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# Mathematical Model for Ions Transport Optimization in an Animal Cell Using MATLAB

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**Abstract**

The increasing world population has raised significant concerns about matching food production to demand, prompting extensive research in scientific fields aimed at enhancing plant and animal productivity. However, some of these advances have introduced unintended consequences, such as uncontrolled cell growth that leads to cancer. Effective regulation of cell size is essential to maintaining organismal health. This study focuses on the utilization of a numerical method to develop a mathematical model that optimizes ion transport within an animal cell as a mechanism to ensure a physiologically healthy cell. Through this model, optimal ion concentrations were identified using MATLAB-SIMULINK. The results were validated using experimental data to ensure that they promote healthy cell growth and stability.The results provide valuable insights for treating disorders associated with abnormal cell growth initiated by unregulated ions concentration. This research contributes to understanding cellular homeostasis and lays the groundwork for future bioengineering applications.

**Keywords:** Optimization, Homeostasis, Partial Differential Equations, Mathematical Modeling, Simulation, Membrane Potential, Optimal Ion Concentration, Ion Channel, Finite Difference Method.
**MSC2010: 00A71.**

# 1 Introduction

## 1.1 Background of the Study: Ion Optimization in Animal Cells

All living things require cellular homeostasis to be maintained, and this is achieved by the basic mechanism of ion control within cells. Ions like sodium $Na^+$, potassium ($K^+$), and calcium ($Ca^{2+}$) are essential for a number of biological functions, including sustaining the membrane potential at rest, and assisting in signal transduction [Alberts B.(2002)], [Matejczyk(2019)]. For life to continue, cells must be able to precisely regulate the amount of these ions across the membrane.

The field of ion dynamics in cells has been studied since the early 1900s, when researchers like Alan Hodgkin and Julius Bernstein made important discoveries that helped to establish the foundation for our knowledge of the electrical characteristics of cells [Hodgkin A. L.(1952)], [Hodgkin





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A. L.(1949)]. A significant turning point in the study of cell membrane concentration gradient maintenance was the 1957 discovery of the $Na^+/K^+ - ATPase$ pump by Jens Christian Skou [C.(1957)].This discovery found that this pump keeps the intracellular content of $K^+$ high and $Na^+$ low in animal cells. The groundbreaking work of Hodgkin and Huxley's in the 1950s gave a quantitative explanation of ion mobility during action potentials [Hodgkin A. L.(1952)]. In order to control ion concentrations, cells employ a number of strategies, such as secondary active transport linked to the movement of other ions, active transport via ATP-powered pumps, and passive diffusion through ion channels. When certain stimuli are met, like voltage changes (for voltage-gated channels) or ligands binding (for ligand-gated channels), ion channels open and close with great selectivity [B.(2001)].

The selective permeability of the membrane distinctions sustains the membrane potential, which is the voltage differential across the cell membrane. For animal cells to operate correctly, membrane potential must be maintained within the ranges of $-60mV to -70mV$ [Alberts B.(2002)], [B.(2001)]. Comprehending ion optimization is crucial for both fundamental biology and its applications in bioengineering. Cell homeostasis disturbances are linked to a variety of cell abnormalities, including cancer [Alberts B.(2002)], [Blaustein M. P.(2016)], [A.C. and S(2011)]. The therapeutic potential of ion channel modulation has been the subject of recent research, wherein development of treatments for conditions related to ion imbalances is ongoing, offering a strategic approach to restoring or maintaining healthy ion levels in cells. The understanding of ions transport in cells has been greatly improved by developments in computer modelling and technology. In order to replicate the movement of ions across the cell membrane, mathematical models have recently been constructed [Matejczyk(2019)], [Cuevas E.(2024)], [J. and X.(2020)]. These models have developed into vital resources for investigating the intricate relationships between diverse ion channels transport and forecasting cellular responses to varied stimuli. Improvement of these models to accurately simulate various conditions in bioengineering, enhancing relevance and applicability in designing artificial cells and optimizing cell-based biotechnological processes is a growing area of interest for current studies. Technological and computational modelling developments keep expanding our knowledge and providing fresh perspectives on the intricate interactions among ions, cellular functions, and health. Building on this rich history, the goal of this work is to get a deeper knowledge of ion optimization and its implications for applied biotechnology.

## 1.2 Literature Review

In animal cells, ion regulation is an essential process that affects many cellular processes as signal transmission, osmotic equilibrium, and membrane potential. Maintaining homeostasis and facilitating cellular functions including ion transport, apoptosis, and the maintenance of a normal cell size depend on the optimization of ion concentrations within the cell [Keener J.(2009)], [Wilfred(2012)]. This overview of the literature delves into the workings of ion channels, how ions are regulated, and the latest developments in ion dynamics modelling in animal cells. Animal cells maintain specific concentrations of ions such as sodium ($Na^+$), potassium ($K^+$), calcium ($Ca^{2+}$), across their membranes. Cells are able to actively manage these ion concentrations due to the selective permeability of the cell membrane, which is regulated by ion channels [B.(2001)]. According to Blaustein et al. (2016) [Blaustein M. P.(2016)], the $Na^+/K^+ - ATPase$ pump is an important component that keeps intracellular $K^+$ and $Na^+$ concentrations high and low, respectively. This allows the cell to perform a variety of tasks, such as preserving the resting membrane potential and promoting secondary active transport. Proteins in the cell membrane called ion channels help ions pass across cell membranes more easily. Two important mechanisms for preserving ion homeostasis are voltage-gated and ligand-gated ion channels. According to Hille and Kandel et al. , cellular signaling pathways closely control the operation of ion channels, guaranteeing that ion concentrations stay optimal under a range of physiological circumstances [B.(2001)], [Kandel E. R.(2012)].Voltage-gated channels are extremely selective for a particular ion, such as $Na^+$,$K^+$ and $Ca^{2+}$ and react to changes in cell membrane potential [Wilfred(2012)]. Based on the predominant ion, these channels can further be separated into families [Alberts B.(2002)], [B.(2001)], [Jeremy

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M B. and S.(2002)]. Voltage-gated $Na^+$ channel are the source of long-lasting action potentials, making them the choice for local anaesthetics. Voltage-gated $Ca^{2+}$ channels control intracellular calcium ions concentrations and, as a result, are in charge of numerous cellular metabolic reactions. Voltage-gated $K^+$ channels make up the biggest and most varied group of voltage-gated ion channels. Their role in producing the resting membrane potential is crucial. Ligand-Gated Ion Channels (LGIC) are trans-membrane proteins that are opened by the binding of a neurotransmitter (carrier protein) [Wilfred(2012)]. The ligands bind to a site which causes structural modifications, that change the permeability of the ion channel, and thus allow only specific ions to pass through. These proteins bind a substance which triggers a change in its own shape, moving the bound molecule from the outside of the cell to its interior or vice versa depending on the gradient. Only one kind of ion can pass through the cell channel in voltage-gated ion channels. An electrical potential difference, known as the membrane potential, is created between the cell's internal and external environments as a result of the ion flux. Many physiological activities, including signal transduction, muscle contraction, growth, motility, hormone secretion, volume regulation, and apoptosis, depend critically on this membrane potential. Animal cells have a resting membrane potential of $-60mVto-70mV$ [Alberts B.(2002)], [B.(2001)], which is mainly sustained by the unequal distribution of $Na^+$ and $K^+$ across the membrane. The equilibrium potential of $K^+$, also known as the Nernst potential, and the activity of the $Na^+/K^+-ATPase$ both affect this potential, which is essential for the operation of excitable cells [Alberts B.(2002)]. By taking into account the permeability and concentration of different ions, the Goldman-Hodgkin-Katz (GHK) equation offers a quantitative description of the membrane potential [E.(1943)], [Hodgkin A. L.(1949)], [Murase and Kitano(2011)]. The control of cell volume is closely related to ion concentrations, especially those of $Na^+$, $K^+$ and $Ca^{2+}$. The concentration of these ions determines the osmotic balance, which in turn affects the size and shape of cells by influencing the passage of water over the membrane. Cells can resist osmotic stress by regulating ion channels through the regulatory volume increase (RVI) and regulatory volume decrease (RVD) pathways [Hoffmann E. K.(1995)], [F.(2007)]. Studies have shown that ion channels functionally participate in cancer progression, which implies that the in-depth understanding of ion concentration optimisation is potentially useful for tumour detection and treatment. [Gerisch A.(2014)], [V'yacheslav L. and P.(2011)], [Yang N. J.(2015)] New developments in mathematical modelling have shed further light on how ions move through animal cells. Frameworks for comprehending the dynamics of ion flow and its influence on cellular activities are provided by models like the Hodgkin-Huxley model for action potentials and the more recent ion flux models [Hodgkin A. L.(1952)], [Keener J.(2009)]. The way in which cells maintain homeostasis under different conditions can be predicted by computational models that mimic the interaction between ion channels, transporters, and cellular metabolism [Coalson and Kournikova(2005)], [Smith N. P.(2017)], [Yang N. J.(2015)]. Studying pathological situations such as cancer, where ion imbalance can cause uncontrolled cell development, makes these models very helpful [Hoffmann E. K.(1995)], [A.C. and S(2011)]. It takes a multifaceted approach incorporating several ion channels to optimize ion concentrations in animal cells. It is essential to comprehend these processes through computational methods in order to clarify the mechanisms that underlie cellular homeostasis. New insights into the regulation of ion homeostasis in health and disease are predicted to result from future research focusing on merging models of ion dynamics based on computational mathematics with experimental evidence. With a focus on important mechanisms, models, and directions for future investigation, this literature review offers a thorough summary of the state of knowledge regarding ion optimization in animal cells.

## 1.3 Objective

The objective of this study is to develop a mathematical model that optimizes ion transport within an animal cell as a mechanism to ensure a physiologically healthy cell.

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### 1.4 Significance of the Study

Ion optimization in animal cells is an important field of study that has implications for fundamental science as well as real-world applications in environmental science, biotechnology, and medicine. First, it deepens our understanding of cellular homeostasis by exploring how ion concentrations within cells are regulated, which is essential for critical cellular functions such as nutrient transport, energy production, and signal transduction. Disruptions in ion homeostasis are linked to diseases such as cancer and neurodegenerative disorders, making this research a potential foundation for new therapeutic approaches. Second, the study advances medical research by identifying potential drug targets in ion channels and transporters, which could aid in developing new or personalized treatments for conditions caused by ion dysregulation. Furthermore, the study contributes to biotechnology by applying principles of ion optimization to synthetic biology, potentially improving the efficiency of engineered cells for use in pharmaceuticals or biofuels and enabling cells to adapt to extreme environments. In addition, the development of computational models in this study enhances our ability to simulate ion dynamics, promoting interdisciplinary research and reducing the need for extensive in vivo testing. Finally, this research offers educational value by providing insights into cellular physiology and ion regulation, laying a foundation for future research in areas like physiology, biophysics, and medical sciences. Overall, this study addresses critical challenges in health, biotechnology, and sustainability, with implications for food security, disease treatment, and environmental adaptation.

## 2 Materials and Methods

### 2.1 Numerical Solution-Finite difference method(FDM)

This numerical approach is appropriate for partial differential equations (PDEs) that take into account spatial fluctuations in ion concentration [Allman E. S.(2013)], [Morton and Mayer(2005)], [Smith J.(2020)]. Time and space must be discretized. This study encounters inhomogeneous PDEs governing the flow of ions across cell membrane with spatial fluctuations in ion concentrations on the inside and outside of the cell. The particular method for this problem is the central finite difference method, with a uniform time step ($n$) and spatial step ($m$). The following partial derivatives are expressed as follows

$$\frac{\partial^2 u}{\partial y^2} = u_{yy} = \frac{u_{i,j+1} - u_{i,j} + u_{i,j-1}}{m^2} \tag{1}$$

$$\frac{\partial u}{\partial y} = u_y = \frac{u_{i,j+1} - u_{i,j}}{n} \tag{2}$$

Let the mesh point at time $t$ be denoted by $n$ . Then the forward difference for the first order derivative with respect to time $t$ will be

$$\frac{\partial u}{\partial t} = u_t = \frac{u_{i+1,j} - u_{i,j}}{n} \tag{3}$$

### 2.2 Mathematical Model for Ion Optimization in Animal Cells

The mathematical model for ion optimization in animal cells involves differential equations that describe the movement of ions across the cell membrane and their interactions with various cellular processes [Allman E. S.(2013)], [Matejczyk(2019)]. Goldman-Hodgkin-Katz equation (GHK)

$$V_m = \frac{RT}{F} ln \frac{P_{Na}[Na^+]_o + P_K[K^+]_o + P_{Ca}[Ca^2+]_o}{P_{Na}[Na^+]_i + P_K[K^+]_i + P_{Ca}[Ca^2+]_i} \tag{4}$$

This formula calculates the resting membrane potential by putting into consideration the permeabilities and concentrations of various ions [Keener J.(2009)], [Murase and Kitano(2011)]. Where

$V_m$ is membrane potential ,$P$ is permeability of the membrane to the ion, and subscripts $o$ and $i$ denote the extra cellular and intracellular concentrations of the ion respectively. Poisson equation This formula determines the electric potential $\psi(x,t)$ that helps us to calculate the ions flux using the Nernst-Planck equation. The electrochemical potential $\psi$ for a given charge distribution can be solved using the Poisson equation, which is stated as follows [Matejczyk(2019)]:

$$\epsilon\Delta^2\psi = \sigma_i z_i e C_i \tag{5}$$

where $i$ is the $i^{th}$ species, $z$ is the ion valence, $e$ is the elementary charge, $C$ is the ion concentration, and $\epsilon$ is the electrolyte permittivity.To discretize this equation the second spatial derivative for the electric potential is expressed using central difference formula as follows

$$-\epsilon\frac{\psi^n(j+1) - 2\psi^n(j) + \psi^n(j-1)}{\Delta^2} = \sum_i z_i e C^n_( j) \tag{6}$$

Nernst-Planck equation The Nernst-Planck equation for the ion concentration $C_i(x,t)$ of ion $i$ species in differential form is given by [Murase and Kitano(2011)], [Coalson and Kournikova(2005)];

$$\frac{\partial C_i}{\partial t} = D_i\nabla^2 C_i - \frac{z_i e}{k_B T}\nabla\cdot(C_i\nabla\phi) \tag{7}$$

Where $D_i$ is the diffusion coefficient of ion $i$, $z_i$ is the valence of ion $i$ ,$e$ is the elementary charge, $k_B$is the Boltzmann constant, $\phi$ is the temperature and is the electric potential. Discretizing this equation using a time step $\Delta t$ and a one-dimensional grid with spacing $\Delta x$ , the spatial derivatives are expressed by use of the central difference formula while temporal derivatives uses the forward difference formula [Morton and Mayer(2005)], [Murase and Kitano(2011)], [Wang C.X. and Z.(2022)] .

$$\begin{aligned}\frac{C_i^{n+1}(j) - C_i^n(j)}{\Delta t} &= \frac{D_i}{\Delta x^2}\left(C_i^n(j+1) - 2C_i^n(j) + C_i^n(j-1)\right) \\ &- \frac{D_i z_i e}{k_B T\Delta x}\cdot\frac{1}{2}\left(C_i^n(j+1)\Psi^n(j+1) - C_i^n(j-1)\psi^n(j-1)\right)t\end{aligned} \tag{8}$$

Where $C_i^n(j)$ is the concentration of ion $i$ at grid point $j$ and time step $n$ and $\psi^n(j)$ is the electric potential at grid point $j$ and time step $n$.

### 2.3 Numerical Simulation and Optimization

Here we formulate the algorithm that finds the minimum of constrained nonlinear multivariable function using MATLAB fmincon [Cohen(2023)], [Paluszek and Thomas(2011)]. This is a nonlinear programming algorithm that finds the minimum of a problem specified by

$$min f(x) \quad \text{such that} \begin{cases} c(x) \leq 0, \\ ceq(x) = 0, \\ A\cdot x \leq b, \\ Aeq\cdot x = beq, \\ lb \leq x \leq ub. \end{cases} \tag{9}$$

Where, $b$ and $beq$ are vectors, $A$ and $Aeq$ are matrices,$c(x)$ and $ceq(x)$ are functions that return vectors, and $f(x)$ is a function that returns a scalar. $f(x)$,$c(x)$, and $ceq(x)$ can be nonlinear functions. $x$,$lb$, and $ub$ can be passed as vectors or matrices. We then construct this problem in MATLAB as follows;

$$x = fmincon(problem) or x = fmincon(fun, x0, A, b, Aeq, beq, lb, ub, nonlcon, options) \tag{10}$$

This algorithm minimizes the objective function $fun$ subject to the following constraints; Homeostasis constraint: To ensure stable internal conditions in an animal cell, the appropriate ions concentration ranges are maintained by employing the following constraint;

$$(ion)_{min} \leq (ion)_{actual} \leq (ion)_{max} \tag{11}$$

Membrane potential constraint:To ensure that the voltage difference across a cell membrane is maintained within physiologically healthy ranges the conditions prescribed by the GHK equation are observed.

$$\begin{aligned} V_m = (RT/F)log(P_K * K_{out} + P_N a * Na_{out} + P_C a * Ca_{out})... \\ /(P_K * K_{in} + P_N a * Na_{in} + P_C a * Ca_{in}) \\ ceq = V_m + 70 \end{aligned} \tag{12}$$

Formulation of the objective function is done by considering the minimum sum of the squared differences between actual and observed ion concentrations, subject to homeostasis and membrane potential constraints. The objective is to minimize the deviation of actual ion concentrations from their observed values. The square is to make the differences positive. We thus write:

$$fun = minimize \sum_{ion} ([ion]_{actual} - [ion]_{observed})^2 \tag{13}$$

The summation is taken over all ions under consideration, with $[ion]_{actual}$ being the actual concentration of a given ion and $[ion]_{observed}$ the observed concentration of that ion. In this problem we start with an initial guess or set of actual concentrations ($x0$). The optimization algorithm then adjusts $x$ iteratively to minimize the objective function [Danjuma T. and T.(2020)] [Bello J. F. and Adinya(2024)], thereby making $x$ converge toward the optimal concentrations.In this problem we determine observed values based on theoretical models and previous experiments that are known to be beneficial for cell function.

## 2.4 Solve the Optimization Problem

To solve the optimization problem a MATLAB code is developed, computation is done by adjusting the ions concentrations in the direction that reduces the objective function while convergence is achieved by looping the code for sufficient iterations.

# 3 Results

## 3.1 Findings and Discussion

The optimal levels of ions concentrations in a cell are presented.

### 3.1.1 Optimal Levels of Concentration of Ions in a cell.

The bar plot below shows the optimal ion concentrations of a cell while the line plot shows the ion concentrations in a healthy cell over time.

Figure 1 displays a bar graph;

Figure 2 displays a line graph

The two graphs displays the optimal ions concentration levels both on the outside and on the inside of a cell. These ion levels ensure that a cell remains physiologically healthy. The line plot shows the optimal ion levels of a cell over time. It shows that as long as physiological constraints are observed the ion concentrations remain optimally constant. The figures for the optimal ion concentration levels are as shown in the table below.

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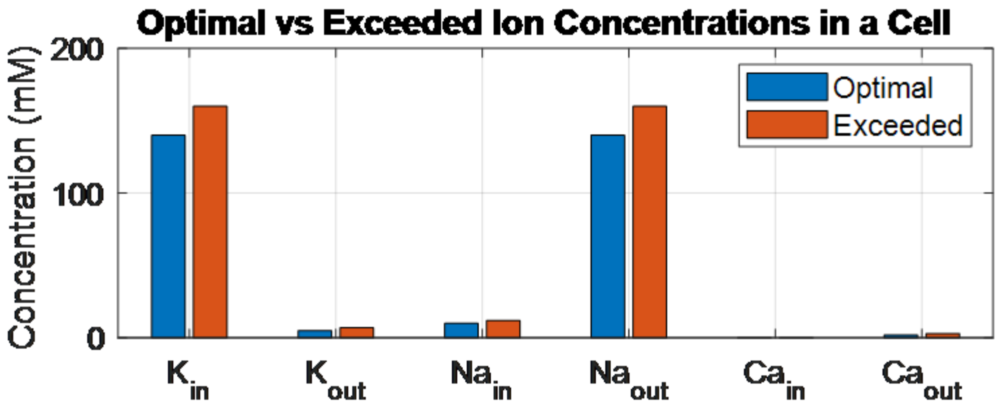


Figure 1: optimal and exceeded ion concentrations 1

| cyan!20yellow!20 Parameters | Intracellular | Extracellular |
|---|---|---|
| cyan!20Potassium($K^+$) | K_in:140.1236mM | K_OUT:3.5mM |
| cyan!20Sodium($Na^+$) | Na_in:10.0054mM | Na_out:139.8965mM |
| cyan!20Calcium($Ca^2$+) | Ca_in:1e-04mM | Ca_out:2mM |

Table 1: optimal ion concentration values

#### 3.1.2 Effect of each ion

Calcium: Elevated intracellular calcium (levels above $1e-04mM$) can lead to uncontrolled cell growth and reduced apoptosis, contributing to cancer progression [Atkins and de Paula(2014)] Potassium: Abnormal potassium levels (levels above $140.1236mM$)can disrupt cell cycle regulation and promote survival in environments where cells would normally undergo apoptosis [J.O.(1995)]. Sodium: Elevated sodium levels (levels above $10.0054mM$) can lead to increased cellular volume and changes in cell shape, contributing to a pro-cancerous environment [House CD(2010)]

### 3.2 Comparison with Other Studies

The results obtained from the mathematical model on optimal ion concentrations are compared with experimental data for validation. The experimental data available from the relevant literature expresses the ion concentrations as a range of values while the data from our model computes an optimal value. According to Alberts et al, 2014 [Alberts B.(2002)] experimental results for sodium, potassium, and calcium ion concentrations in an animal cell which are also corroborated by Gadsby and Nakao, 2003) [Gadsby D. C.(2003)] are stated as follows:

Sodium ($Na^+$)
Intracellular Concentration: Approximately $5-15mM$
Extracellular Concentration:Approximately $135-145mM$

Potassium ($K^+$)
Intracellular Concentration:Approximately $120-150mM$
Extracellular Concentration: Approximately $3.5-5mM$

Calcium ($Ca^{2+}$)
Intracellular Concentration:Approximately $100nM$ (resting)

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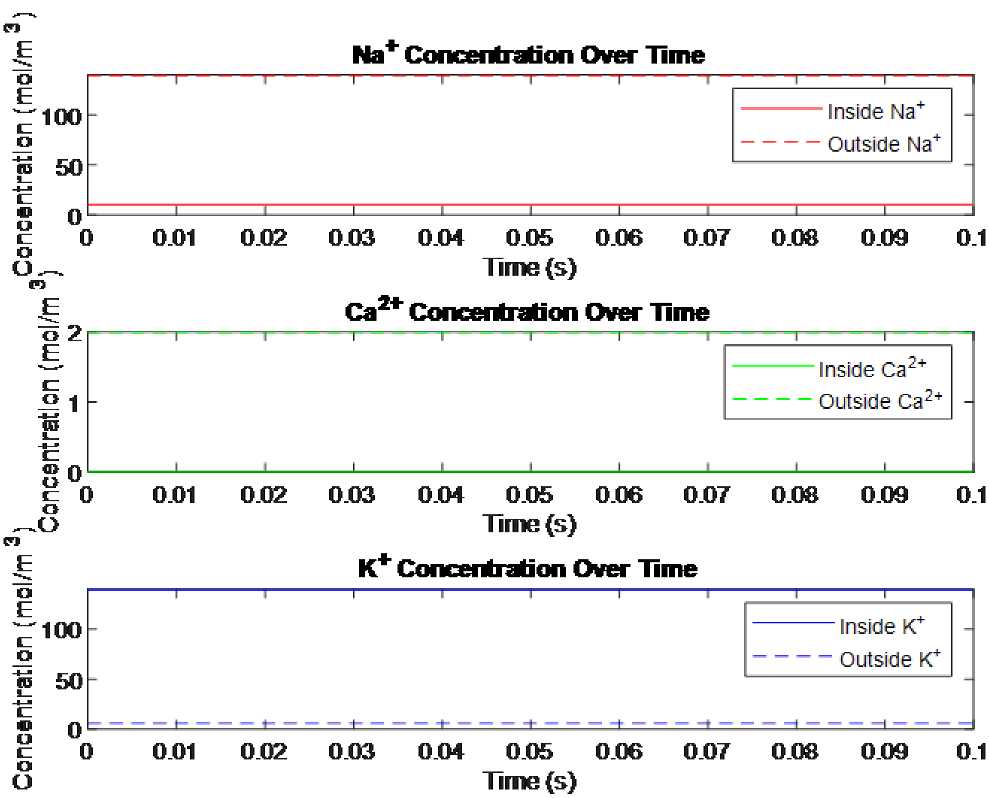


Figure 2: optimal and exceeded ion concentrations

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Extracellular Concentration:Approximately $1 - 2mM$
According to our model, the optimal ions concentration levels are listed as follows:

$K_{in}$ : $140.1236mM$
$K_{out}$ : $3.5mM$
$Na_{in}$ : $10.0054mM$
$Na_{out}$ : $139.8965mM$
$Ca_{in}$ : $1e - 04mM$
$Ca_{out}$ : $2mM$

| gray!30 Optimal values | Experimental bounds | | | |
|---|---|---|---|---|
| yellow!20 Parameter | X | LB | UB | cyan!20LB$\leq X \leq UB$ |
| K_in | 140.1236 | 120 | 150 | cyan!20Yes |
| K_out | 3.5 | 3.5 | 5 | cyan!20Yes |
| Na_in | 10.0054 | 5 | 15 | cyan!20Yes |
| Na_out | 139.8965 | 135 | 145 | cyan!20Yes |
| Ca_in | 0.0001 | 0.0001 | 0.0001 | cyan!20Yes |
| Ca_out | 2 | 1 | 2 | cyan!20Yes |

Table 2: Analysis of the results

The analysis in the table 2 shows that the model results $(X)$ are within the experimental bounds, this gives our results validity and justification.These values are also supported by the literature benchmark especially the work of Hille, 2001 and Cohen, 2023 [B.(2001)] [Cohen(2023)].

## 3.3 Limitations

It is important to recognize the limitations of this study. First, the results are purely theoretical, derived from simulations and models rather than an experimental investigation. Furthermore, the study's consideration of animal-specific cells may have limited the conclusions' application to other biological systems. Lastly, other ions that might also be important for cellular functions were not included in the analysis; only sodium, calcium, and potassium ions were considered. These limitations imply that additional experimental verification and wider ion inclusion are required for more thorough understanding.

## 3.4 Conclusion

In this research construction of a mathematical model that investigates the optimal levels of ions concentration in relation to the electrochemical gradient in a cell using a numerical computational technique was undertaken. Ions transport governing equations were used to create a suitable model and finite difference numerical method used to solve the problem. Matlab software was used for simulation and visualization of the results. The optimization of the ion concentrations of $K^{+}$,$Na^{+}$ , and $Ca^{2+}$ using the mathematically developed model determined the optimal levels of these ions in an animal cell. By determining these optimal ion levels, the model effectively controls desired cell growth, reducing the need for costly and time-consuming trial-and-error methods. This approach to ion concentration optimization is crucial for understanding how cells regulate their internal environment and maintain stability under varying conditions. The model's applications extend to the medical field, where it can be used to develop treatments for conditions related to ion imbalances, offering a strategic approach to restoring or maintaining healthy ion levels in cells. Furthermore, this model can be improved to accurately simulate various conditions in bioengineering, enhancing its relevance and applicability in optimizing cell-based biotechnological processes. This research thus contributes to a deeper understanding of cellular ion regulation and presents practical applications that are valuable in both medical and biotechnological fields. It is however important for more studies to be carried out to improve this mathematical model by adding biological components like genetic regulation and environmental effects in order to improve the model's precision and suitability for use in a variety of biological systems.

### 3.5 Recommendations

The following recommendations are made in order to optimize the potential of this mathematical model in scientific research and practical applications, with a view to advancing fields that rely on precise control of cellular processes.

1. Cost Reduction: By forecasting ideal circumstances for desired cell sizes without requiring a great deal of trial and error, the model can be used in research and industrial operations to optimize resources and lower the cost of manufacturing and trials.

2. Cell Growth Regulation: This model can be applied to control and accurately regulate ion concentrations and in effect cell growth during cell culture procedures. By increasing the yield of cells with the desirable sizes, this can boost the effectiveness of biotechnological applications for industrial applications.

3. Development of Targeted Therapies: Application of this model to create treatment plans that alter ion concentrations to regulate cell proliferation in diseases like cancer where cell size is a crucial component is recommended.

4. Validation Studies: To guarantee the model's resilience and dependability, we recommend thorough experimental validation process on various cell kinds and situations. This will support the model's establishment as a common tool in academic and professional contexts.

5. Educational Tool: This model can be used to instruct students in the connection between ion concentrations and cell physiology in academic settings. This provides a practical orientation for Students who may find it easier to comprehend the fundamentals of mathematical models and cellular biology.

6. Training for Research: This model can be included in research training courses for scientists and technicians working in bio-informatics and cell biology to provide them advanced knowledge of mathematical modelling and how it applies to biological processes.

7. Ethical Considerations: We recommend strict adherence to ethical standards when using this model to manipulate ion concentrations to change cell sizes, especially when it comes to fields like synthetic biology and genetic engineering or experimental treatments.

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