

# Assessing Asset Value Changes Using a System of Stochastic Models with Constant Terms and Periodic Drift Coefficients

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## Abstract

The Stochastic Differential Equation (SDE) is well known prevailing mathematical tools used for the estimation of asset values over time. In particular, this paper considered stochastic systems with prominence on variations of stock parameters. These problems were solved analytically by implementing the Itô's method of solution where precise measures were given on the assessments of asset values. Therefore, the impressions on Tables for investors in financial markets were analyzed to demonstrate empirically the behavior of asset values when volatility increases. Also the expectations of each independent solution were obtained graphically. From the analysis we deduce that ; increase in volatility decreases the value of assets, increase in volatility shows rise and fall in the assessments of asset values due to periodic parameter incorporated in the model, and finally describes the average value of the asset over time as it affects financial markets in time varying investments. This work presents a unique contribution and first-time approach that is, assessing asset values by modeling stock drift coefficients, with some constants and periodic event parameters.

**Keywords:** Stock Prices, Financial Markets, Stochastic Analysis, Drift and Volatility.

**MSC2010:** 35A01, 65L10, 65L12, 65L20, 65L70.

## 1 Introduction

In finance generally, investments are ventures associated with risk which cannot be avoided. Risk is a factor to understand with respect to investment portfolios. It influences and is contributory to fluctuation or variations in returns on the stock and portfolio. All the same, because of the risk involved in the management of investment portfolios, it is necessary to provide the investor with a Mathematical framework for investment decisions. In general, risk is an established factor in financial investment as well as in other human activities. Against this backdrop, insurance companies deem it pertinent for financial outcomes, lives, properties etc. to be insured.

In several areas of science and engineering, the design, correct evaluation, and analysis of real-life systems should consider the prospect of “white noise”. This refers to random forces that influence the system or inaccurate assessments or analysis in the system. Uncertainty is integral to the mathematical formulation of several real-life occurrences like stock market fluctuations, noise in population systems and irregular fluctuations in communication networks with respect to observed signals. Certain analytical errors can lead to buying assets such as stocks out of anxiety. In fact, some financial analysts, who invest in financial market are unsure of the behavior of the stock market; face the challenge of not knowing which stocks to buy or sell for profit maximization. Hence both financial analyst and potential investors require relevant information on the prediction of stock price behavior [1].

In order to help investors and owners of corporations take optimal decisions on the level of their investment in stock market [2], researchers are curious and fascinated about studying the behaviour of the unstable market variables. However, the price evolution of risky assets are generally modelled as a path or track of risky assets and are considered as diffusion process defined on several basic probability space, with the Geometric Brownian motion as the main tool used as the established reference model [3].

Many researchers have modelled stock market prices with diverse approaches and obtained interesting results. For example, [4] studied systems of SDEs for economic investment whose rate of returns and asset valuation follows series price index, with periodic and multiplicative effects. In the research of [5], they studied the concept of asset value with delay parameter in the model and defined conditions which governed asset values as a result of delay parameter.

Stochastic analysis of the behaviour of stock prices was studied by [2], and results showed that the proposed model was efficient for predicting stock prices. Similarly, [6] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE). In the research, the drift and volatility coefficients for the stochastic differential equations were obtained and the Euler-Maruyama method for system of SDEs was utilized to invigorate the stock prices. [7] developed the geometric Brownian motion and looked at the exactness of the model with comprehensive analysis of assumed data. [8] studied the stochastic modelling of stock prices applying a method of Brownian motion model to explain the stock price time series.

Yet [3] studied stochastic model of the fluctuation of stock market price, conditions for finding out the equilibrium price, adequate conditions for robust stability and convergence to equilibrium of the growth rate of the value function of shares. However, [9] looked at a stochastic model of price changes on the floor of the stock market. In the work of [10], the equilibrium price and the market growth rate of shares were determined.

Past researches for example, [6] examined a stochastic model of several selected stocks in the Nigerian Stock Exchange (NSE) where the Euler-Maruyama method for system of (SDE) was utilized to invigorate the stock prices and result revealed that stock1 yielded the best returns on investment compared to stock2, stock3 and stock4. The merit of this research over [6] is that the present research, models the effect of growth rates on stock market price estimation or forecast, with respect to volatility and the drift. Extensive work on stock market behaviour has been done as in [11–15] and [16–23]. More so, the aim of this paper is to develop stochastic models with constant terms and periodic drift coefficients in assessing asset value changes.

It is understandable that [4] has considered systems of SDEs for economic investments whose rate of returns and asset valuation follows series price index, periodic and multiplicative effect; while [5] considered the concept of asset values with delay parameter in the model and defined conditions which governed asset values. The advantage of this paper over [4] and [5] respectively is that, this present paper models the constant terms and periodic drift coefficients in assessing asset value changes where the effects of volatility and other stock variables were critically examined. Further more, the tables, graphs and other stock quantities were discussed. The work assessed asset values by modeling stock drift coefficient with some constants and periodic events parameters, which is a unique contribution to studies about stock market fluctuations. Our novel idea compliments previous efforts in this area .

The rest of the paper is set out as follows: Section 2.1 presents material and methods, results

and discussion are given in Section 3 and paper is concluded in Section 4.1.

## 2 Mathematical Preliminaries and Methods

### 2.1 Material and Methods

A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that  $(\Omega, \mathcal{F}, \mathbb{P})$  is a probability space with filtration  $\{f_t\}_{t \geq 0}$  and  $W(t) = (W_1(t), W_2(t), \dots, W_m(t))^T$ ,  $t \geq 0$  an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of

$$dX(t) = f(t, X(t)) dt + g(t, X(t)) dZ(t), \quad 0 \leq t \leq T.$$

[24]

$$X(0) = x_0.$$

where  $T > 0$ ,  $x_0$  is an n-dimensional random variable and coefficient functions are in the form  $f : [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  and  $g : [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$  SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S)) dS + \int_0^t g(S, X(S)) dZ(S)$$

Where  $dX, dZ$  are terms known as stochastic differentials.

**Theorem 1.1 (Itô's Lemma).** Let  $f(S, t)$  be a twice continuously differentiable function on  $[0, \infty) \times A$  and let  $S_t$  denote an Itô's process

$$dS_t = a_t dt + b_t dZ(t), \quad t \geq 0$$

Applying the Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higher order terms (h.o.t.)}$$

So, ignoring h.o.t and substituting for  $dS_t$  we obtain

$$\begin{aligned} dF_t &= \frac{\partial F}{\partial S_t} (a_t dt + b_t dz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (a_t dt + b_t dz(t))^2 \\ &= \frac{\partial F}{\partial S_t} (a_t dt + b_t dz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b_t^2 dt. \\ &= \left( \frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b_t^2 \right) dt + \frac{\partial F}{\partial S_t} b_t dz(t). \end{aligned}$$

More so, given the variable  $S(t)$  denotes stock price, then following GBM implies, the function  $F(s, t)$ , Itô's lemma gives:

$$dF = \left( \mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} \right) dt + \sigma S \frac{\partial F}{\partial S} dz(t).1001[18]and1001[19]$$

Nevertheless, the stochastic analysis on the variations of stock drift and it's influences in financial markets, is considered. The volatility dynamics and other drift coefficients of stock prices was taken to be constant throughout the trading days. The initial stock price which is assumed to follow different trend series was categorized the entire origin of stock dynamics is found in a complete probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  with a finite time investment horizon  $T > 0$ . Therefore, we have the following system of stochastic differential equations below;

$$dX_t = -\beta \mu X_t dt + \sigma X_t dW_t^1 \quad (2.1)$$

$$dX_\phi = K \tanh(X_\phi) dt + \sigma X_\phi dW_t^2 \quad (2.2)$$

$$dX_\varpi = (-\beta\alpha + K \tanh(X_\varpi)) X_\varpi dt + \sigma X_\varpi dW_t^3 \quad (2.3)$$

where  $\mu$  is an expected rate of returns on stock,  $\sigma$  is the volatility of the stock,  $dt$  is the relative change in the price during the period of time and  $W_t^1 = W_t^2 = W_t^3$  is a Wiener process  $\beta$   $K$  are constants and  $\tanh$  is periodic events parameter.

## 2.2 Method of Solution

The proposed model (2.1-2.3) consist of a system of variable coefficient problem of stochastic differential equations whose solutions are not trivial. we solve equations independently as follows using Itô's theorem 1.1:

From (2.1) let  $f(X_t, t) = \ln X_t$

Taking the partial derivative yields

$$\frac{\partial f}{\partial X_t} = \frac{1}{X_t}; \quad \frac{\partial^2 f}{\partial X_t^2} = \frac{-1}{X_t^2}; \quad \frac{\partial f}{\partial t} = 0 \quad (2.4)$$

Using Itô's lemma, we have:

$$df(X_t, t) = \sigma X_t \frac{\partial f}{\partial X_t} dW_t^1 + \left( -\beta\mu X_t \frac{\partial f}{\partial X_t} + \frac{1}{2} \sigma^2 X_t^2 \frac{\partial^2 f}{\partial X_t^2} + \frac{\partial f}{\partial t} \right) dt \quad (2.5)$$

Substituting (2.4) into (2.5) gives

$$\begin{aligned} df(X_t, t) &= \sigma X_t \frac{1}{X_t} dW_t^1 + \left( -\beta\mu X_t \frac{1}{X_t} + \frac{1}{2} \sigma^2 X_t^2 \left( \frac{-1}{X_t^2} \right) + 0 \right) dt \\ &= \sigma \frac{X_t}{X_t} dW_t^1 + \left( -\beta\mu X_t \frac{X_t}{X_t} - \frac{1}{2} \sigma^2 X_t^2 \right) dt = \sigma dW_t^1 + \left( -\beta\mu - \frac{1}{2} \sigma^2 \right) dt \\ &= \left( -\beta\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t^1 \end{aligned} \quad (2.6)$$

Integrating the above expression, we have

$$\int_0^t d \ln X_t = \int_0^t df(X_u, u) = \int_0^t \left( -\beta\mu - \frac{1}{2} \sigma^2 \right) du + \int_0^t \sigma dW_t^1 \quad (2.7)$$

$$\ln X_t - \ln X_0 = \left[ -\beta\mu u - \frac{1}{2} \sigma^2 u \right]_0^t + [\sigma W_u]_0^t = \ln \left( \frac{X_t}{X_0} \right) = \left( -\beta\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t^1$$

Taking  $\ln$  on both sides gives

$$X_t = X_0 e^{(-\beta\mu - \frac{1}{2} \sigma^2) t + \sigma W_t^1} \quad (2.8)$$

From (2.2) let  $f(X_\phi, t) = \ln X_\phi$

Taking the partial derivative yields

$$\frac{\partial f}{\partial X_\phi} = \frac{1}{X_\phi}, \quad \frac{\partial^2 f}{\partial X_\phi^2} = \frac{-1}{X_\phi^2}, \quad \frac{\partial f}{\partial t} = 0 \quad (2.9)$$

Applying Itô's lemma yields,

$$df(X_\phi, t) = \sigma X_\phi \frac{\partial f}{\partial X_\phi} dW_t^2 + \left( K \tanh X_\phi \frac{\partial f}{\partial X_\phi} + \frac{1}{2} \sigma^2 X_\phi^2 \frac{\partial^2 f}{\partial X_\phi^2} + \frac{\partial f}{\partial t} \right) dt \quad (2.10)$$

Substituting (2.9) into (2.10) gives

$$\begin{aligned}
 df(X_\phi, t) &= \sigma X_\phi \frac{1}{X_\phi} dW_t^2 + \left( K \tanh X_\phi \frac{1}{X_\phi} + \frac{1}{2} \sigma^2 X_\phi^2 \left( -\frac{1}{X_\phi^2} \right) + 0 \right) dt \\
 &= \sigma \frac{X_\phi}{X_\phi} dW_t^2 + \left( K \tanh X_\phi \frac{X_\phi}{X_\phi} - \frac{1}{2} \sigma^2 X_\phi^2 \right) dt \\
 &= \sigma dW_t^2 + \left( K \tanh - \frac{1}{2} \sigma^2 \right) dt \\
 &= \left( K \tanh - \frac{1}{2} \sigma^2 \right) + \sigma dW_t^2 dt.
 \end{aligned} \tag{2.11}$$

Integrating the above expression, we have

$$\int_0^t d \ln X_\phi = \int_0^t df(X_\phi, u, u) = \int_0^t \left( K \tanh - \frac{1}{2} \sigma^2 \right) du + \int_0^t \sigma dW_t^2. \tag{2.12}$$

That is,

$$\ln X_\phi - \ln X_0 = \left[ K \tanh u - \frac{1}{2} \sigma_u^2 \right]_0^t + [\sigma W_u^2]_0^t$$

Therefore

$$\ln \left( \frac{X_\phi}{X_0} \right) = \left[ K \tanh - \frac{1}{2} \sigma^2 \right] t + \sigma W_t^2.$$

Taking ln on both sides, gives

$$X_\phi = X_0 e^{(K \tanh - \frac{1}{2} \sigma^2)t + \sigma W_t^2} \tag{2.13}$$

From (2.3), let

$$f(X_\varpi, t) = \ln X_\varpi$$

Taking the partial derivative, yields

$$\frac{\partial f}{\partial X_\varpi} = \frac{1}{X_\varpi}; \quad \frac{\partial^2 f}{\partial X_\varpi^2} = \frac{-1}{X_\varpi^2}; \quad \frac{\partial f}{\partial t} = 0 \tag{2.14}$$

Applying Itô's lemma yields,

$$\begin{aligned}
 df(X_\varpi, t) &= \sigma X_\varpi \frac{\partial f}{\partial X_\varpi} dW_t^3 \\
 &+ \left( (-\beta \alpha + K \tanh) X_\varpi \frac{\partial f}{\partial X_\varpi} + \frac{1}{2} \sigma^2 X_\varpi^2 \frac{\partial^2 f}{\partial X_\varpi^2} + \frac{\partial f}{\partial t} \right) dt
 \end{aligned} \tag{2.15}$$

Substituting (2.14) into (2.15), gives

$$\begin{aligned}
 df(X_\varpi, t) &= \sigma X_\varpi \frac{1}{X_\varpi} dW_t^3 \alpha \\
 &+ \left( (-\beta \alpha + K \tanh) X_\varpi \frac{1}{X_\varpi} + \frac{1}{2} \sigma^2 X_\varpi^2 \left( \frac{-1}{X_\varpi^2} \right) + 0 \right) dt \\
 \sigma \frac{X_\varpi}{X_\varpi} dW_t^3 + \left( (-\beta \alpha + K \tanh) X_\varpi \frac{X_\varpi}{X_\varpi} - \frac{1}{2} \sigma^2 X_\varpi^2 \right) dt &= \left( (-\beta \alpha + K \tanh) - \frac{1}{2} \sigma^2 \right) dt \\
 &= \left( (-\beta \alpha + K \tanh) - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t^3.
 \end{aligned} \tag{2.16}$$

Integrating the above expression yields,

$$\int_0^t dInX_{\varpi} = \int_0^t df(X_{\varpi}u, u) = \int_0^t \left( (-\beta\alpha + Ktanh) - \frac{1}{2}\sigma^2 \right) du + \int_0^t \sigma dW_t^3. \quad (2.17)$$

That is,

$$InX_{\varpi} - InX_0 = \left[ (-\beta\alpha + Ktanh)u - \frac{1}{2}\sigma_{\varpi}^2 \right]_0^t.$$

Therefore

$$In \left[ \frac{X_{\varpi}}{X_0} \right] = \left[ (-\beta\alpha + Ktanh) - \frac{1}{2}\sigma^2 \right] t + \sigma W_t^3.$$

Taking ln on both sides, gives

$$X_{\varpi} = X_0 e^{((- \beta \alpha + K \tanh \frac{1}{2} \sigma^2) t + \sigma W_t^3)} \quad (2.18)$$

The expected value of the solutions (2.8),(2.13) and (2.18) is given as

$$EX_t(t) = X_0 e^{(-\beta\mu - \frac{1}{2}\sigma^2)t + \sigma W_t^1} = X_t(0) = e - \beta\mu t \quad (2.19)$$

$$EX_{\phi}(t) = X_0 e^{(K \tanh - \frac{1}{2}\sigma)t + \sigma W_{\phi}^1} = X_{\phi}(0) = e - K \tanh t \quad (2.20)$$

$$EX_{\varpi}(t) = X_0 e^{((- \beta \alpha + K \tanh) - \frac{1}{2}\sigma^2)t + \sigma W_{\varpi}^1} = X_{\varphi}(0) = e(-\beta\alpha + K \tanh)t \quad (2.21)$$

### 3 Results and Discussion

This Section presents the table of results for equations whose solutions are in (2.8),(2.13),(2.18) and graphical solutions of (2.19-2.21). Parameter values were chosen to reflect the actual characteristics of the asset to ensure the valuation model is realistic and capture the true value of the asset. Hence the following parameter values were used in the simulation :

$$X_t = X_0 \left( -\beta\mu - \frac{1}{2}\sigma^2 \right) t + \sigma W_t^1, \beta = 0.01, \mu = 0.25, W_t^1 = W_t^2 = W_t^3 = 1$$

Table 1: The effect of volatility on the assessment of asset values when time is fixed on the SDE model with drift coefficient

$X_0$	Time(t)	Volatility( $\sigma$ )	$X_t$	Volatility( $\sigma$ )	$X_t$
60.77	4.0000	0.5	60.1653	1.2	11.21133
	4.0000	0.6	53.3619	1.4	4.8408
	4.0000	0.7	45.4811	1.6	1.8168
	4.0000	0.8	37.2293	1.8	0.5695
50.25	4.0000	0.5	50.7550	1.2	9.2721
	4.0000	0.6	45.0157	1.4	4.0029
	4.0000	0.7	38.3598	1.6	1.4726
	4.0000	0.8	30.7845	1.8	0.4616
40.10	4.0000	0.5	39.7009	1.2	7.3992
	4.0000	0.6	35.2116	1.4	3.1943
	4.0000	0.7	30.0054	1.6	1.11751
	4.0000	0.8	24.5663	1.8	0.3684

$$X_\phi = X_o e^{(K \tanh - \frac{1}{2} \sigma^2)t} + \sigma W_t^2, K = 1.0000, h = 25, W_t^2 = 1$$

Table 2: The effect of volatility on the assessment of asset values when time is fixed on the SDE model with periodic drift coefficient

$X_0$	Time(t)	Volatility( $\sigma$ )	$X_t$	Volatility( $\sigma$ )	$X_t$
60.77	4.0000	0.5	392.4040	1.2	73.1367
	4.0000	0.6	348.0298	1.4	31.5700
	4.0000	0.7	296.5758	1.6	79.2258
	4.0000	0.8	242.8126	1.8	64.8641
50.25	4.0000	0.5	324.4743	1.2	60.4759
	4.0000	0.6	287.7818	1.4	26.1049
	4.0000	0.7	245.2351	1.6	65.5109
	4.0000	0.8	200.7789	1.8	53.6353
40.10	4.0000	0.5	258.9337	1.2	48.2604
	4.0000	0.6	229.6527	1.4	20.8320
	4.0000	0.7	195.7000	1.6	52.2784
	4.0000	0.8	160.2236	1.8	42.8015

$$X_\varpi = X_o e^{((- \beta \alpha + K \tanh - \frac{1}{2} \sigma^2)t + \sigma W_t^3)}, \beta = 0.01, h = 25, K = 1.0000$$

Table 3: The effect of volatility on the assessment of asset values when time is fixed on the SDE model with constant terms and periodic drift coefficient

$X_0$	Time(t)	Volatility( $\sigma$ )	$X_t$	Volatility( $\sigma$ )	$X_t$
60.77	4.0000	0.5	388.5026	1.2	72.4075
	4.0000	0.6	344.5720	1.4	31.2601
	4.0000	0.7	293.6224	1.6	78.4358
	4.0000	0.8	240.4000	1.8	64.2187
50.25	4.0000	0.5	321.2483	1.2	59.8719
	4.0000	0.6	284.9225	1.4	25.8486
	4.0000	0.7	242.7929	1.6	64.8577
	4.0000	0.8	198.7840	1.8	53.1017
40.10	4.0000	0.5	256.3593	1.2	47.7792
	4.0000	0.6	227.3710	1.4	20.6274
	4.0000	0.7	193.7512	1.6	51.7571
	4.0000	0.8	158.6316	1.8	42.3757

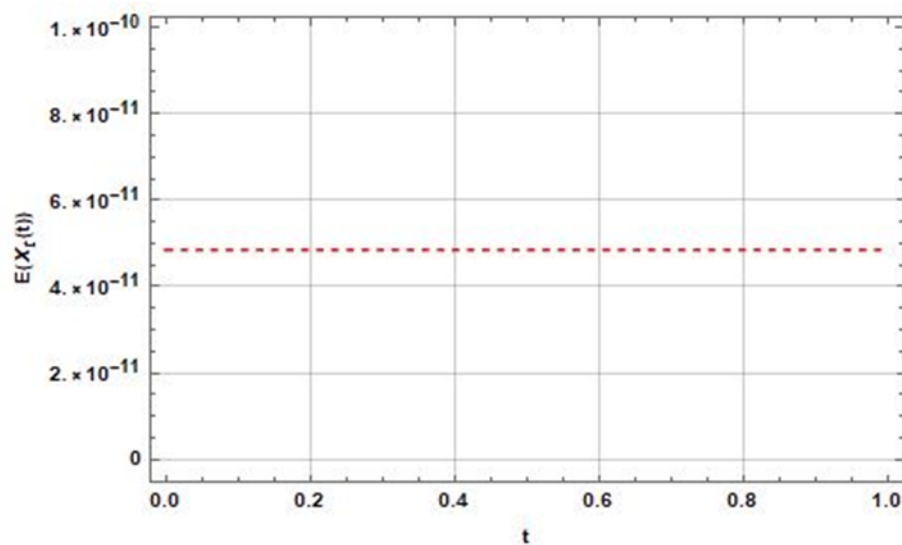


Figure 1: The effect of expected negative drift coefficient on financial market against time



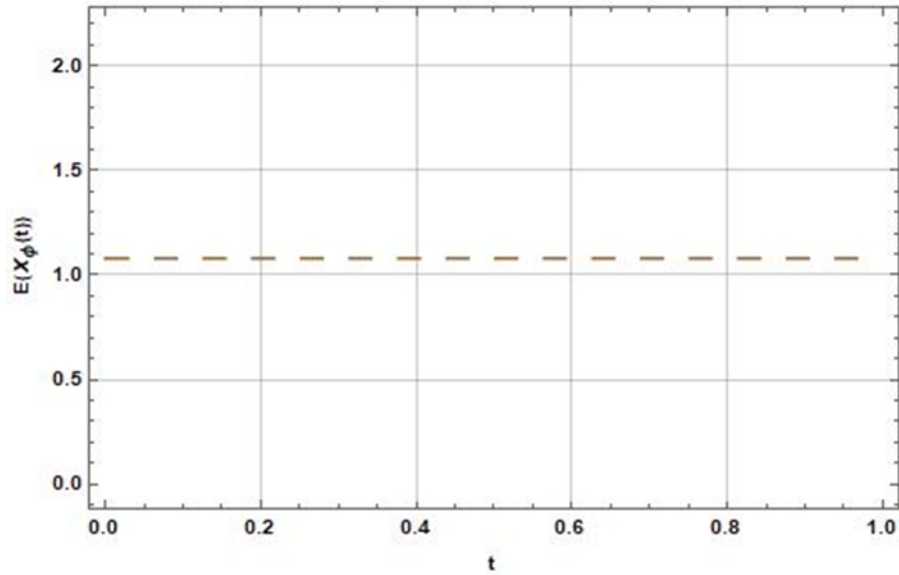


Figure 2: The effect of expected periodic drift coefficient on financial market against time

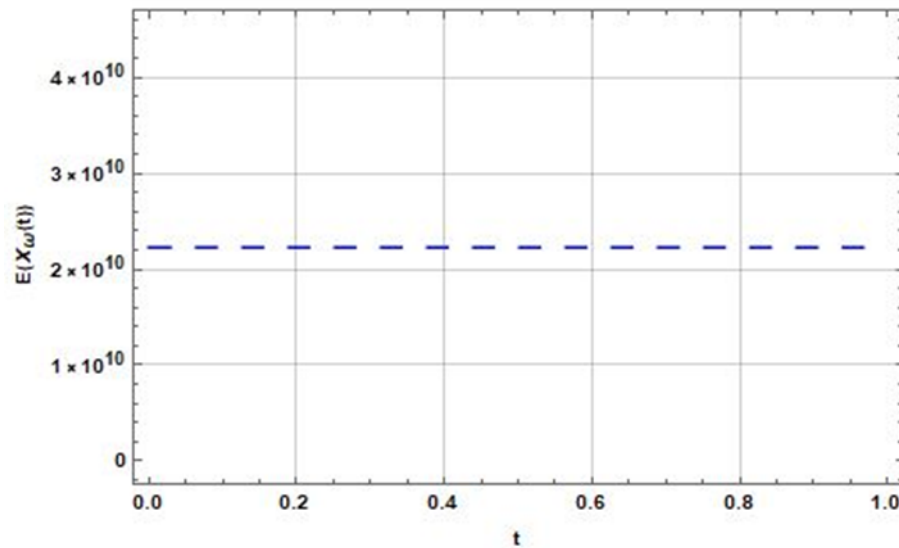


Figure 3: The effect of expected constant terms with periodic function drift coefficient on financial market against time

The following findings were made. From tables 1, 2 and 3; it can be seen that increase in volatility decreases the value of asset; this means that the asset is more likely to experience large price swings in either direction. When this happens, the asset's value can be said to have decreased because its price is more uncertain. This makes the asset less desirable for investors, who typically prefer assets that have stable, predictable prices. As a result, increased volatility can lead to a decline in the value of an asset [25]. It is important to note that volatility can increase for a variety of reasons, such as economic uncertainty, political turmoil. However, columns 6 of tables 2 and 3 respectively, shows rise and fall in asset values as volatility increases. This refers to the fluctuation

of prices for different types of financial assets, such as stocks, bond, or commodities. These price fluctuations can be driven by a number of factors such as economic conditions, interest rates, or investor sentiment. When prices rise, this is known as an upswing or a bull market. Conversely, when prices of asset fall, this can have a significant impact on the economy, affecting both businesses and consumers. For businesses, changes in asset prices can affect their cash flow and ability. Though, Figures 1,2 and 3 describe the average value of the asset price over time this value is calculated by taking the average of the SDE solution at different points in time. It can be thought of as the expected value of the market at any given time and it can be used to predict future market movement. For instance, if the expected mean is increasing over time, it means that the market is expected to move up, conversely, if the expected mean is decreasing, it means that the market is expected to move down. All these are informative to every investor.

#### 4.1 Conclusion

This paper considered stochastic systems with prominence on variations of stock parameters by implementing the Itô's method of solution where precise measures were given on the assessments of asset values. Therefore, the impressions on Tables for investors in financial markets were analyzed to demonstrate empirically the behavior of asset values when volatility increases. From the analysis we deduce that; increase in volatility decreases the value of assets, increase in volatility shows rise and fall in the assessments of asset values due to periodic parameter incorporated in the model, and finally describes the average value of the asset over time as it affects financial markets in time varying investments. Therefore, for further study, we recommend stability analysis on stochastic differential equations in the assessment of stock parameters for a future study.

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