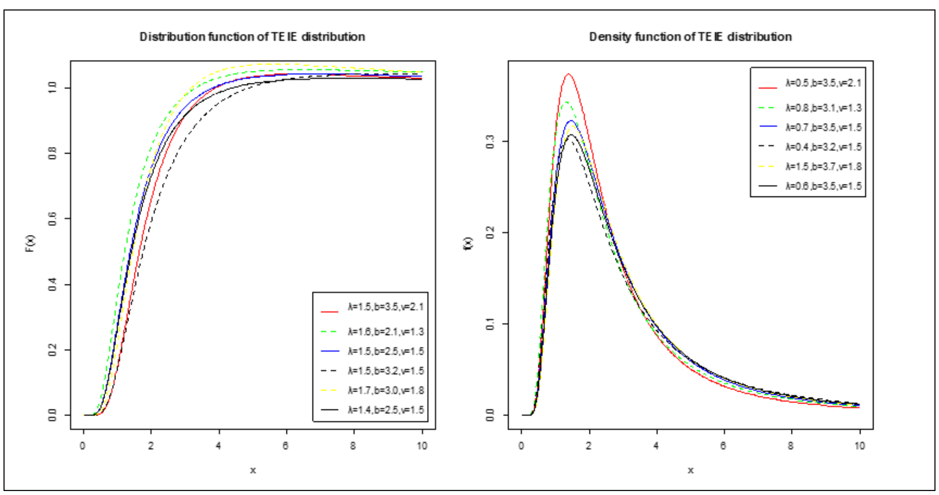

Figure 1: Graph of the distribution and density function of TEIE distribution

The hazard rate function or instantaneous failure rate, which is an important quality characterizing life phenomenon defined by $h(x) = \frac{f(x)}{S(x)}$. The hazard rate function for TEIE is given by The survival function, S(x), is defined as $S(x) = 1 - F(x)$. The survival function of the TEIE distribution is given by

$$S(x) = 1 - (1+\lambda)\left(1 - \left[1 - e^{-bx^{-1}}\right]^{v}\right) + \lambda\left(1 - \left[1 - e^{-bx^{-1}}\right]^{v}\right)^{2}, \tag{2.7}$$

and

The hazard rate function or instantaneous failure rate, which is an important quality characterizing life phenomenon defined by $h(x) = \frac{f(x)}{S(x)}$. The hazard rate function for TEIE is given by

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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$$h(x) = \frac{bvx^{-2}e^{-bx^{-1}}\left[1-e^{-bx^{-1}}\right]^{v-1}\left[(1+\lambda)-2\lambda\left(1-\left[1-e^{-bx^{-1}}\right]^{v}\right)\right]}{1-(1+\lambda)\left(1-\left[1-e^{-bx^{-1}}\right]^{v}\right)+\lambda\left(1-\left[1-e^{-bx^{-1}}\right]^{v}\right)^{2}} \tag{2.8}$$

the graphs of S(x) and h(x) of the TEIE distribution is given in figures 2.0 as

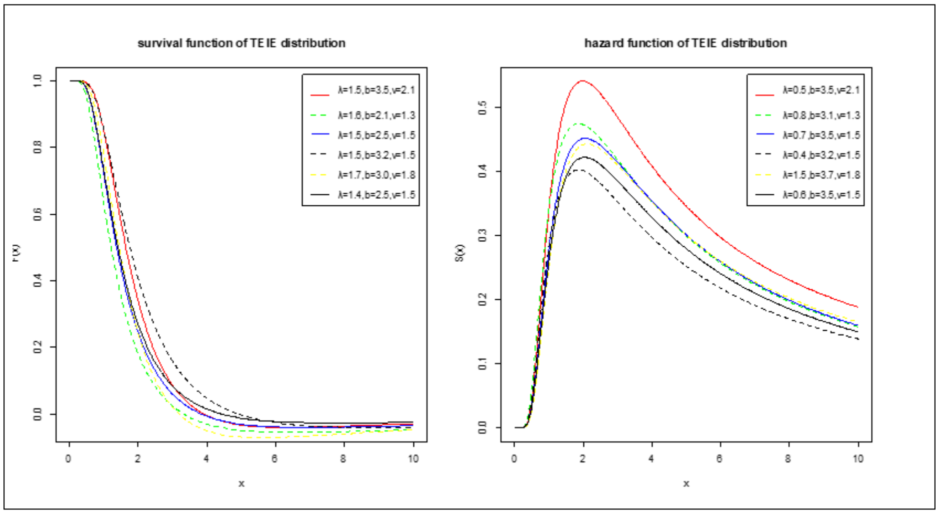

Figure 2: Graph of the survival and hazard function of the TEIE distribution

The hazard function of TEIE distribution exhibits an increasing, decreasing and inverted bathtub failure rate which demonstrate its flexibility and applicability in modeling survival data of different failure patterns.

3 Moments of TEIE distribution

The n^{th} moment of a TEIE for random variable X can be obtained as

$$E(X^n) = \int_{-\infty}^{\infty} x^n f(x)dx \tag{3.1}$$

Putting (6) in (10), we have

$$E(X^n) = bv\int_{-\infty}^{\infty} x^{n-2}e^{-bx^{-1}}\left[1-e^{-bx^{-1}}\right]^{v-1}\left[(1+\lambda)-2\lambda\left(1-\left[1-e^{-bx^{-1}}\right]^{v}\right)\right]dx \tag{3.2}$$

By algebraic manipulation, we have

$$E(X^n) = M_1 + M_2 + M_3 \tag{3.3}$$

where,

$$M_1 = bv(1+\lambda)\int_{-\infty}^{\infty} x^{n-2}e^{-bx^{-1}}\left[1-e^{-bx^{-1}}\right]^{v-1}dx \tag{3.4}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
11(2), 2025, Pages 45 - 54
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$$M_2 = -2bv\lambda \int_{-\infty}^{\infty} x^{n-2} e^{-bx^{-1}} \left[1 - e^{-bx^{-1}}\right]^{v-1} dx \tag{3.5}$$

and

$$M_3 = 2bv\lambda \int_{-\infty}^{\infty} x^{n-2} e^{-bx^{-1}} \left[1 - e^{-bx^{-1}}\right]^{v-1} \left(1 - \left[1 - e^{bx^{-1}}\right]^{v}\right) dx \tag{3.6}$$

Applying Taylor series expansion into (12), (13), and (14), finally we have

$$M_1 = v(1+\lambda) \sum_{i=0}^{\infty} (-1)^i \binom{v-1}{i} [i+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right) \tag{3.7}$$

$$M_2 = 2v\lambda \sum_{j=0}^{\infty} (-1)^j \binom{v-1}{j} [j+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right), \tag{3.8}$$

and

$$M_3 = 2v\lambda \sum_{k=0}^{\infty} (-1)^k \binom{2v-1}{k} [k+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right), \tag{3.9}$$

Combining equations (15)−(17), we obtain the n^{th} moment of TEIE as

$$\begin{aligned} E(X^n) = v(1+\lambda) \sum_{i=0}^{\infty} (-1)^i \binom{v-1}{i} [i+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right) \\ +2v\lambda \sum_{j=0}^{\infty} (-1)^j \binom{v-1}{j} [j+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right) \\ +2v\lambda \sum_{k=0}^{\infty} (-1)^k \binom{2v-1}{k} [k+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right) \end{aligned} \tag{3.10}$$

3.1 Moment generating function of TEIE distribution

The moment generating function of the TEIE distribution can be derived using the relation

$$M_x(t) = E\left(e^{tX}\right) = \sum_{n=0}^{\infty} e^{tX} f(x) dx = \sum_{k=0}^{\infty} \frac{t^n}{n!} E(X^n) \tag{3.11}$$

Then substituting (15), (16), and (17) into (18), we obtain the moment generating function of TEIE as

$$\begin{aligned} M_x(t) = v(1+\lambda) \sum_{k=0}^{\infty} (-1)^i \frac{t^n}{n!} \binom{v-1}{i} [i+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right) \\ +v\lambda \sum_{j=n=0}^{\infty} (-1)^i \frac{t^n}{n!} \binom{v-1}{j} [j+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right) \\ +2v\lambda \sum_{k=n=0}^{\infty} (-1)^k \frac{t^n}{n!} \binom{2v-1}{k} [k+1]^{\frac{n}{b}-1} \Gamma\left(1 - \frac{n}{b}\right) \end{aligned} \tag{3.12}$$

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International Journal of Mathematical Sciences and Optimization: Theory and Applications
11(2), 2025, Pages 45 - 54
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3.2 Random Number Generation and Parameter Estimation of TEIE

Here, we use one of the methods of inversion to generate random numbers for the TEIE as

$$(1+\lambda)\left(1-\left[1-e^{-bx^{-1}}\right]^{v}\right)+\lambda\left(1-\left[1-e^{-bx^{-1}}\right]^{v}\right)=u$$

Where $u \backsim U(0,1)$, this yield

$$x=\left|\frac{-b}{\left(1-\left[\frac{1+2\lambda-u}{2\lambda+1}\right]^{-\frac{1}{v}}\right)}\right| \tag{3.13}$$

Random number are generated by using (21), where the parameters a, b, and λ are known.

4 Maximum likelihood Estimates of the parameters

The maximum likelihood approach is used to estimates the unknown parameters of the distribution. Let $\underline{x}=x_1\ldots,x_n$ represent a random sample obtained from the TEIE distribution. The likelihood function $L(x;\zeta)$ and the log-likelihood function $\log L(x;\zeta)=l(x;\zeta)$ corresponding are respectively given as

$$L(x;\zeta)=bv\prod_{i=1}^{n}x^{-2}e^{bx^{-1}}\left[1-e^{bx^{-1}}\right]^{v-1}\left[(1+\lambda)-2\lambda\left(1-\left[1-e^{bx^{-1}}\right]^{v}\right)\right] \tag{4.1}$$

and

$$l=\log(b)+\log(v)-2\log\sum_{i=1}^{n}x_i+b\sum_{1=1}^{n}{x_i}^{-1}+(v-1)\sum_{i=1}^{n}\log\left[1-e^{-bx_i{}^{-1}}\right]$$
$$+\log\sum_{i=1}^{n}\left[(1+\lambda)-2\lambda\left(1-\left[1-e^{-bx_i{}^{-1}}\right]^{v}\right)\right] \tag{4.2}$$

The maximum likelihood estimates (MLEs) of the parameters b, and v, taking as ,$\hat{b}$, $\hat{v}$ and $\hat{\lambda}$ are given by $\hat{\zeta}=(\hat{b},\hat{v},\hat{\lambda})$ which makes $L(x;\hat{\lambda})$ or $l(x;\hat{\lambda})$. We differentiate (43) with respect to b,v, and λ to obtain the score vector $\left(J_b=\frac{\partial l}{\partial b},J_v=\frac{\partial l}{\partial v},J_\lambda=\frac{\partial l}{\partial\lambda}\right)^T$. The element of the score vector is given by

$$J_b=\frac{n}{b}+\sum_{i=1}^{n}{x_i}^{-1}-(v-1)\sum_{i=1}^{n}\frac{{x_i}^{-1}}{\left[1-e^{-bx_i{}^{-1}}\right]}-2\lambda v\sum_{i=1}^{n}\frac{{x_i}^{-1}e^{-bx_i{}^{-1}}}{b\left[1-e^{-bx_i{}^{-1}}\right]}\left(1-\left[1-e^{-bx_i{}^{-1}}\right]^{v}\right), \tag{4.3}$$

$$J_v=\frac{n}{v}+\sum_{i=1}^{n}\left[1-e^{-bx_i{}^{-1}}\right]+2\sum_{i=1}^{n}\frac{\lambda\left(1-\left[1-e^{-bx_i{}^{-1}}\right]^{v}\right)\log\left[1-e^{-bx_i{}^{-1}}\right]}{\left[(1+\lambda)+2\lambda\left(1-\left[1-e^{-bx_i{}^{-1}}\right]^{v}\right)\right]}, \tag{4.4}$$

$$J_\lambda=\frac{n}{\lambda}+\sum_{i=1}^{n}\frac{1-\lambda\left(1-\left[1-e^{-bx_i{}^{-1}}\right]^{v}\right)}{\left[(1+\lambda)+2\lambda\left(1-\left[1-e^{-bx_i{}^{-1}}\right]^{v}\right)\right]}. \tag{4.5}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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4.1 APPLICATION

In this section, we compare the fit of the TEIE model and some other competing models using breast cancer data sets. We measure how well the TEIE distribution performs compared to the Transmuted Inverse Exponential (TIE) and the Inverse Exponential distribution (IE). The data represent 121 breast cancer patients' survival times during a specific period from 1929 to 1938. The data has been studied by [28]. The observations are listed as follows: 0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4,10.3, 11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7,16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1,30.0, 31.0, 31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0,38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0, 41.0, 42.0, 43.0,43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0, 47.0, 48.0, 49.0, 51.0,51.0, 51.0, 52.0, 54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0,111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0,154.0. For each model, we obtained the estimate of the parameters by using the maximum likelihood method and assessed the goodness-of-fit by using the following information criteria: Akaike information criterion (AIC), Consistent Akaike information criterion (CAIC), Cramer-von Mises (CM) statistic. In general, the smaller the value of the information criteria, the better the model fit to the data. Table 1 represent the exploratory data analysis of the breast cancer data which shows that the data is positively skewed, over-dispersed, and leptokurtic. Figure 3.0 is the Box and violin plot for the cancer data which also shows that the data is positively skewed. The Total Test on Time (TTT) plot is given in Figure 4.0 which indicates that the cancer data exhibits non-monotone (bathtub) failure rate.

Table 1.0 Exploratory data analysis of the breast cancer data

Data	n	Range	Lower quartile	Median	Upper quartile	mean	Var.	Skew.	Kurt.
Data	121	153.70	17.50	40.0	60.0	46.33	1244.46	1.04	3.40

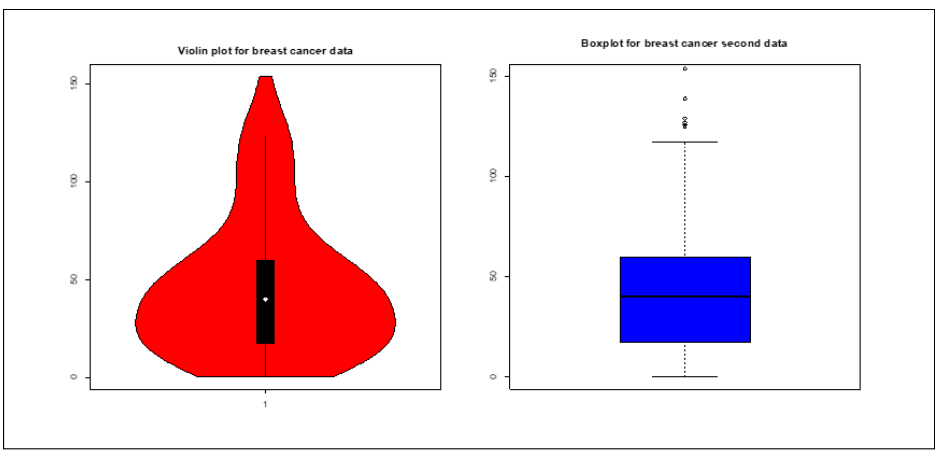

Figure 3: Graph of the Violin plot and Box plot for breast cancer data

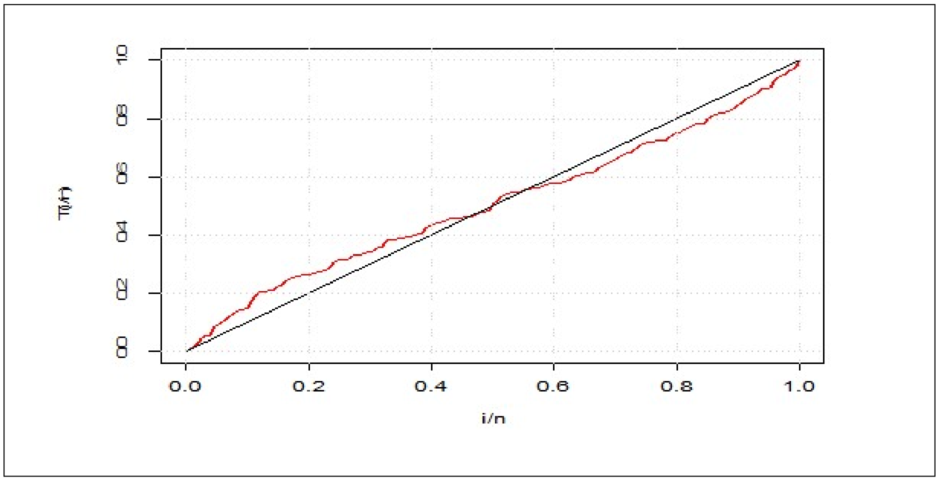

Figure 4: TTT plot for breast cancer data

Table 2. MLEs and standard error (in braces) and measures of goodness of fit for the breast cancer data.

	Parameters Estimation			*Measures of goodness of fit*				
Model	b	v	λ	l	*AIC*	*CAIC*	*HQIC*	*CVM*
TEIE	5.105 (0.046)	0.724 (0.072)	−0.933 (0.821)	640.376	1286.751	1286.956	1290.157	1.4034
TIE	−0.939 (0.142)	− (−)	7.358 (0.345)	645.798	1295.596	1295.698	1297.867	1.4128
IE	10.321 (0.938)	− (−)	− (−)	677.279	1356.558	1356.592	1357.694	1.9313

It will be observed from the values of AIC, CAIC, HQIC, and CVM for the TEIE, TIE and the IE in Table 2 that TEIE gives lower values than that of TIE and the IE distribution. Hence, the TEIE distribution gives a better fit than the other two distributions based on the data considered in this study.

5 Conclusion

The Transmuted Exponentiated Inverse Exponential distribution has been generated and their parameters are estimated. The developed distribution is applied to breast cancer data and it is compared with Transmuted Inverse Exponential and the Inverse Exponential distribution and leads to better fit than the other two distributions. Hence the new Transmuted Exponentiated Inverse Exponential distribution can be applied in the field of medicine.

References

[1] Osagie, S. A., and Osemwenkhae, J. E."Exponentiated Lomax Weibull distribution with properties and Applications". International Journal of Mathematical Sciences and Optimization: Theory and Applications 6.2(2020), pp.814-833.

International Journal of Mathematical Sciences and Optimization: Theory and Applications
11(2), 2025, Pages 45 - 54
https://doi.org/10.5281/zenodo.16740353

[2] Ogunde, A. A.,Oseghale, O. I., Phillips, S. A., and Omosigho, D. O. " Gamma Generalized Power Lomax: A Novel Distribution in Modelling Covid-19 Data". International Journal of Mathematical Sciences and Optimization: Theory and Applications 9.1(2023), pp. 138-143.

[3] Ogunde, A. A, Fayose, S. T., Ajayi, B., and Omosigho, D. O. "Properties, Inference, and Applications of Alpha Power Extended Inverted Weibull Distribution". International Journal of Statistics and Probability 9.6 (2020).

[4] Ogunde, A. A., Fatoki, O. and Oseghale, O. I. (2017a). "On the Application of Transmuted Inverted Weibull distribution". Global Journal of Science Fronttier Research 17. 6(2017).

[5] Arowolo, O. T., Ogunsanya, A. S., Ekum, M. I., Oguntola T. O., and Ukam, J. B. "Exponentiated Weibull Inverse Rayleigh distribution". International Journal of Mathematical Sciences and Optimization: Theory and Applications 9.1(2023), pp. 104-122.

[6] Oguntunde, P. E., Adebowale O. A., and Enahoro A. O. "On the Flexibility of the Transmuted Inverse Exponential Distribution". Proceedings of the World Congress on Engineering 2017 Vol I WCE 2017, July 5-7, 2017, London, U.K.

[7] Ogunde, A. A., Omosigho, D. O., and Ajayi, B. :Structural Properties of Marshall-Olkin Extended Inverse Exponential Distribution". Mathematical Theory and Modeling 10.2(2020).

[8] Adetunji, K. I. and Jolayemi, E. T. "The weighted Exponentiated inverted exponential distribution". International Journal of Statistics and Applied Mathematics 6.1(2021), pp.45-50.

[9] Laoye, V. E. "Type II Topp-Leone Exponentiated Exponential Distribution: Properties and Application to Failure data". International Journal of Mathematical Research 16.6(2025), pp.39-47.

[10] Ashour, S. K. and M. A. Eltehiwy, M. A. "Transmuted Lomax distribution". American Journal of Applied Mathematics and Statistics 1.6(2013), pp.121-127.

[11] Khan, M.S. and King, R. "Transmuted Generalized Inverse Weibull Distribution". Journal of Applied Statistical Science 20(2013), pp.15-31.

[12] Ahmad, A., Ahmad, S. P., and Ahmed, A. "Transmuted inverse Rayleigh distribution: A generalization of then inverse Rayleigh distribution". Mathematical Theory and Modeling 4.7(2014), pp.90-98.

[13] Merovci, F., and Puka, I. "Transmuted Pareto distribution". Probability Statistical Forum 07, (2007), pp. 1-11.

[14] Elbatal, I. "Transmuted Modified Inverse Weibull Distribution: A Generalization Of The Modified Inverse Weibull Probability Distribution". International Journal of Mathematical Archeive 4(2013),pp.117-129.

[15] Afify, A. Z., Nofal, Z. M. and Butt, N. S. "Transmuted complementary Weibull geometric distribution". Pakistan Journal of Statistics and Operation Research 10.4(2014), pp.435-454.

[16] Oguntunde, P. E. and Adejumo, A. O. "The transmuted inverse exponential distribution". International Journal Advance Statistics and Probability 3.1(2015), pp.1-7.

[17] Merovci, F. "Transmuted Rayleigh Distribution". Austrian Journal of Statistics, 42(2013), pp.21-32.

[18] Elbatal, I. and Aryal, G. On the transmuted additive Weibull distribution. Australian Journal of Statistics 42.2(2013), pp.117-132.

[19] Aryal, G.R. "Transmuted Log-Logistic Distribution". Journal of Statistics Application and Probability 2(2013), pp.11-20.

[20] Mahmoud, M.R. and Mandouh, R.M. (2013). Transmuted Frechet Distribution. Journal of Applied Science and Research, 9, 5553-5561.

[21] Afify, A.Z., Nofal, Z.M. and Yousof, H.M. "The Transmuted Weibull Lomax Distribution: Properties and Application". Pakistan Journal of Statistics and Operation research 11.2(2015), pp.135-152.

[22] Shaw, W.T., and Buckley, I.R.C. (2007). The Alchemy of Probability Distributions: Beyond Gram-Charlier & Cornish-Fisher Expansions, and Skew-Normal or Kurtotic-Normal Distributions. 2007, Research report.

[23] Aryal, G. R., and Tsokos, C. P. "Transmuted Weibull distribution: A generalization of the Weibull probability distribution". European Journal of Pure Applied Mathematics 4(2011), pp.89-102.

[24] Hussian, M. A. "Transmuted exponentiated gamma distribution: A generalization of the exponentiated gamma probability distribution". Applied Journal of Mathematical Science, 8.27(2014), pp.1297-1310.

[25] Lucena, S.E.F., Silva, A.H.A. and Cordeiro, G.M. "The Transmuted Generalized Gamma Distribution: Properties and Application". Journal of Data Science 13(2015), pp.187-206.

[26] Ogunde, A. A, Omosigho, D. O., and Fatoki, O. (2017). "On Statistical Properties of the Exponentiated Transmuted Inverted Weibull Distribution". Journal of Statistics: Advances in Theory and Applications 18.2(2017), pp.143-164.

[27] Ogunde, A. A., Ibraheem, A. G., Audu, A. T. "Performance rating of transmuted Nadarajah and Haghighi exponential distribution: an analytical approach". Journal of Statistics: Advances in Theory and Applications 17.2(2017), pp.137-151.

[28] Tahir, M.H., Cordeiro, G.M., Mansoor, M., Zubair, M. "The Weibull-Lomax distribution: properties and applications". Hacetepe Journal of Mathematics and Statistics 44(2015), pp.461–480.