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# Diffusion - Reaction Model for Infectious Diseases

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**Abstract**

Infectious diseases abound in the world, affecting and even claiming the lives of many of their victims. Many researches have been conducted with the goal of proffering how to curtail or control the spread of the diseases. A diffusion-reaction model is herein proposed for the spread and control of an infectious disease. A numerical approach is considered for the solution of the proposed model. With the aid of data generated from an instance of meningitis outbreak, the rate of spread of the disease and the quantity of treatment, to apply to control the disease, are determined.

**Keywords:** Infectious diseases, Diffusion-reaction, Finite difference method, meningitis, Data-driven approach.
**MSC2010: 00A71.**

## 1 Introduction

Infectious diseases are communicable diseases that bring about infection. They can be caused by bacteria, viruses, fungi or parasitic pathogens. The study of the causes, developments, effects, the regularity and irregularity of the spread of infectious diseases do call to mind the realistic regulatory importance for the control and inhibition of their spread [1]. Infectious diseases, which include COVID- 19, Meningitis, Dengue, Gonococcal Infection (Gonorrhea), Hepatitis A, B, C, D, and E, are significant causes of human and animal death.

One of the infectious diseases of epidemiological significance the world is yet to get rid of is Meningitis. The disease has infected hundreds of millions of people and claimed the lives of hundreds of thousands worldwide [2]. Some background information about this disease are provided below:

Meningitis causes a part of the brain, called the meninges, to swell. Meninges is the protective spine and brain membranes, that protect both the spinal cord and the brain which comprises of three membranes, namely: Dura mater, Arachroid mater and Pia Mater membrane, see the human meninges in Figure 1. The inflammation (swelling) of the spine and brain membrane is caused by viral, bacterial, fungal or parasitic pathogens which invades the cerebrospinal fluid (CSF) and circulating through the Central Nervous System (CNS) engulfs the whole body system.

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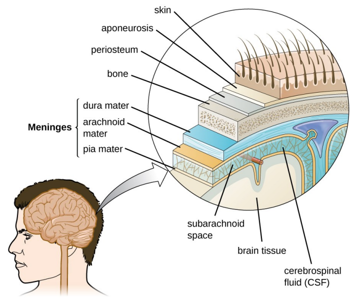


Figure 1: The Graphical Representation of Human Meninges.Source: Google Search Engine

This swelling results in a number of illnesses such as fever, cold hands and feet, vomiting and or stomach disorder (Diarrhea), confusion and irritability, severe muscles and joints pain, severe headache, stiff neck, light sensitivity or dislike of bright light, and convulsion or seizure. According to [3], these symptoms are usually observed between 2 to 10 days of the infection.

Meningitis is of major public health concern in sub-Saharan Africa as it is responsible for the occurrence of epidemic meningitis in the African Meningitis Belt, an area which comprise of 26 countries extending from Senegal in the West to Ethiopia in the East, and with an estimated population of about 500 million, [4] and [5].

The nineteen northern states of Nigeria, including the Federal Capital Territory, are located within this region. Over the years, the epidemic has been distributed across these states and in the recent past, the belt has widened to include some southern states namely: Oyo, Cross River, Imo, Anambra, Enugu and Ebonyi States. This region in Nigeria, see Figure 2, is known as Meningitis Belt of Nigeria, [6].

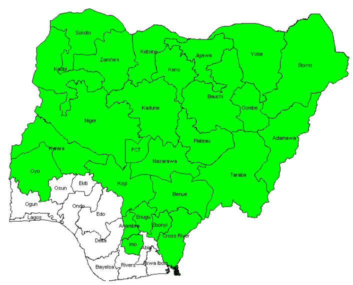

Figure 2: States in the Meningitis Belt of NigeriaSource: World Health Organization. (2023)

There are essentially three principles employed in the prevention of Meningitis, namely: vaccination, health education and personal hygiene such as frequent hand washing with soap and running clean water, keeping the environment clean, sleeping in well ventilated rooms, avoidance of overcrowded environments, non-sharing of personal body items with others (towel, handkerchief, brush, utensils etc.), resting adequately and engaging in regular physical exercise.

Vaccination is an effective ways to protect against Meningitis. There are vaccines for three types ofn bacterial Meningitis, namely: Neisseria meningitidis vaccine, Streptococcus pneumoniae (otherwise called Pneumococcal conjugate vaccine) and Haemophilus influenzae type-b (or Pentavalent vaccine). The last two vaccines are available through the routine immunisation program for children under five years of age, [7].

Meningitis is treated with antibiotics (such as Penicillin, Ampicillin, Chloramphenicol and Ceftriaxone), and antifungal (such as Amphotericin B, Itraconazole and Fluconazole). There is a steroid or reactive vaccination (Dexamethasone) that helps control the swelling and pressure in the skull.

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History tells that there are evidences that shows that Meningitis was introduced to Nigeria from Niger Republic through Jibia, a border town in Katsina State which is only 30 kilometres from Maradi in the year 1970. Since then, several major epidemics of Meningitis have occurred in Northern Nigeria. The outbreak do occur in the hot, dry and dusty season when the absolute humidity used to be low (around December - June), possibly related to drying and damage to the nasopharyngeal mucosa, and subsides with the rainy season, and may re-emerge the following dry season, [4].

## 2 Literature Review

Many researches have been conducted on Meningitis, but majorly on theoretical considerations such as different classes of humans exposed to the disease. [8] reported that following 2021˘2022 outbreaks, Zamfara achieved approximately 90 % outbreak preparedness via health worker training, expanded routine immunization from 666 to 700 facilities, and migrant-targeted vaccine strategies. [9] analyzed 7,140 suspected cases with 553 deaths during Dec 2016 – May 2017, attack rate 155/100,000, case fatality highest in children aged 5 – 9 (10 %).According to [10] a qualitative study across three senatorial zones identifying gaps in symptom awareness and vaccine-seeking behavior was conducted and it was recommended that intensified community awareness and vaccination campaigns are critically needed.

Mathematically, the main existing model for studying infectious diseases is the SIR model, where S denotes susceptible, I, infectious and R, recovered or removed, [11]. In this model the whole population is compartmentalised into sub-populations, wherein each compartment has its unique property. This model is based on Kermack and McKendrick's SIR model which forms the foundation of modern disease modeling. The flow diagram for the SIR model is as given in Figure 3.

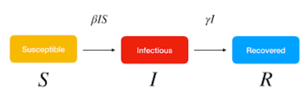


Figure 3: The Graphical Representation of SIR diagram.Source: Google Search Engine

Taking the change in each compartment with respect to time, we have

$$\frac{\partial S}{\partial t} = -\beta SI \tag{2.1}$$

$$\frac{\partial I}{\partial t} = \beta SI - \gamma I \tag{2.2}$$

$$\frac{\partial R}{\partial t} = \gamma I \tag{2.3}$$

where $\beta$ is the rate at which susceptible population become infected, known as the transmission rate, $\gamma$, the recovery rate.

On addition, equations (1), (2) and (3) give 0. This assumes that the population is constant. For this major assumption which is not consistent in real-life, due to the fact that meningitis has become an endemic disease within the population, with population growth and decay not equal we seek for an alternative method of modelling.

## 3 Proposed Model

In any disease outbreak, a lot of empirical data are generated such as number of suspected cases, confirmed cases and death. The assessment of the spread of meningitis and the effectiveness of the control is directly based on these data. Thus, a mathematical model that has an in - capacity to deal with these two aspects is called Diffusion-Reaction Model, which is proposed for studying the spread and control of meningitis in this work.

Basically, diffusion occurs when a disease spreads, or pours out, from region of higher concentration, temperature or pressure to a region of lower concentration, temperature or pressure, [3]. Herein, the diffusion process is modelled mathematically as a two-dimensional equation which is time dependent, since the environment of the outbreak is a land mass where humans live.

$$\frac{\partial u}{\partial t} = p\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3.1}$$

where $u$ as the disease phenomenon (meningitis) under study is a function of $x$ coordinate, $y$ coordinate of the affected location, for example, Bakura local goverment area of Zamfara State with $(78.68396, 0.01497)$ and $t$ - time, and $p$ is the rate of spread of disease.

Reaction is a response to something that involves taking action, or an action taken in response to something. A reactive measure could be vaccination, chemoprophylaxis, use of antibiotics, quarantine, isolation, public education or enlightenment. For the purpose of this study, the focus is on vaccination.

It is pertinent to know that without any intervention, the disease will continue to ravage the source community and even spread in to other localities. Once there is a reaction to curtail the spread, the reactive phase, which is both diffusion and control (vaccination) processes occur simultaneously. Thus, the mathematical model for this phase evolves as presented in equation (5).

$$\frac{\partial u}{\partial t} = p\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + q(u) \tag{3.2}$$

where $p$ is the rate of spread of the disease and $q$ is the rate (or quantity) of vaccination.

### 3.1 Case Study - Meningitis Outbreak of 2016 - 2017 in Zamfara State

Our case studty is Zamfara State of Nigeria with latitude $12.1667^0 N$ and longitude $6.5000^0 E$, having an area of $38,418$ square meters.


Figure 4: Zamfara Location on the map of Nigeria.Source: Google Search Engine

Between December 2016 and June 2017, a total of $14,280$ suspected meningitis cases were reported across 23 of the 36 states in Nigeria. The descriptive characterization of the $2016 - 2017$ $CSM$ outbreak in Nigeria as reported by Njidda et al 2021 were $1,145$ deaths ($CFR = 8$ % ) among suspected cases with Zamfara and Sokoto States been the epicenter of the outbreak. A total of $7,140$ suspected meningitis cases, out of which 553 deaths were reported in Zamfara State between $13th$ December, 2016 and $15th$ June, 2017 across Kaura Namoda, Bakura, Birnin Magagi, Bungudu, Gusua, Maradun, Tsafe, and Shinkafi local government areas [12]. Discretizing the region of some local government headquarters in Zamfara with Kaura Namoda as the centre establishes that the mesh generated is irregular mesh.

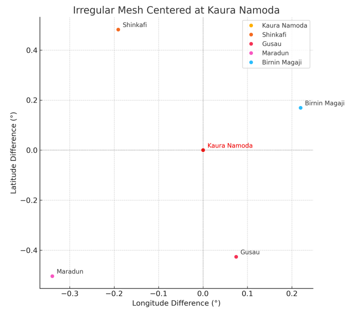


Figure 5: Irregular Mesh Centered At Kaura NamodaSource: Generated using Python code

# 4 Model Solution

The basic assumption for this model is that in epidemiological situations, data are generated. These data become the variables of interest which are not continuous but discrete. This is because time variable is always continuous but once time is divided into periods, it is then treated as a discrete quantity. From these scenarios, the solution would be cumbersome to be treated analytically. Thus, the numerical approach is adopted for the solution of the model by utilizing the finite difference representation for the data generated for the study which is a case of meningitis outbreak in Zamfara State between December 2016 and June 2017. The details are provided in Section 4.1.

There are two types finite difference methods, namely, finite difference with regular mesh and finite difference with irregular mesh. From the sketch of selected locations in the study area, see Figure 4, it is clear that the technique with irregular mesh is better fitted.

The following finite difference representations were made up of:

$$\frac{\partial u}{\partial t} = \frac{u_{i,j}^{(l+1)} - u_{i,j}^{(l)}}{k} \tag{4.1}$$

$$\frac{\partial^2 u}{\partial x^2} = b\left[h_2(u_{i+1,j}) - H^{12}(u_{i,j}) + h_1(u_{i-1,j})\right] \tag{4.2}$$

$$\frac{\partial^2 u}{\partial y^2} = d\left[g_2(u_{i,j+1}) - G^{12}(u_{i,j}) + g_1(u_{i,j-1})\right] \tag{4.3}$$

Where $h_1$, $h_2$, $g_1$, and $g_2$ are the step sizes on the $x$ and $y$ coordinates. $H^{12} = h_1 + h_2$; $G^{12} = g_1 + g_2$; $b = \frac{1}{aH^{12}}$; $a = h_1 \times h_2$; $d = \frac{1}{cG^{12}}$; $c = g_1 \times g_2$.

A total of 7,140 suspected meningitis cases and 553 deaths were reported in Zamfara State between 13*th* December, 2016 and 15*th* June, 2017. Given below is the number of suspected and confirmed cases.

Table 1: The number of Suspected cases and Confirmed cases according to [12]

| Onset month | Suspected cases | Confirmed cases |
|---|---|---|
| December | 17 | 0 |
| January | 180 | 9 |
| February | 340 | 38 |
| March | 2174 | 27 |
| April | 4031 | 51 |
| May | 353 | 33 |
| June | 45 | 27 |

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The table below contains the number of suspected cases, number of deaths and its rate in the selected *LGAs* in Zamfara state.

Table 2: Number of Death and its rate in some LGAs [12]

| LGA | Suspected cases | No of Deaths | Death Rate (%) |
|---|---|---|---|
| Kaura Namoda | 671 | 63 | 9.4 |
| Bakura | 555 | 62 | 11.2 |
| Birnin Magagi | 643 | 36 | 5.6 |
| Bungudu | 262 | 9 | 3.4 |
| Gusau | 857 | 48 | 5.6 |
| Maradun | 544 | 26 | 4.8 |
| Tsafe | 167 | 29 | 17.4 |
| Shinkafi | 1428 | 115 | 8.1 |
| Anka(Other) | 2013 | 165 | 8.2 |

The total sum of death recorded in Zamfara state according to [12] is 553. To curtail the spread of Meningitis outbreaks, governments and partner agencies should consider investing in targeted vaccination of the most vulnerable communities. However, sustainable control and elimination of meningitis outbreaks in the Africa meningitis belt may depend on the extension of the highly successful preventive MenAfriVac campaign framework to include multivalent conjugate meningococcal vaccine that covers the serogroups with the potential to cause outbreaks, [13] and [14].

Below is the table for the approximated values of suspected cases of each LGAs per month:

Table 3: Values of Suspected cases for each *LGAs* per month

| LGA | Dec. | Jan. | Feb. | Mar. | Apr. | May | Jun. | Total |
|---|---|---|---|---|---|---|---|---|
| Kaura Nanoda | 2 | 17 | 32 | 204 | 379 | 33 | 4 | 671 |
| Bakura | 1 | 14 | 26 | 169 | 313 | 28 | 4 | 555 |
| Birnin Magagi | 1 | 16 | 31 | 196 | 363 | 32 | 4 | 643 |
| Bugudun | 1 | 6 | 12 | 80 | 148 | 13 | 2 | 262 |
| Gusau | 2 | 22 | 41 | 261 | 484 | 42 | 5 | 857 |
| Maradun | 1 | 14 | 26 | 165 | 307 | 27 | 4 | 544 |
| Tsafe | 1 | 4 | 8 | 51 | 94 | 8 | 1 | 167 |
| Shinkafi | 3 | 36 | 68 | 435 | 806 | 71 | 9 | 1428 |
| Anka(Others) | 5 | 51 | 96 | 613 | 1137 | 99 | 12 | 2013 |
| Total | 17 | 180 | 340 | 2174 | 4031 | 353 | 45 | 7140 |

There is a need to know the distance between Kaura Namoda and a few selected local goverments, which are: Shinkafi, Maradun, Gusua, and Birnin Magagi. Using the population, the latitude, the longitude, the distance between Kaura Namoda and other LGAs, the difference in latitude and longitude of Kaura Namoda and other LGAs, the $x$ and $y$ coordinates of other LGAS using Kaura Namoda, the table below was generated.

The population, their respective latitude and longitude, of the selected LGAs in Zamfara State as at 2016 are given in Table 4.

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Table 4: The Population, latitude and longitude of The Selected $LGAs$ in Zamfara As At 2016

| LGA | Population | Lat. (N) | Long. (E) | Dist. Btw KN and other LGA(km) | Diff. in Lat. | Diff. in Long. |
|---|---|---|---|---|---|---|
| Kaura Namoda | $393,000$ | 12.5937 | 6.5865 | 0 | 0 | 0 |
| Bakura | $257,700$ | 12.6046 | 5.8765 | 78.69 | 0.0109 | 0.7100 |
| Birnin Magagi | $253,500$ | 12.3445 | 6.8659 | 147.75 | 0.2492 | 0.2794 |
| Bungudu | $356,200$ | 12.2653 | 6.5543 | 36.29 | 0.3284 | 0.0322 |
| Gusau | $528,400$ | 12.1702 | 6.6641 | 48 | 0.4235 | 0.0776 |
| Maradun | $285,800$ | 12.7137 | 6.2310 | 37.15 | 0.1200 | 0.3555 |
| Tsafe | $367,600$ | 11.9553 | 6.9182 | 79.48 | 0.6384 | 0.3317 |
| Shinkafi | $187,200$ | 13.0376 | 6.512 | 59.8 | 0.4439 | 0.0745 |
| Anka(Others) | $197,800$ | 12.1083 | 5.9333 | 89.49 | 0.4854 | 0.6532 |
| Total | $2,827,200$ | – | – | – | – | – |

With the information above, the $x$ and $y$ coordinates of the selected $LGAs$ were calculated. These values, the notation of the selected $LGAs$ and their suspected values at $l = 0(1)6$ is given below.

Table 5: The notation of the selected $LGAs$ and their suspected values at $l = 0(1)6$

| L.G.A | l=0 | l=1 | l=2 | l=3 | l=4 | l=5 | l=6 | g or h |
|---|---|---|---|---|---|---|---|---|
| $u_{i,j}$ | 2 | 17 | 32 | 204 | 379 | 33 | 4 | - |
| $u_{i+1,j}$ | 1 | 16 | 31 | 196 | 363 | 32 | 4 | $h_1 = 147.74824$ |
| $u_{i,j-1}$ | 2 | 22 | 41 | 261 | 484 | 42 | 5 | $g_2 = 0.35479$ |
| $u_{i-1,j}$ | 1 | 14 | 26 | 165 | 307 | 27 | 4 | $h_2 = 37.14928$ |
| $u_{i,j+1}$ | 3 | 36 | 68 | 435 | 806 | 71 | 9 | $g_1 = 0.4633$ |

## 5 Numerical Consideration

This section focuses on determination of the rate of spread and the efficacy of the control or treatment employed.

Crank & Nilcoson finte difference approximation is considered. [5] proposed a method that replaced both $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2}$ by the mean of their finite difference representations on the $(l)^{th}$ and $(l+1)^{th}$ time rows in the irregular mesh by

$$\frac{u_{i,j}^{l+1} - u_{i,j}^{l}}{k} = \frac{p}{2}\left[\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)_{i,j}^{l+1} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)_{i,j}^{l}\right]$$

$$\begin{aligned}\frac{u_{i,j}^{l+1} - u_{i,j}^{l}}{k} &= \frac{p}{2}\left[b\left(h_2(u_{i+1,j}^{l+1}) - H^{12}(u_{i,j}^{l+1}) + h_1(u_{i-1,j}^{l+1})\right) + d\left(g_2(u_{i,j+1}^{l+1}) - G^{12}(u_{i,j}^{l+1}) + g_1(u_{i,j-1}^{l+1})\right)\right] \\ &+ \frac{p}{2}\left[b\left(h_2(u_{i+1,j}^{l}) - H^{12}(u_{i,j}^{l}) + h_1(u_{i-1,j}^{l})\right) + d\left(g_2(u_{i,j+1}^{l}) - G^{12}(u_{i,j}^{l}) + g_1(u_{i,j-1}^{l})\right)\right]\end{aligned}$$

The suspected cases are reported monthly; hence $k = 1$.

$$\begin{aligned}(u_{i,j}^{l+1} - u_{i,j}^{l}) &= \frac{p_l}{2}\left[b\left(h_2(u_{i+1,j}^{l+1} + u_{i+1,j}^{l}) - H^{12}(u_{i,j}^{l+1} + u_{i,j}^{l}) + h_1(u_{i-1,j}^{l+1} + u_{i-1,j}^{l})\right)\right] \\ &+ \frac{p_l}{2}\left[d\left(g_2(u_{i,j+1}^{l+1} + u_{i,j+1}^{l}) - G^{12}(u_{i,j}^{l+1} + u_{i,j}^{l}) + g_1(u_{i,j-1}^{l+1} + u_{i,j-1}^{l})\right)\right]\end{aligned}$$

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The value of $p$ and $q$ are generated from the equation above and given below

$$p_l = \frac{2(u_{i,j}^{l+1} - u_{i,j}^l)}{b\begin{bmatrix} h_2(u_{i+1,j}^{l+1} + u_{i+1,j}^l) - H^{12}(u_{i,j}^{l+1} + u_{i,j}^l) + h_1(u_{i-1,j}^{l+1} + u_{i-1,j}^l) \\ +d(g_2(u_{i,j+1}^{l+1} + u_{i,j+1}^l) - G^{12}(u_{i,j}^{l+1} + u_{i,j}^l) + g_1(u_{i,j-1}^{l+1} + u_{i,j-1}^l)) \end{bmatrix}} \tag{5.1}$$

and

$$q_l = \frac{2(u_{i,j}^{l+1} - u_{i,j}^l) - p_l\begin{bmatrix} b(h_2(u_{i+1,j}^{l+1} + u_{i+1,j}^l) - H^{12}(u_{i,j}^{l+1} + u_{i,j}^l) + h_1(u_{i-1,j}^{l+1} + u_{i-1,j}^l)) \\ +d(g_2(u_{i,j+1}^{l+1} + u_{i,j+1}^l) - G^{12}(u_{i,j}^{l+1} + u_{i,j}^l) + g_1(u_{i,j-1}^{l+1} + u_{i,j-1}^l)) \end{bmatrix}}{u_{i,j}^l} \tag{5.2}$$

## 5.1 Numerical Results

Using equations (3) and (4), with the given values of other parameter we solve for $p_l$ and $q_l$

When $l = 0$

$$p_0 = \frac{2(u_{i,j}^1 - u_{i,j}^0)}{\begin{bmatrix} b\left(h_2(u_{i+1,j}^1 + u_{i+1,j}^0) - H^{12}(u_{i,j}^1 + u_{i,j}^0) + h_1(u_{i-1,j}^1 + u_{i-1,j}^0)\right) \\ +d\left(g_2(u_{i,j+1}^1 + u_{i,j+1}^0) - G^{12}(u_{i,j}^1 + u_{i,j}^0) + g_1(u_{i,j-1}^1 + u_{i,j-1}^0)\right) \end{bmatrix}}$$

$$p_0 = \frac{2(17-2)}{\begin{bmatrix} 9.85363 \times 10^{-7}\left(37.14928(16+1) - 184.89752(17+2) + 147.74824(14+1)\right) \\ +7.43663\left(0.35479(36+3) - 0.81809(17+2) + 0.4633(22+2)\right) \end{bmatrix}}$$

Further simplification, we have

$$p_0 = 0.428601$$

and

$$q_0 = \frac{2(u_{i,j}^1 - u_{i,j}^0) - p_0\begin{bmatrix} b(h_2(u_{i+1,j}^1 + u_{i+1,j}^0) - H^{12}(u_{i,j}^1 + u_{i,j}^0) + h_1(u_{i-1,j}^1 + u_{i-1,j}^0)) \\ +d(g_2(u_{i,j+1}^1 + u_{i,j+1}^0) - G^{12}(u_{i,j}^1 + u_{i,j}^0) + g_1(u_{i,j-1}^1 + u_{i,j-1}^0)) \end{bmatrix}}{u_{i,j}^0}$$

Where $(u_{i,j}^1 - u_{i,j}^0) = (17-2)$, $(u_{i+1,j}^1 + u_{i+1,j}^0) = (16+1)$,$(u_{i,j}^1 + u_{i,j}^0) = (17+2)$, $(u_{i-1,j}^1 + u_{i-1,j}^0) = (14+1)$,$(u_{i,j+1}^1 + u_{i,j+1}^0) = (36+3)$ and $(u_{i,j-1}^1 + u_{i,j-1}^0) = (22+2)$.

$$q_0 = \frac{2(17-2) - (0.4286)\begin{bmatrix} 9.85363 \times 10^{-7}(37.14928(17)) - 184.89752(19) + 147.74824(15) \\ +7.43663\left(0.35479(39) - 0.81809(19) + 0.4633(24)\right) \end{bmatrix}}{2}$$

Further simplification, we have

$$q_0 = 4.2 \times 10^{-5}$$

Similarly, the computations of $p_l$ and $q_l$ are continued for $l \geq 1$ and the solution is summarized in the table below

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Table 6: Values of $p_l$ and $q_l$

| $l$ | $p_l$ | $q_l$ |
|---|---|---|
| 0 | 0.428601 | $4.2 \times 10^{-5}$ |
| 1 | 0.15516 | $3.694062 \times 10^{-6}$ |
| 2 | 0.369157 | $-2.347865 \times 10^{-5}$ |
| 3 | 0.152557 | $-4.9395 \times 10^{-6}$ |
| 4 | -0.427256 | $2.196198 \times 10^{-7}$ |
| 5 | -0.392141 | $2.151515 \times 10^{-7}$ |

### 5.2 Remarks on Numerical Results

The required vaccine for the total population of the selected *LGAs* in Zamfara at the onset of the epidemic, if there was an immediate response at the end of December should be

$$\begin{aligned} Dosages &= 2827200 \times 500 \times 4.2 \times 10^{-5} \\ &= 59371.2 \end{aligned}$$

Where $0.5ml$ is the dosage of vaccine injection per patient, 2827200 was the total number of population of the selected *LGAs*. This way the necessary number of dosage if there was a prompt reaction as at the end of each month was calculated. The result is summarized in the table below

Table 7: The necessary number of dosage to be administered

| $l$ | Rate of spread | Rate of reaction | Proposed Dosage | Actual Vaccination Strategy |
|---|---|---|---|---|
| 0 | 0.428601 | $4.2 \times 10^{-5}$ | 59,371 | Not applicable |
| 1 | 0.15516 | $3.694062 \times 10^{-6}$ | 5,222 | No |
| 2 | 0.369157 | $-2.347865 \times 10^{-5}$ | 0 | No |
| 3 | 0.152557 | $-4.9395 \times 10^{-6}$ | 0 | No |
| 4 | -0.427256 | $2.196198 \times 10^{-7}$ | 310 | vaccination* |
| 5 | -0.392141 | $2.151515 \times 10^{-7}$ | 304 | vaccination* |

* Dosage not specified.

These dosages are the minimum required to curb the spread of the outbreak. The negative sign signifies a state of in-activeness or no need for reaction.

## 6 Conclusion

The exercise of controlling meningitis outbreak has been shown to comprise the estimation of the spread and as well as the inclusion of a control strategy. Therefore a Diffusion - reaction model was proposed and it is observed that this model involved the use of data through a numerical approach to estimate the parameters for spread of the disease and the effectiveness of the control measure.

The proposed model is also suitable even for any infectious disease.

## 7 Recommendation

The inclusion of numerical scheme to solve diffusion-reaction model which is data driven is novel and further research is recommended for a better, sufficient, accurate, and more robust data driven - numerical solution to both industrial and health-wise problem.

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