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Fixed Point Theorems for Some Iteration Processes with Generalized Zamfirescu Mappings in Uniformly Convex Banach Spaces

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Abstract

The paper establish fixed point theorems for some iteration processes in uniformly convex Banach spaces with generalized Zamfirescu mappings. Our results improve a multitude of recent results in literature. .

Keywords: Generalised Zamfirescu Mappings, Fixed Point, Uniformly Convex Banach Spaces, Iterative Scheme.
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1 Introduction

Let (Q, d) be a complete metric space, $T : Q \to Q$ be a selfmap of Q. The set $F_T = \{x \in Q : Tx = x\}$ is the set of fixed point of T in X. Let Q be a non-empty closed and convex subset of a Banach space X.

A Banach space is a complete normed linear space. A Banach space $(X, \|.\|)$ is said to be uniformly convex, if for every $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x, y \in X$ with $\|x\| \leq 1$, $\|y\| \leq 1$ and $\|x - y\| \geq \epsilon$ then $\|\frac{1}{2}(x + y)\| \leq 1 - \delta$.

The following result is due to Zamfirescu [1] in 1972.

Theorem 1.1. **[1]** Let (X, d) be a complete metric space and $T : X \to X$ be a mapping for

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which there exist real numbers α, β and γ satisfying $0 \leq \alpha < 1$, $0 \leq \beta < 0.5$ and $0 \leq \gamma < 0.5$ such that for each $x, y \in X$ at least one of the following is true:

(Z_1) $d(Tx, Ty) \leq \alpha d(x, y)$,

(Z_2) $d(Tx, Ty) \leq \beta[d(x, Tx) + d(y, Ty)]$,

(Z_3) $d(Tx, Ty) \leq \gamma[d(x, Ty) + d(y, Tx)]$.

Let

$$\delta = max\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}.$$

where δ satisfy $0 \leq \delta < 1$.
A mapping T satisfying the contractive conditions (Z_1), (Z_2) and (Z_3) in Theorem 1.1 above is called Zamfirescu operator.
Remark 1: For the proof of Theorem 1.1 (see [2]). Also shown below:

If Z_1 holds then we have

$$\begin{aligned}
d(x_2, x_1) &= d(Tx_1, Tx_0) \\
&\leq \alpha d(x_1, x_0) \\
d(x_3, x_2) &= d(Tx_2, Tx_1) \\
&\leq \alpha^2 d(x_1, x_0) \\
\\
d(x_{n+1}, x_n) &\leq \alpha^n d(x_1, x_0), \quad \forall = 1, 2, 3, \ldots \\
d(x_{n+p}, x_n) &= d(x_n, x_{n+p}), \\
&= \sum_{k=n}^{n+p-1} d(x_{k+1}, x_k). \\
\\
d(x_{n+1,n}) &= d(x_n, x_{n+1}) \\
&\leq \alpha d(x_n, x_{n-1}) \\
\\
d(x_{n+2}, x_{n+1}) &= d(Tx_{n+1}, Tx_n) \\
&\leq \alpha^2 d(x_n, x_{n-1}) \\
\\
d(x_{n+3}, x_{n+2}) &= d(Tx_{n+2}, Tx_{n+1}) \\
&\leq \alpha^3 d(x_n, x_{n-1}) \\
\\
d(x_{n+k}, x_{n+k-1}) &= d(Tx_{n+k-1}, Tx_{n+k-2}) \\
&\leq \alpha^k d(x_n, x_{n-1}) k \in N^*
\end{aligned}$$

If (Z_2) holds, then we have

$$\begin{aligned}
d(Tx,Ty) &\leq \beta[d(x,Tx)+d(y,Ty)]\\
&\leq \beta d(x,Tx)+\beta d(y,x)+\beta d(x,Tx)+\beta d(Tx,Ty)\\
&= 2\beta d(x,Tx)+\beta d(x,y)+\beta d(Tx,Ty)\\
d(Tx,Ty)-\beta d(Tx,Ty) &\leq 2\beta d(x,Tx)+\beta d(x,y)\\
(1-\beta)d(Tx,Ty) &\leq 2\beta d(x,Tx)+\beta d(x,y)\\
d(Tx,Ty) &\leq (\frac{2\beta}{1-\beta})d(x,Tx)+(\frac{\beta}{1-\beta})d(x,y)\ 0\leq\beta<0.5
\end{aligned}$$

denoting

$$\delta = max\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}. \tag{1.1}$$

where δ satisfy $0 \leq \delta < 1$. (1.2)
Using Z_2 for $x, y \in X$, then we obtain

$$\begin{aligned}
d(Tx,Ty) &\leq (\frac{2\beta}{1-\beta})d(x,Tx)+(\frac{\beta}{1-\beta})d(x,y)\\
d(Tx,Ty) &\leq 2(\frac{\beta}{1-\beta})d(x,Tx)+(\frac{\beta}{1-\beta})d(x,y)
\end{aligned}$$

Let $\delta = \frac{\beta}{1-\beta}$,

$$d(Tx,Ty) \leq 2\delta d(x,Tx)+\delta d(x,y). \tag{1.3}$$

If Z_3 holds, from theorem 1.1, suppose $0 \leq \delta < 1$ then

$$\begin{aligned}
d(Tx,Ty) &\leq \gamma[d(x,Ty),d(y,Tx)]\\
&\leq \gamma d(x,Ty)+[d(y,x)+d(x,Ty)+d(Ty,Tx)]\\
(1-\gamma)d(Tx,Ty) &\leq 2\gamma d(x,Ty)+\gamma d(y,x)\\
d(Tx,Ty) &\leq d(x,Ty)+\frac{\gamma}{1-\gamma}d(x,y)\ 0\leq\gamma<0.5
\end{aligned}$$

In a similar manner, Let consider,

$$\begin{aligned}
d(Tx,Ty) &\leq (\frac{2\gamma}{1-\gamma})d(x,Ty)+(\frac{\gamma}{1-\gamma})d(x,y)\\
(Tx,Ty) &\leq 2(\frac{\gamma}{1-\gamma})d(x,Ty)+(\frac{\gamma}{1-\gamma})d(x,y)
\end{aligned}$$

Let $\delta = \frac{\gamma}{1-\gamma}$, to obtain

$$d(Tx,Ty) \leq 2\delta d(x,Ty)+\delta d(x,y). \tag{1.4}$$

Remark 2: Suppose $(X, \|.\|)$ is a normed linear space, then (1.3) becomes

$$\|Tx-Ty\| \leq 2\delta\|x-Tx\|+\delta\|x-y\|, \tag{1.5}$$

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for all $x, y \in X$ and where $0 \leq \delta < 1$ is defined by (1.1)

Bosede used (1.5) to find the fixed point theorems for some iteration processes with generalized Zamfirescu mappings in uniformly convex Banach spaces (see [3]).
Our aim in this paper is to establish some fixed point theorems for some iteration processes with generalized Zamfirescu mappings in uniformly convex Banach spaces. The generalized Zamfirescu mapping defined in this paper is better than the one defined by Ciric (2003) and Bosede (2009) (see [4,5]). We shall use an iterative scheme (2.3) and the contraction mapping as indicated below:
Let X be a non-empty closed convex subset of a normed linear space X and $T : X \to X$ a self map of X, then

$$\|Tx - Ty\| \leq (2\delta\|x - Tx\| + \delta\|x - y\|)e^{\|x-Tx\|} \tag{1.6}$$

where $0 \leq \delta < 1$ is defined by (1.1) and e^x denotes the exponential function of $x \in X$

2 Preliminaries

Suppose Q be a closed and convex subset of a Banach space X and $T : Q \to Q$ a mapping which satisfies the condition

$$d(x, Tx) + d(y, Ty) \leq ad(x, y) \tag{2.1}$$

$\forall\ x, y \in Q, where\ 2 \leq a < 4.$
Let $x_0 \in Q$ be arbitrary and sequence $\{x_n\}_{n=1}^{\infty}$ be a sequence defined by

$$x_{n+1} = \frac{1}{2}(x_n + Tx_n), n = 0, 1, 2, \ldots \tag{2.2}$$

Mann Iterative Scheme [6]
Let $x_0 \in Q$ be arbitrary, $\{x_n\}_{n=0}^{\infty}$ be a sequence defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n, \quad n = 0, 1, 2, \ldots \tag{2.3}$$

satisfying conditions $0 \leq \alpha_n \leq 1$ and $\sum_{n=0}^{\infty} \alpha_n(1 - \alpha_n) = \infty$.
Ishikawa Iterative Scheme [7]
Let $x_0 \in Q$ be arbitrary, $\{x_n\}_{n=0}^{\infty}$ be a sequence defined by

$$\begin{aligned} y_n &= (1 - \beta_n)x_n + \beta_n Tx_n, \\ x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n Ty_n, \quad n = 0, 1, 2, \ldots \end{aligned} \tag{2.4}$$

with $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ being sequences of real number, satisfying the following conditions $\sum_{n=0}^{\infty} \alpha_n = \infty$, $\sum_{n=0}^{\infty} \beta_n = \infty$ and $0 \leq \alpha_n, \beta_n \leq 1$.
Noor Iterative Scheme [8]
Let $x_0 \in Q$ be arbitrary, $\{x_n\}_{n=0}^{\infty}$ be a sequence defined by

$$\begin{aligned} z_n &= (1 - \gamma_n)x_n + \gamma_n Ty_n \\ y_n &= (1 - \beta_n)x_n + \beta_n Tx_n, \\ x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n Tz_n, \quad n = 0, 1, 2, \ldots \end{aligned} \tag{2.5}$$

with $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ being sequences of real number, satisfying the following conditions $\sum_{n=0}^{\infty} \alpha_n = \infty$, $\sum_{n=0}^{\infty} \beta_n = \infty$, $\sum_{n=0}^{\infty} \gamma_n = \infty$ and $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$.

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Using the contractive condition (2.1) and iteration (2.2), Ciric in [4] established the following results:

Theorem 2.1 Let Q be a closed and convex subset of a Banach space X with the norm $\|x\| = d(x, 0)$, $x \in Q$ and a mapping $T : Q \to Q$ which satisfies the contractive definition (2.1). Let $x_0 \in$ Q be arbitrary sequence $\{x_n\}_{n=0}^{\infty}$ defined iteratively by (2.2). Then, T has at least one fixed point.

Remark 3: The proof of Theorem 2.1 is contained in [4]. Our aim in this paper is to establish fixed point theorems for Ishikawa (2.4) and Mann (2.3) iterative schemes respectively in uniformly convex Banach spaces for the class of Zamfirescu mappings, using the contractive definition (1.6)

Remark 4: The contractive definition (1.6) is well defined. Iterative scheme (2.4) used in our result is more general than iterative scheme (2.3) used by Bosede in [3], iterative scheme (2.2) used by Ciric and others in literature (see [3–5, 9–29]).

Suppose $\beta_n = 0$, $\forall n \in N$ in the Ishikawa Iteration (2.4), we obtain iteration (2.3) which is the iteration used by Bosede, also suppose $\alpha_n = \frac{1}{2}$, $\forall\ n \in N$ in the Ishikawa iteration (2.3), we obtain iteration (2.2) used by Ciric [4] in Theorem 2.1 above.

The following result establishes a fixed point theorem for Mann iteration process for the class of Zamfirescu mappings in uniformly convex Banach spaces.

3 Main Result

Theorem 3.1

Suppose $T : Q \to Q$ be a selfmap, Q be a closed and convex subset of a uniformly convex Banach space X, satisfying contractive condition (1.6). Let $x_0 \in Q$ be arbitrary and a sequence of $\{x_n\}_{n=0}^{\infty}$ defined iteratively by (2.3) converges strongly to the fixed point of T.

Proof

Considering iterative scheme (2.3), contractive definition (1.6) and triangular inequalities, for any arbitrary $x_0 \in Q$, we have

$$\begin{aligned}
\|x_{n+1} - p\| &= \|(1-\alpha_n)x_n + \alpha_n Tx_n - p\| \\
&= \|(1-\alpha_n)x_n + \alpha_n Tx_n - \alpha_n p - (1-\alpha_n)p\| \\
&= \|(1-\alpha_n)(x_n - p) + \alpha_n(Tx_n - p)\| \\
&= \|(1-\alpha_n)(x_n - p) + \alpha_n(Tx_n - p)\| \\
&\leq \|(1-\alpha_n)\|x_n - p\| + \alpha_n\|Tx_n - p\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n\|Tx_n - Tp\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n\|Tp - Tx_n\| \\
&\leq \|(1-\alpha_n)\|x_n - p\| + \alpha_n(2\delta\|p - Tp\| + \delta\|p - x_n\|)e^{\|Tp-p\|}\| \\
&\leq \|(1-\alpha_n)\|x_n - p\| + \alpha_n(2\delta\|p - p\| + \delta\|x_n - p\|)e^{\|p-p\|}\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n(2\delta(0) + \delta\|x_n\|x_n - p\|)e^{0}\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n(0 + \delta\|x_n - p\|)\| \\
&\leq \|(1-\alpha_n + \alpha_n\delta)\|x_n - p\|\| \qquad (3.1)
\end{aligned}$$

where $0 \leq \alpha_n < 1$, $0 \leq \delta < 1$ and $0 \leq (1 - \alpha_n + \alpha_n\delta) < 1$, gives

$$\|x_{n+1} - p\| \leq \|x_n - p\| \qquad (3.2)$$

This shows that $\|x_n - p\|$ is a monotone decreasing sequence. From (3.2) $\|x_n - p\| \to 0$ as $n \to \infty$, then, iterative scheme (2.3) converges strongly to the fixed point of T.

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Hence, completes the proof.
Remark 5: The iterative scheme (2.4) used in our next result is more general than the iterative scheme (2.2) used by Ciric (1990). Suppose $\beta_n = 0 \; \forall \; n \in N$ in (2.4), we obtain an iterative scheme (2.3).
Remark 6: The contraction definition used in (1.6) is more general than (1.5) used by Bosede (2009) in the following sense:
Suppose $e^0 = 1$ in (1.6), we obtain (1.5) $\forall \; x, y \in X$
The following result establish a fixed point theorem for Ishikawa iteration process for the class of Zamfirescu mappings in uniformly convex Banach spaces.
Theorem 3.2
Suppose $T : Q \to Q$ be a selfmap and Q be as in Theorem 3.1 above. Let $x_0 \in Q$ be arbitrary and a sequence $\{x_n\}_{n=0}^{\infty}$ be defined by (2.4) iteratively. Hence, Ishikawa iterative scheme converges strongly to the fixed point of T.

Proof
From Theorem 1.1, it was established that T has a unique fixed point in Q, that is $p = Tp$ such that $p \in Q$. This study need to show that the sequence $\|x_n - p\|$ is monotone decreasing. Considering the iterative scheme (2.4), contractive definition (1.6) and triangular inequalities. For arbitrary $x_0 \in Q$, we obtain

$$\begin{aligned}
\|x_{n+1} - p\| &= \|(1-\alpha_n)x_n + \alpha_n T y_n - p\| \\
&= \|(1-\alpha_n)x_n + \alpha_n T y_n - \alpha_n p - (1-\alpha_n)p\| \\
&= \|(1-\alpha_n)(x_n - p) + \alpha_n (T y_n - p)\| \\
&= \|(1-\alpha_n)(x_n - p) + \alpha_n (T y_n - p)\| \\
&\leq \|(1-\alpha_n)\|x_n - p\| + \alpha_n \|T y_n - p\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n \|T y_n - Tp\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n \|Tp - T y_n\| \\
&\leq \|(1-\alpha_n)\|x_n - p\| + \alpha_n (2\delta\|p - Tp\| + \delta\|p - y_n\|)e^{\|Tp - p\|}\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n (2\delta\|p - p\| + \delta\|y_n - p\|)e^{\|p - p\|}\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n (2\delta(0) + \delta\|y_n - p\|)e^{0}\| \\
&= \|(1-\alpha_n)\|x_n - p\| + \alpha_n (0 + \delta\|y_n - p\|)\| \\
&\leq \|(1-\alpha_n)\|x_n - p\| + \alpha_n \delta \|y_n - p\| \qquad (3.3)
\end{aligned}$$

However, $y_n = (1-\beta_n)x_n + \beta_n T x_n$

$$\begin{aligned}
\|y_n - p\| &\leq \|(1-\beta_n)x_n + \beta_n T x_n - p\| \\
&= \|(1-\beta_n)(x_n - p) + \beta_n (T x_n - p)\| \\
&\leq \|(1-\beta_n)\|x_n - p\| + \beta_n \|T x_n - p\| \\
&= \|(1-\beta_n)\|x_n - p\| + \beta_n \|T x_n - Tp\| \\
&= \|(1-\beta_n)\|x_n - p\| + \beta_n \|Tp - T x_n\| \\
&\leq \|(1-\beta_n)\|x_n - p\| + \beta_n (2\delta\|p - Tp\| + \delta\|p - x_n\|)e^{\|Tp - p\|}\| \\
&= \|(1-\beta_n)\|x_n - p\| + \beta_n (2\delta\|p - p\| + \delta\|x_n - p\|)e^{\|p - p\|}\| \\
&= \|(1-\beta_n)\|x_n - p\| + \beta_n (2\delta(0) + \delta\|x_n - p\|)e^{0}\| \\
&= \|(1-\beta_n)\|x_n - p\| + \beta_n (0 + \delta\|x_n - p\|)\| \\
&= \|(1-\beta_n + \beta_n\delta)\|x_n - p\| \qquad (3.4)
\end{aligned}$$

where $0 \leq \beta_n < 1, \;\; 0 \leq \delta < 1$ and $0 \leq (1 - \beta_n + \beta_n \delta) < 1$, gives

$$\|y_n - p\| \leq \|x_n - p\| \qquad (3.5)$$

Substituting (3.5) into (3.3) gives

$$\begin{aligned}\|x_{n+1}-p\| &\leq \|(1-\alpha_n)\|x_n-p\| + \alpha_n\delta\|x_n-p\| \\ &= \|(1-\alpha_n+\alpha_n\delta)\|x_n-p\|\end{aligned}$$

Also, by observing that $0 \leq \alpha_n \leq 1$, $0 \leq \delta < 1$ and since $0 \leq (1-\alpha_n+\alpha_n\delta) < 1$, gives

$$\|x_{n+1}-p\| \leq \|x_n-p\| \tag{3.6}$$

This shows a monotone decreasing sequence such that $\|x_n-p\|$ is monotone decreasing. From (3.6) observe that $\|x_n-p\| \to 0$ as $n \to \infty$, then, iterative scheme (2.4) converges strongly to the fixed point of T.
This completes the proof.

References

[1] Zamfirescu T.; *Fix Point Theorem in Metric spaces*: Arch. Math., Vol. 23, (1972), pp. 292-298.

[2] Berinde V. ; *Iterative approximation of fixed points for pseudo-contractive operators*: Seminar on Fixed point Theory,Vol.3,(2002), pp.209-216.

[3] Bosede A.O.; *On fixed point theorems for Mann and Ishikawa iteration processes in the class of Zamfirescu mappings in Uniformly convex Banach spaces.*

[4] Ciric L.B.; *Fixed point theory* : Construction Mapping Principle, FME Press.Beograd (2003).

[5] Bosede A.O ; *Noor Iterations Associated with Zamfirescu mappings in Uniformly convex Banach spaces*: Fasciculli Mathematici,Vol. 42, (2009), pp. 29-38.

[6] Mann W.R.; *Mean value method in Iteration* : Proc. Amer. Math.Soc,Vol. 44, (1953), pp. 506-510.

[7] Ishikawa S.; *Fixed point by a new iteration method* : Proc. Amer.Math. Soc., Vol. 44(1), (1974), pp. 147-150.

[8] Noor, M. A., *New approximation schemes for general variational inequalities* J. Math. Anal. Appl., 251 (2000), pp. 217-229

[9] Akewe, H. ; *The stability of a Modified Jungck-Mann Hybrid Fixed Point Iteration Procedure*, International Journal of Mathematical Sciences and Optimizatios: Theory and Applications,Vol. 1, (2016), pp. 95-104.

[10] Berinde V. ; *On the convergence of Ishikawa iteration in the class of quasi-contractive operators*: Acta Math.Univ.Comenianae.Vol. LXXIII (1), (2004), pp.119-120.

[11] Bosede A.O., Akinbo G.;*On the continuous dependence of fixed points for the Picard and Mann iteration processes for a class of ϕ-Contractions*:International Journal of Contemporary Mathematical Sciences,(2006).

[12] Bosede A.O.; *Some Common fixed point Theorems in Normed Linear Spaces*: Acta Univ. Palacki. Olomuc.Fac.rer.nat. Mathematics, Vol.49(1), (2010), pp. 19-26.

[13] Bosede A.O.; *Strong Convergence Results for the Jungck-Ishikawa and Jungck Mann Iteration processes* : Bulletin of Mathematical Analysis and Application,Vol. 2(3), (2010), pp. 65-73.

[14] Bosede A.O. and Akinbo G.; *Some stability theorems associated with A-distance and E-distance in uniform spaces*: Acta Universities Apulensis,Vol. 26, (2011), pp. 121-128.

[15] Bosede A.O.; *Strong convergence of Noor iteration for a general class of function* : Bulletin of Mathematical Analysis and Applications, Volume Issue (2011),pp. 140-145.

[16] Bosede A.O. and Rhoades B.E. ; *Stability of Picard and Mann iterations for a general class of functions* : Journal of advaced Mathematical studies,Vol. 3(2), (2010), pp. 1-3.

[17] Goebel K., Kirk W. and T. Shimi ; *A fixed point theorem in uniformly convex spaces* : Bull. U.M.E, Vol.(4)7, (1976), pp. 67-75.

[18] Imoru C.O. and Olatinwo M.O. ; *On the stability of picard and Mann iteration processes* : carpathian,J. Math., Vol. (19)(2),(2003), pp. 155-160.

[19] Imoru C.O., Olatinwo M.O., Akinbo G. and Bosede A.O. ; *On a version of the Banach's fixed Point theorem, General Mathematics* : Vol. 16(1), (2008), pp. 25-32.

[20] Omidire O.J. ; *Common Fixed point of some certain generalized contractive conditions in convex Metric space settings*, International Journal of Mathematical sciences and Optimization: Theory and Applications, Vol. 10 (3),(2024), pp.1-9.

[21] Osilike M.O.;*Stability Results for Fixed Point Iteration Procedure*: Applied Mathematics E-Notes,Vol. 6, (2006), pp. 289-293.

[22] Rafiq A.; *On the convergence of a three step Iteration Process in the class of quasi-Contractive Operators* : Acta Math. Acad. Paedagog Ntiregyhaziensis, Vol.22, (2006), pp. 305-309.

[23] Rhoades B.E.; *Fixed Point Iteration Using Infinite Matrices* :Trans. Amer.Math. Soc.,Vol. 196, (1974), pp. 161-176.

[24] Rhoades B.E.; *Fixed Point Theorems and Stability Results for Fixed point Iteration Methods* : J.Math.Anal. Appl., Vol. 56(2), (1976), pp. 741-750.

[25] Weng X.L. ; *Fixed point iteration for local strictly pseudo contractive mapping* : Proc. Amer.Math. Soc., Vol.103,(1991), pp. 727-731.

[26] Rus I.A., Petrusel A and Petrusel G. ; *Fixed Point Iteration Procedures* : Indian J.Pure Appl., Vol. 56(2), (1976), pp. 741-750.

[27] Xu H.K. ; *Inequalities in Banach spaces with applications* : Nonlinear Analysis, THeory and Applications, Vol. 16(20), (1991), pp. 1127-1138.

[28] Xu B. and Noor M.A.; *Fixed point iterations for asymptotically nonexpansive mappings in Banach spaces* : Journal of Mathematical Analysis and Applications, Vol. 267,(2002), pp.444-453.

[29] Zeider E. ; *Nonlinear Functional Analysis and it's application* : fixed point theorems, Springer Verlag, New York, Inc., Vol. 43, (1986), pp. 1-909.