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Dufour and Soret Effects on Convective Flow of Chemically-Reacting Fluid with Heat Generation, Variable Surface Temperature and Concentration

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Abstract

Combined effects of Dufour, Soret, dissipation, thermal radiation on heat and mass transfer of free convective flow of a viscous incompressible, chemically-reacting and electrically conducting fluid past an impulsively moving vertical plate adjacent to a non-Darcy porous regime in the presence of heat generation, variable surface temperature and concentration conditions have been investigated. The governing non-linear partial differential equations of the flow field were solved using the implicit Crank-Nicolson finite difference method. The velocity, temperature and concentration profiles were studied for different values of thermophysical parameters such as thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, Dufour number, Soret number, and chemical reacting parameter. Also, numerical values of Skin friction coefficient, local Nusselt number and Sherwood number were computed and analyzed.

Keywords: Dufour (Du), Soret (Sr), Convective flow, Chemically Reacting fluid, Dissipation, Thermal radiation, MHD, Heat and mass transfer, Heat Generation, Variable Surface Temperature and Concentration.
MSC2010: 35Q20, 32W30.

1 Introduction

The processes involving high temperatures, radiation, conduction, convection, and mass transfer play a very important role in the design of pertinent equipment involving nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. Because of this, much relevant research has been carried out by many researchers. Among them, Chamkha

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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[1] studied non-Darcy fully developed mixed convection in a porous medium channel with heat generation/absorption and hydromagnetic effects. Chamka et al [2] studied radiation effects on the free convection flow past a semi-infinite vertical plate with mass transfer. Mahajan and Gebhart [3] reported the influence of viscous heating dissipation in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Prasad and Reddy [4] considered the radiation and mass transfer effects on a two-dimensional flow past an impulsively started isothermal vertical plate. Shanker et al [5] studied radiation and mass transfer effects on unsteady magneto-hydrodynamics (MHD) free convective fluid flow embedded in a porous medium with heat generation/absorption.

Radiation and Darcy effects on unsteady MHD heat and mass transfer flow of a chemically reacting fluid past an impulsively started vertical plate with heat generation were discussed by Suneetha and Bhaskar [6]. Muthucumarawamy and Kumar [7] investigated heat and mass transfer effects on moving vertical plates in the presence of thermal radiation. Latha et al [8] studied finite difference analysis on an unsteady mixed convection flow past a semi-infinite vertical permeable moving plate in the presence of radiation and viscous dissipation.

Gangadhar and Reddy [9] investigated chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction. Ghosh and Beg [10] presented a theoretical analysis of radiative effects on transient free convective heat transfer past a hot vertical surface in porous media.

The interaction of radiation with hydromagnetic flow has become industrially more prominent wherever high temperatures occur. Due to this fact, Takhar et al [11] analyzed the radiation effects on MHD free convection flow past a semi-infinite vertical plate using the Runge-Kutta Merson quadrature. Chaudhary et al [12] studied the radiation effect with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface with Ohmic heating. Prasad et al [13] studied the transient radiative hydromagnetic free convection flow past an impulsively started vertical plate with variable heat and mass flux. Viscous mechanical dissipation is very important in geophysical flows and also in certain industrial operations and is usually characterized by Eckert number. In view of this, Suneetha et al [14] studied thermal radiation effects on MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. Ramachandra et al [15] reported radiation and mass transfer effects on unsteady MHD-free convection flow past a heated vertical plate in a porous Darcy medium with viscous dissipation.

The influence of viscous dissipation and radiation on unsteady MHD-free convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction was studied by Israel-Cookey et al [16]. Mohiddin et al [17] investigated MHD free convection boundary layer flow past a vertical cone with uniform heat and mass flux. Zuecco [18] used the Network Simulation Method (NSM) to study the effects of viscous dissipation and radiation on unsteady MHD free convection flow past a vertical porous plate. In Rajput et al [19] radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer were examined. Rajput and Kumar [20] also investigated the rotation effect on MHD flow past an impulsively started vertical plate with variable mass diffusion while Idowu and Falodun [21] investigated Soret and Dufour effects on MHD heat and mass transfer of Walter's-B viscoelastic fluid over a semi-infinite vertical plate using spectral relaxation analysis. The influence of Dufour and Soret on unsteady MHD convective heat and mass transfer flow in non-Darcy porous medium was investigated by Gbadeyan et al. [22]. Also, the collective effects of dissipation, radiation, Dufour and soret on heat and mass transfer of chemically reacting fluid with heat generation were studied in [23]. Ebiwareme et al. [24] used Laplace Adomian Decomposition Method to investigate the effects of Soret and Dufour on chemically reacting free convective fluid flowing over a vertical plate along with viscous dissipation. Also the impact of Soret, Dufour and radiation on Laminar Flow of a rotating liquid past a porous plate via chemical reaction was studied by Kumar et al. [?].

The aim of this study is to address critical gaps in existing research by examining the combined effects of Dufour, Soret, dissipation and thermal radiation on heat and mass transfer of free convective flow of a Viscous incompressible chemically-reacting and electrically conducting fluid past an

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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impulsively started vertical plate embedded in a non-Darcy porous regime in the presence of heat generation with variable surface temperature and concentration conditions. The study has applications in the design of pertinent equipment in areas such as nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles.

2 Problem Formulation

A two-dimensional unsteady laminar natural flow of a viscous incompressible chemically reacting, conducting, radiating, dissipating and electrically-conducting fluid past an impulsively started moving vertical plate embedded in a porous regime in the presence of heat generation with variable surface temperature and concentration is studied. The fluid is hydromagnetic and assumed to be absorbing-emitting, grey, and non-scattering.

The x'- axis is taken along the plate in the upward direction and the y'- axis is taken normal to it. At time $t' \leq 0$, it is assumed that the plate and the fluid are at the same temperature T'_∞ and concentration level C'_∞ everywhere in the fluid. At time $t' > 0$, the plate starts moving impulsively in the vertical direction with constant velocity u_0 against the gravitational field. Also at $t' > 0$, it is assumed that the variable temperature and concentration near the plate are raised to $T' = T'_\infty + (T'_w - T'_\infty)x^m$ and $C' = C'_\infty + (C'_w - C'_\infty)x^n$ respectively and are maintained constantly thereafter. In the energy equation, radiative heat flux is analysed using the Rosseland diffusion approximation. The geometry of the model is depicted in Figure 1.

Figure 1: Geometry of the flow model

Following Brewster [26] the Rosseland diffusion flux is given as

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T'^4}{\partial y'} \tag{2.1}$$

where σ_s is the Stefan-Boltzmann constant while k_e is the mean absorption coefficient. Temperature differences within the flow are sufficiently small, so Equation (1) can be linearized by expanding T'^4 into Taylor's series about T'_∞ assumed to be (after neglecting higher-order terms), given as

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{2.2}$$

In view of Equation (2), Equation (1) becomes

$$q_r = -\frac{16\sigma_s T'^3_\infty}{3k_e}\frac{\partial T'}{\partial y'} \tag{2.3}$$

Under the usual Bousinessq's approximation, the basic boundary layer equations of the model are given [4,14] as:

Continuity Equation

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{2.4}$$

Momentum Equation

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu'\frac{\partial^2 u'}{\partial y'^2} - \frac{u'}{k'}(\nu + bu') - \frac{\sigma B_0^2}{\rho}u' \tag{2.5}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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Energy Equation

$$\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\alpha\frac{\partial^2 T'}{\partial y'^2}+\frac{\nu}{C_p}[\frac{\partial u'}{\partial y'}]^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'}+\frac{Q_0}{\rho C_p}(T'-T'_\infty)+\frac{D_m k_T}{\rho C_p}\frac{\partial^2 C'}{\partial y'^2} \tag{2.6}$$

Now using Equation (3), Equation (6) reduces to

$$\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\alpha\left(1+\frac{16\sigma_s T'^3_\infty}{3k_e\rho C_p}\right)\frac{\partial^2 T'}{\partial y'^2}+\frac{Q_0}{\rho C_p}(T'-T'_\infty)$$
$$+\frac{D_m k_T}{\rho C_p}\frac{\partial^2 C'}{\partial y'^2} \tag{2.7}$$

Species Equation

$$\frac{\partial C'}{\partial t'}+u'\frac{\partial C'}{\partial x'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2}-K_1(C'-C'_\infty) \tag{2.8}$$

where u', v' are the fluid velocity components along x', y' directions respectively, t' is the time, g the acceleration due to gravity, β the volumetric coefficient of thermal expansion, β^* the volumetric coefficient expansion of concentration, T' the temperature of the fluid in the boundary layer, C' the species concentration in the boundary layer, T'_w the temperature at the wall temperature layer, T'_∞ the free stream temperature far away from the plate, C'_w the concentration at the wall, C'_∞ the free stream concentration in the fluid far away from the plate, k' the permeability of the porous medium, B_0 magnetic induction, ν the kinematic viscosity of the grey fluid, α thermal diffusivity, ρ the density of the fluid, C_p the specific heat at constant pressure, q_r the radiation heat flux, Q_0 the heat generation/absorption constant, D the species diffusion coefficient and K'_1 the chemical reaction parameter, and k_T thermal diffusion ratio. Equation (9) is the corresponding initial and boundary conditions of the model given as follows:

$$\begin{aligned}
&t'\le 0: \quad u'=0,\quad v'=0,\quad T'=T'_\infty,\quad C'=C'_\infty \quad \text{for all} \quad x'\ge 0\\
&t'>0: \quad u'=u_0,\quad v'=0,\quad T'=T'_\infty+(T'_w-T'_\infty)x^m,\quad C'=C'_\infty+(C'_w-C'_\infty)x^n \quad \text{at} \quad y'=0\\
&\quad u'=0,\quad v'=0,\quad T'=T'_\infty,\quad C'=C'_\infty \quad \text{as} \quad x'=0,\\
&\quad u'=0,\quad T'=T'_\infty,\quad C'=C'_\infty \quad \text{as} \quad y'=\infty
\end{aligned} \tag{2.9}$$

Equations (4), (5), (7), (8) and the boundary and initial conditions, Equation (9), are transformed into a dimensionless form using the dimensionless quantities in Equation (10) [4,14].

$$\begin{aligned}
&X=\frac{x'u_0}{\nu},\quad Y=\frac{y'u_0}{\nu},\quad U=\frac{u'}{u_0},\quad V=\frac{v'}{u_0},\quad t=\frac{t'u_0^2}{\nu},\\
&T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\quad C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\quad Pr=\frac{\nu}{\alpha},\quad Sc=\frac{\nu}{D},\quad Re=\frac{u_0L}{\nu},\\
&Da=\frac{k'}{L^2},\quad Fs=\frac{b}{L},\quad Gr=\frac{g\beta\nu(T'_w-T'_\infty)}{u_0^3},\quad Gm=\frac{g\beta^*\nu(C'_w-C'_\infty)}{u_0^3},\\
&M=\frac{\sigma B_0^2\nu}{\rho u_0^2},\quad Ec=\frac{u_0^2}{C_p(T'_w-T'_\infty)},\quad N=\frac{16\sigma_s T'^3_\infty}{3k_e k},\quad Q=\frac{Q_0\nu^2}{\rho C_p u_0^2},\\
&K=\frac{K_1\nu}{u_0^2},\quad Sr=\frac{D_m k_T(T'_w-T'_\infty)}{\nu\rho C_p(C'_w-C'_\infty)},\quad Du=\frac{D_m k_T(C'_w-C'_\infty)}{\rho C_p\nu(T'_w-T'_\infty)}
\end{aligned} \tag{2.10}$$

Now, Equations (4), (5), (7), (8) and (9) are transformed to dimensionless form as given in Equations (11), (12), (13), (14) and (15).

International Journal of Mathematical Sciences and Optimization: Theory and Applications
12(1), 2026, Pages 32 - 43
https://doi.org/10.5281/zenodo.20384913

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2.11}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = Gr\,T + Gm\,C + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{R_e^2\,D_a} - \frac{F_s}{D_a R_e}U^2 - MU \tag{2.12}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = E_c\left[\frac{\delta U}{\delta Y}\right]^2 + \frac{1}{Pr}\left[1 + \frac{4}{3N}\right]\frac{\partial^2 T}{\partial Y^2} + QT + D_u\frac{\partial^2 C}{\partial Y^2} \tag{2.13}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} + Sr\frac{\partial^2 T}{\partial Y^2} - KC \tag{2.14}$$

$$\begin{aligned} t \le 0: &\quad U = 0, \quad V = 0, \quad T = 0, \quad C = 0 \quad \text{for all} \quad X, Y \\ t > 0: &\quad U = 1, \quad V = 0, \quad T = X^m, \quad C = X^n \quad \text{at} \quad Y = 0 \\ &\quad U \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty. \end{aligned} \tag{2.15}$$

where X and Y are dimensionless coordinates, U and V are dimensionless velocities, t is dimensionless time, T is the dimensionless temperature function, C is the dimensionless concentration function, N is the conduction radiation heat transfer parameter, Pr = Prandtl number, Sc = Schmidt number, R_e = Reynold number, Da = Darcy number, F_s = Forchheimer (non-Darcy) inertia numbers, Gr = thermal Grasshof number, Gm = species Grasshof number, M = magnetic field parameter, Ec = Eckert number, N = radiation parameter, Q = dimensionless heat generation/arbsorption coefficient, K = chemical reaction parameter, Sr = Soret number, and Du = Dufour number.

Following Modest [27], the local values of Skin friction, Sherwood number and Nusselt number are respectively, given as:

$$\tau_x = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{2.16}$$

$$Sh_x = -X\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{2.17}$$

$$Nu_x = -X\left(\frac{\partial T}{\partial Y}\right)_{Y=0} \tag{2.18}$$

and their corresponding average values are defined as

$$\overline{\tau_x} = -\int_0^1 \tau_x\,dX \tag{2.19}$$

$$\overline{Sh_x} = -\int_0^1 Sh_x\,dX \tag{2.20}$$

$$\overline{Nu_x} = -\int_0^1 Nu_x\,dX \tag{2.21}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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3 Method of Solution

To solve the unsteady coupled partial differential Equations (11), (12), (13) and (14) with the corresponding boundary conditions, Equation (15), we employed the implicit Crank-Nicolson finite difference scheme.

4 Solution to the Problem

In this study, the region of integration is considered as a rectangle with $X_{max} = 1$ and $Y_{max} = 14$, where $Y_{max} = 14$ corresponds to $Y = \infty$, which lies very well outside the momentum energy and concentration boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations so that two of the boundary conditions of Equation (15) are satisfied. Here, the subscript i designates the grid point along the X- direction, j along the Y- direction and the superscript n along the t-direction. The appropriate mesh sizes considered for the calculation are $\Delta X = 0.05$, $\Delta Y = 0.25$, and the time step is taken as $\Delta t = 0.01$. The finite difference equations corresponding to the transient coupled non-linear partial differential Equations (11), (12), (13) and (14) are given as follows

$$\frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n} + U_{i,j-1}^{n+1} - U_{i-1,j-1}^{n+1} + U_{i,j-1}^{n} - U_{i-1,j-1}^{n}}{4\Delta X} + \frac{V_{i,j}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j}^{n} - V_{i,j-1}^{n}}{2\Delta Y} = 0 \tag{4.1}$$

$$\frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta t} + \frac{U_{i,j}^{n}}{2(\Delta X)}\left[U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n}\right] + \frac{V_{i,j}^{n}}{4(\Delta Y)}\left[U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^{n} - U_{i,j-1}^{n}\right]$$
$$= \frac{Gr}{2}\left[T_{i,j}^{n+1} + T_{i,j}^{n}\right] + \frac{Gm}{2}\left[C_{i,j}^{n+1} + C_{i,j}^{n}\right] + \frac{U_{i,j-1}^{n+1} - 2U_{i,j}^{n+1} + U_{i,j+1}^{n+1} + U_{i,j-1}^{n} - 2U_{i,j}^{n} - U_{i,j-1}^{n}}{2(\Delta Y)^2} -$$
$$\frac{1}{2Re^2 Da}\left[U_{i,j}^{n+1} + U_{i,j}^{n}\right] - \frac{F_s}{2ReDa}U_{i,j}^{n}\left[U_{i,j}^{n+1} + U_{i,j}^{n}\right] - \frac{M}{2}\left[U_{i,j}^{n+1} + U_{i,j}^{n}\right] \tag{4.2}$$

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} + U_{i,j}^{n}\frac{T_{i,j}^{n+1} - T_{i-1,j}^{n+1} + T_{i,j}^{n} - T_{i-1,j}^{n}}{2(\Delta X)} + V_{i,j}^{n}\frac{T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1} + T_{i,j+1}^{n} - T_{i,j-1}^{n}}{4\Delta Y}$$
$$= E_c\left[\frac{U_{i,j+1}^{n} - U_{i,j}^{n}}{2(\Delta Y)}\right]^2 + \frac{1}{Pr}\left[1 + \frac{4}{3N}\right]\frac{T_{i,j+1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j-1}^{n+1} + T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{2(\Delta Y)^2}$$
$$+ \frac{Q}{2}\left[T_{i,j}^{n+1} + T_{i,j}^{n}\right] + Du\frac{C_{i+1,j}^{n+1} - 2C_{i,j}^{n+1} + C_{i-1,j}^{n+1} + C_{i+1,j}^{n} - 2C_{i,j}^{n} + C_{i-1,j}^{n}}{2(\Delta Y)^2} \tag{4.3}$$

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} + U_{i,j}^{n}\frac{C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^{n} - C_{i-1,j}^{n}}{2(\Delta X)} + V_{i,j}^{n}\frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j+1}^{n} - C_{i,j-1}^{n}}{4(\Delta Y)}$$
$$= \frac{1}{Sc}\left[C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^{n+1} + C_{i,j+1}^{n} - 2C_{i,j}^{n} + C_{i,j-1}^{n}\right]$$
$$+ Sr\frac{T_{i,j+1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j-1}^{n+1} + T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{(2(\Delta Y)^2} - \frac{K}{2}\left[C_{i,j}^{n+1} + C_{i,j}^{n}\right] \tag{4.4}$$

The finite difference Equations (22), (23), (24) and (25) are now written in tri-diagonal matrix system as follows

$$-V_{i,j-1}^{n+1} + V_{i,j}^{n+1} = \beta_1 \tag{4.5}$$

$$-A_2U_{i,j-1}^{n+1} + B_2U_{i,j}^{n+1} + D_2U_{i,j+1}^{n+1} = \beta_2 \tag{4.6}$$

$$A_3T_{i,j-1}^{n+1} + B_3T_{i,j}^{n+1} + D_3T_{i,j+1}^{n+1} = \beta_3 \tag{4.7}$$

$$A_4C_{i,j-1}^{n+1} + B_4C_{i,j}^{n+1} + D_4C_{i,j+1}^{n+1} = \beta_4 \tag{4.8}$$

During any one-time step, the coefficients $U_{i,j}^n$ and $V_{i,j}^n$ appearing in the difference equations are treated as constants. The values of C, T, U and V at time level $n+1$ using the known values at previous time level n are calculated as follows:
The finite difference Equation (26) at every internal nodal point on a particular i-level constitutes a tri-diagonal system of equations which is solved by using the Thomas algorithm as discussed in Carnahan [28], as well as Gbadeyan et al [23]. Hence the values of C are known at every internal nodal point on a particular i at $(n+1)^{th}$ time level. Similarly, the values of T are calculated from Equation (25). Using the values of C and T at $(n+1)^{th}$ time level in Equation (24), the values of U at $(n+1)^{th}$ time level is found similarly. Then the values of V are calculated explicitly using Equation (23) at every particular i-level at $(n+1)^{th}$ time level. This process is repeated for various i-levels. Thus, the values of C, T, U and V are known at all grid points in the rectangular region at $(n+1)^{th}$ time level.
The truncation error in the finite difference approximation is of $0(\Delta t^2 + \Delta X + \Delta Y^2)$ and it tends to zero as Δt, ΔX, and ΔY tend to zero. Hence the scheme is compatible and unconditionally stable [15]. Hence, stability and compatibility ensure the convergence of the scheme.

5 Results and Discussion

In order to get a physical insight into the problem various computations have been performed to study the effects of the controlling thermofluid and hydrodynamics parameters (such as Prandtl number (Pr), Schmidt number (Sc), Thermal Grashof number (Gr), Species Grashof number (Gm), etc) on the dimensionless velocity (U), temperature (T) and concentration (C) profiles. These results are shown in Figures 2 - 13. To ascertain the accuracy of the method used in this study, results from the present study are compared with those from previous study [14] as shown in Figure 2. It is observed that the present results are in good agreement with the previous study. Default values of the thermophysical parameters are specified as follows: $Pr = 0.71$ (Air), $Sc = 0.6$ (Hydrogen), $N = 3.0$, $M = 1.0$, $Gr = 10$, $Gm = 10$, $K = 0.5$, and $t = 0.4$. All graphs therefore correspond to this value unless otherwise indicated.
Figure 3 represents the velocity profile due to variations in Gr, Gm, and Sc. From the figure, it is observed that the velocity accelerates due to enhancement in the thermal buoyancy force (Gr) which shows free convection effects. The solutal Grashof number (Gm) is the ratio of the species buoyancy force to the viscous hydrodynamic force. It is observed in the figure that the velocity increases with a considerable rise in the species buoyancy force (Gm). In both cases observed, the outcome demonstrate that as thermal Grashof number (Gr) or Species Grashof number (Gm) increases there is a rapid rise in the velocity near the surface of the vertical plate which then descends smoothly to the free stream velocity. This corroborates the findings of Gbadeyan et al. [23]. It is also observed in Figure 3 that velocity increases as Schmidt's number increases.
Figures 4 and 5 illustrate the effects of Dufour and Soret on velocity profiles. In Figure 4, it is observed that as Dufour number (Du) increases, the velocity decreases. Similarly in Figure 5 as Soret number (Sr) increases, velocity is found to decrease. Hence, an increase in both Dufour number and Soret number brings about a decrease in the velocity profile which supports the conclusion of Idowu et al. [21].

International Journal of Mathematical Sciences and Optimization: Theory and Applications
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The effect of the radiation parameter (N) on the velocity is shown in Figure 6. N is the relative contribution of conduction heat transfer to thermal radiation transfer. As N increases, a considerable increase is observed in the velocity profile. Figure 7 depicts the combined effects of Gr, Gm, N on temperature profile and it is observed that an increase in Gr, Gm, N respectively brings about an appreciable increase in the temperature. The effect of Dufour on temperature is considered in Figure 8 and it is found that as Dufour number increases, the temperature profile decreases.

The effects of temperature exponent (m), Concentration exponent (n) and Eckert number (Ec) on temperature are displayed in Figure 9. An increase in the values of m or n leads to an increase in temperature. As Ec decreases, temperature increases. Figure 10 depicts the effect of Soret (Sr) on the Concentration profile. As Sr increases, Concentration is found to increase. Figure 11 shows the effects of temperature exponent (m), Concentration exponent (n) and Sc on Concentration. Concentration increases with an increase in m or n or Sc. The effects of the magnetic parameter (M) and Prandtl number (Pr) on temperature are illustrated in Figure 12. The application of a transverse magnetic field to an electrically conducting flow gives rise to a resistive type of force known as Lorentz force. This force tends to slow down the motion of fluid in the boundary layer and increase its temperature. It is observed that as M increases, temperature increases. Prandtl number (Pr) is the ratio of momentum diffusivity to thermal diffusivity. It is observed that the temperature decreases as Pr increases which agrees with the result of Sobamowo et al [29].

Figure 13 shows the effects of M, N and Sr on concentration profiles and it is observed that an increase in M, N, and Sr brings about a considerable increase in concentration profiles.

Figure 2: Effect of Thermal Grashof number (Gr), Schmidt number (Sc), and Radiation parameter (N) on the velocity profile

Figure 3: Velocity profile for various Thermal Grashof number (Gr), Solutal Grashof number (Gm) and Schmidt number (Sc)

Figure 4: Effect of Dufour number (Du) on velocity profile

Figure 5: Effect of Soret number (Sr) on velocity profile

Figure 6: Effect of Radiation parameter (N) on Velocity profile

Figure 7: Effect of Thermal Grashof number (Gr), Solutal Grashof number (Gm), and Radiation parameter (N) on the Temperature profile

Figure 8: Effect of Dufour on the Temperature profile

Figure 9: Effect of Different values of Temperature Exponent (m), Concentration Exponent (n) and Eckert number (Ec) on temperature profile

Figure 10: Effect of Soret number (Sr) on Concentration profile

Figure 11: Effect of Temperature exponent (m) Concentration exponent (n) and Schmidt number (Sc) on Concentration profile

Figure 12: Effect of Magnetic parameter (M) and Prandtl number (Pr) on temperature profile

Figure 13: Effect of Magnetic parameter (M), Radiation parameter (N) and Soret number (Sr) on concentration (C) profile

Table 1 shows the effects of various values of pertinent flow parameters on local skin friction, local Nusselt number and local Sherwood number. It is shown in Table 1 that the buoyancy parameter

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has a great impact on the fluid flow behaviour. Also, it is observed that an increase in thermal buoyancy greatly affects the hydrodynamic boundary layer by causing an increase in the local skin friction. The impact of the thermal Grashof number on the local Nusselt and Sherwood numbers are negligible. This is due to the fact that the thermal buoyancy effect influences the hydrodynamic boundary layer and the local skin friction alone.

Table 1: Effects of various values of pertinent flow parameters on local Skin friction, local Nusselt number and local Sherwood number

Gr	Fs	Ec	M	K	Gm	Pr	Da	Sc	N	Du	Sr	Cf	Nu_x	Sh_x
0.2	2.0	0.1	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.432802	0.693582	0.863351
0.4	2.0	0.1	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	2.798571	0.693582	0.863351
0.6	2.0	0.1	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	4.164340	0.693582	0.863351
2.0	0.0	0.1	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	0.478450	0.358210	1.727901
2.0	0.5	0.1	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	0.300001	0.358210	1.566720
2.0	1.0	0.1	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	0.261026	0.358210	1.426828
2.0	2.0	0.3	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.452494	0.667991	0.413351
2.0	2.0	0.5	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.472286	0.667991	0.413351
2.0	2.0	0.7	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.492078	0.616552	0.413351
2.0	2.0	0.1	5.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.711398	0.819928	0.869715
2.0	2.0	0.1	10.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.548293	0.819928	0.869715
2.0	2.0	0.1	15.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.299002	0.819928	0.869715
2.0	2.0	0.1	1.0	0.0	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.479274	0.965382	0.802305
2.0	2.0	0.1	1.0	0.5	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.402758	0.965382	0.802305
2.0	2.0	0.1	1.0	1.0	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.293709	0.965382	0.802305
2.0	2.0	0.1	1.0	0.3	0.0	0.71	0.6	0.61	0.5	3.0	2.0	2.034923	0.299182	0.796815
2.0	2.0	0.1	1.0	0.3	0.5	0.71	0.6	0.61	0.5	3.0	2.0	3.841286	0.299182	0.796815
2.0	2.0	0.1	1.0	0.3	1.0	0.71	0.6	0.61	0.5	3.0	2.0	5.647649	0.299182	0.796815
2.0	2.0	0.1	1.0	0.3	2.0	1.0	0.6	0.61	0.5	3.0	2.0	1.307347	0.803890	0.861241
2.0	2.0	0.1	1.0	0.3	2.0	2.0	0.6	0.61	0.5	3.0	2.0	0.946103	1.591925	0.861241
2.0	2.0	0.1	1.0	0.3	2.0	7.0	0.6	0.61	0.5	3.0	2.0	0.737310	3.177160	0.861241
2.0	2.0	0.1	1.0	0.3	2.0	0.71	0.1	0.61	0.5	3.0	2.0	2.798571	0.798218	0.179684
2.0	2.0	0.1	1.0	0.3	2.0	0.71	0.2	0.61	0.5	3.0	2.0	5.530110	0.798218	0.179684
2.0	2.0	0.1	1.0	0.3	2.0	0.71	0.3	0.61	0.5	3.0	2.0	8.261649	0.798218	0.179684
2.0	2.0	0.1	1.0	0.3	2.0	0.71	0.6	0.28	0.5	3.0	2.0	1.724911	0.582991	0.655337
2.0	2.0	0.1	1.0	0.3	2.0	0.71	0.6	0.38	0.5	3.0	2.0	1.607209	0.582991	0.716995
2.0	2.0	0.1	1.0	0.3	2.0	0.71	0.6	0.61	0.5	3.0	2.0	1.426589	0.582991	0.870145

6 Conclusion

The study examined the effects of thermal radiation, dissipation, Dufour, Soret, heat and mass transfer of free convective flow of a viscous incompressible electrically conducting and chemically reacting fluid past an impulsively started moving vertical plate adjacent to a non-Darcy porous regime in the presence of heat generation, surface variable temperature and concentration. Rosseland diffusion on flux model was used to simulate the radiative heat flux [26].
From the study, it can be concluded that:

(i) As the value of each of Gr, Gm and Sc increases respectively, the velocity profile of the fluid increases.

(ii) An increase in the value of each of Du and Sr respectively, results in a decrease in velocity profile.

(iii) As the value of N increases, a considerable increase is observed in the velocity profile.

(iv) An increase in the value of Gr, Gm and N respectively, increased the temperature profile.

(v) An increase in the value of Du brought about a decrease in temperature profile.

(vi) An increase in the value of m or n leads to an increase in temperature while a decrease in the value of Ec results in an increase in temperature.

(vii) Concentration increased as each of the values of m or n or Sc increased.

(viii) Temperature increased as the value of M increased while temperature decreased when the value of Pr increased.

(ix) Increase in each of the values of M, N and Sr brought about an increase in both velocity and concentration profiles.

(x) The impact of the thermal Grashof number on local Nusselt and Sherwood numbers is negligible because the thermal buoyancy effect influences the hydrodynamic boundary layer and skin friction alone.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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