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# Klein-4 Group of Constructing Distinct Sets of Mutually Orthogonal Latin Squares of Order

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**Abstract**

The concept of mutually orthogonal Latin squares (MOLS) have been studied extensively since Euler's pioneering work in 1782. The construction of MOLS of various orders have since been the subject of considerable research. Researchers have utilized different algebraic structures to construct MOLS, while some emphasized on matrices and maximal partial spreads in finite projective spaces. However, the basis of the aforementioned methods were the provision of systematic approach to ensure orthogonality. A new method of constructing distinct set of mutually orthogonal Latin squares (MOLS) of order 4 called Klein-4 Group is presented in this paper. Therein the symmetries of rectangle (isomorphic to Klein-4 group $C_2 \times C_2$) is leveraged on to aid the construction of 4 distinct complete sets of MOLS. The design could open ways for its application in cryptography, generation of one time password (OPT) and in cyber environments such as Internet of Things (IoT) needing lightweight cybersecurity measures.

**Keywords:** Latin squares, Mutually orthogonal latin squares, Symmetric group, Klein-4 group.
**MSC2010: 62K10, 05B15, 20B30, 20B35.**

## 1 Introduction

The concepts of mutually orthogonal Latin squares (MOLS) have been studied extensively since Euler's pioneering work Euler [1] with the problem of arrangement of 6 regiments, 6 officers with 6 distinct ranks in a square of order 6 with an anticipation that no rank or regiment be repeated in a row or a column. Bose et al. [2] disproved Euler's conjecture for orders greater than 6 and demonstrated the existence of orthogonal Latin squares for various orders and opened way for further studies however, Euler's conjecture still holds for order $n = 2, 6$. The existence and construction of MOLS of various orders have since been a subject of considerable mathematical research. Jungnickel [3] utilized algebraic structures such as finite fields and groups to construct MOLS,





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emphasizing that the methods often involve difference matrices and maximal partial spreads in finite projective spaces and also provides a systematic approach to ensure orthogonality. Recent studies have further explored the characterization of symmetric groups and semigroups Edeghagha et al. [4] and Imam et al. [5], as well as the algebraic properties of quasigroups which are fundamentally linked to Latin squares constructions. Also, block designs and projective planes are integral in constructing MOLS. These designs are instrumental in ensuring that each pair of Latin squares meets the orthogonality criterion. Colbourn et al. [6] provides extensive methodologies for constructing and analyzing combinatorial designs, including MOLS. Importantly, Stinson [7] investigated modern computational methods involving algorithmic searches and exhaustively checking for the construction of MOLS. These approaches leverage on advanced computational power to verify orthogonality. Another important breakthrough on the existence and construction of MOLS was a generalized method of constructing Mutually orthogonal latin squares (MOLS) of the prime orders by Saka et al. [8], by generating functions defined on the finite fields. This method however, provides the opportunity for deep research into the structural properties of MOLS. Detailed examples were provided for better illustrations. Saka et al. [9] also developed a method of constructing mutually orthogonal Latin squares for prime order via the cyclic subgroup of the symmetric group $S_k$ with a generator as permutations of the 2-permutation of generating set of the Dihedral group $D_k$ and 2-permutation generating set of $S_k$. The method is efficient for any prime order.

Moore [10] obtained the maximum cardinality of MOLS of order n is and that this upper bound is achieved if and only if n is a prime power. Pachamuthua [11] introduced the construction of MOLS through (22) and (32) by Galois Field theory with definitions and parameter relationships of Balanced Incomplete Block Design. Bose et al. [2] and Bose et al. [12] attended to the problem of searching for the sets of mutually orthogonal Latin squares of $n = 4t$, where $4t - 1$ is a prime power with orthogonal mappings module $G(2, 2t)$. The search produced orthogonal mappings that simplified the process of attaining balanced incomplete block with design parameters $v = 4t - 1 = b, r = k = 2t - 1, X = t - 1$. The projective planes was studied with different equivalent notion on MOLS by Owens [13]. In twenty century, Parker [14] demonstrated and prove the existence of $N(35) \geq 5$. This development was indeed a breakthrough to disproof Euler's conjecture on orthogonal Latin squares, particularly in relation to non-prime power orders like 35. The resolvable incomplete block designs of $v = p^2$ t with $p$ being a prime number and prime power were constructed byKhare et al. [15] and Hinkelmann et al. [16] respectively by leveraging on the successive diagonalizing algorithm. Saka [17] observed that the successive diagonalizing algorithm generalized the mitigate the construction tediousness for $v > 16$. Also, Bush [18] generalized the notion of MOLS and called it extension of orthogonal array of unity index and considered the array as a class of polynomials with coefficients in a finite Galois field for $s$ being a prime or power of prime. The construction of balanced incomplete block designs, and partially balanced incomplete block designs with $k = 2, 3, 4, 5$, and two distinct Near-Resolvable BIBDs were achieved by Saka et al. [19] with leverage on Mutually Orthogonal Latin Squares (MOLS) of order 7 and imposition of some algebraic properties. Saka et al. [20], A generalized resolvable balanced incomplete block designs whose parameter combinations were $k$ with the cyclic subgroup of the symmetric group $S_k$.

Over the times, many researches concentrate on construction of MOLS and its application to agricultural sector. However, this paper uses some algebraic structure to construct four distinct sets of mutually orthogonal latin square (MOLS) of order 4 and this opens ways for its application in cryptography, generation of one time password (OTP) and many other internet mechanisms needing security and protections.

Two Latin squares $L_1$ and $L_2$ of the same order, say $n$, are mutually orthogonal if every ordered pair $(i, j)$, $1 \leq i, j \leq n$, appears exactly once when $L_1$ and $L_2$ are superimposed on each other.

The following are examples of mutually orthogonal Latin square of order 4 as seen from their superimposition $S(L_1, L_2)$ :

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$L_1 =$

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1 | 0 | 3 | 2 |
| 2 | 3 | 0 | 1 |
| 3 | 2 | 1 | 0 |

, $L_2 =$

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 3 | 2 | 1 | 0 |
| 1 | 0 | 3 | 2 |
| 2 | 3 | 0 | 1 |

*and* $S(L_1, L_2) =$

| | | | |
|---|---|---|---|
| (0,0) | (1,1) | (2,2) | (3,3) |
| (1,3) | (0,2) | (3,1) | (2,0) |
| (2,1) | (3,0) | (0,3) | (1,2) |
| (3,2) | (2,3) | (1,0) | (0,1) |

## 2 Preliminaries

Our method shall employ the use of symmetric groups in abstract algebra. Hence, we shall do some preliminary discussion on this. Consider a non-empty set $X$. The **symmetric group** on $X$, i.e. $S_X$ under composition of mappings is the group of all permutations of $X$. A subgroup of $S_X$ is called a permutation group on $X$.

A bijection $X \simeq Y$ induces isomorphism $S_X \cong S_Y$. If $|X| = n$, $S_X$ is denoted by $S_n$ and called the *symmetric group of degree* $n$.

A permutation $\delta \in S_n$ can be expressed in the form

$$\begin{pmatrix} 1 & 2 & \cdots & n \\ \delta(1) & \delta(2) & \cdots & \delta(n) \end{pmatrix},$$

consisting of two rows of integers; the first row has integers $1, 2, \cdots, n$ often (but not necessarily) in their natural order, and the second row has $\delta(i)$ below $i$ for each $i = 1, 2, \cdots, n$. This is called a two-row notation for a permutation. One-row notation for a permutation is called a *cycle*. Note that the image of $\delta$ is defined as Im $\delta = \{\delta(1), \delta(2), \cdots, \delta(n)\} = [\delta(1), \delta(2), \cdots, \delta(n)]$.

Let $\delta \in S_n$. Consider a list of distinct integers $y_1, \cdots, y_r \in \mathbb{N}$ such that

$$\begin{aligned} &\delta(y_i) = y_{i+1}, \qquad i = 1, \cdots, r-1, \\ &\delta(y_r) = y_1, \\ &\delta(y) = y \text{ if } y \notin \{y_1, \cdots, y_r\}, \end{aligned}$$

then $\delta$ is called a cycle of length $r$ and denoted by $(y_1 \cdots y_r)$.

**Remark 2.1.** *A cycle of length 2 is called a transposition. If $\delta(y) = y$, we say $\delta$ does not move $y$. A cycle of length 1 is the identity mapping $I$ or $e$.*

Let $A = (a_{ij})$ be an $n \times m$ and $\delta \in S_n$. If $B$ is an $n \times m$ matrix gotten from $A$ by replacing the entries of the rows $1, 2, \cdots, n$ of $A$ with the entries of the rows $\delta(1), \delta(2), \cdots, \delta(n)$ of $A$ respectively. Then, this will be expressed as $B = A\left(R_{\text{Im }\delta}\right) = A\left(R_{\delta(1), \delta(2), \cdots, \delta(n)}\right)$.

## 3 Main Results

We shall now state and prove the result which will serve as our method of construction of MOLSs of order 4.

**Theorem 3.1.** *Suppose $Y = \{I, A, B, C\}$ where $I, A, B, C \in S_4$ such that $A^2 = B^2 = C^2 = I$ and $Y$ forms a group under composition ($\circ$) of maps. That is, $Y = \{I, A, B, C \in S_4 \mid A^2 = B^2 = C^2 = I\}$ such that $|Y| = 4$. Then, form a matrix $Y_0$ whose rows are the second rows of the 2-row format of the elements of $Y$. Consider the set $L = \{A \in S_4 : A^3 = I\}$ such that $L_i = \langle A_i \mid A_i \in L \rangle = \{L_{ij}\}_{j=1}^3 \leq S_4$ for $i = 1, 2, 3, 4$ and let $\mathcal{L}_i = \{Im\ L_{ij}\}_{j=1}^3$ for $i = 1, 2, 3, 4$. Then, $\mathcal{M}_i = \{M_{ij}\}_{j=1}^3 = Y_0(\mathcal{L}_i) = \left\{Y_0\left(R_{L_{ij}(1), L_{ij}(2), \cdots, L_{ij}(4)}\right)\right\}_{j=1}^3$ is a set of MOLS for each $i = 1, 2, 3, 4$.*

*Proof.* Let $S_4$ be the symmetric group of degree 4 and let $I$ be the identity permutation therein

$$I = (1) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \tag{3.1}$$

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An instance of this is when $Y = C_2 \times C_2 \trianglelefteq S_4$, the Klein four-group, whose elements are namely the even transpositions; $Y = \{I = (1),\ A = (12)(34),\ B = (13)(24),\ C = (14)(23)\}$. The multiplication table of $(Y, \circ)$ is shown in Table 1.

Table 1: Group formed by the symmetries of a rectangle i.e. $C_2 \times C_2$

| ∘ | I | A | B | C |
|---|---|---|---|---|
| I | I | A | B | C |
| A | A | I | C | B |
| B | B | C | I | A |
| C | C | B | A | I |

Note that in 2-row format,

$$A = (12)(34) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix},\ B = (13)(24) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \text{ and } C = (14)(23) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

Now, form a matrix $Y_0$ whose rows are the second rows of the 2-row format of the elements of $Y$. That is,

$$Y_0 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}. \tag{3.2}$$

Consider the set

$$L = \{A \in S_4 : A^3 = I\} = \{I, (234), (243), (123), (132), (124), (142), (134), (143)\} \tag{3.3}$$

Note that as shown in Table 2,

$$L_1 = \{L_{1i}\}_{i=1}^3 = \{I, (234), (243)\} = \langle(234)\rangle = \langle(243)\rangle \leq S_4 \tag{3.4}$$
$$L_2 = \{L_{2i}\}_{i=1}^3 = \{I, (123), (132)\} = \langle(123)\rangle = \langle(132)\rangle \leq S_4 \tag{3.5}$$
$$L_3 = \{L_{3i}\}_{i=1}^3 = \{I, (124), (142)\} = \langle(124)\rangle = \langle(142)\rangle \leq S_4 \tag{3.6}$$
$$L_4 = \{L_{4i}\}_{i=1}^3 = \{I, (134), (143)\} = \langle(134)\rangle = \langle(143)\rangle \leq S_4 \tag{3.7}$$

Table 2: Cyclic subgroup $L_i$ of $S_4$ of order 3 for $i = 1, 2, 3, 4$

| * | $I$ | $L_{i2}$ | $L_{i3}$ |
|---|---|---|---|
| $I$ | $I$ | $L_{i2}$ | $L_{i3}$ |
| $L_{i2}$ | $L_{i2}$ | $L_{i3}$ | I |
| $L_{i3}$ | $L_{i3}$ | $I$ | $L_{i2}$ |

Now, $\mathcal{M}_i = \{M_{ij}\}_{j=1}^3 = Y_0(\mathcal{L}_i) = Y_0\left(\{\text{Im } L_{ij}\}_{j=1}^3\right) = \left\{Y_0\left(R_{L_{ij}(1), L_{ij}(2), \cdots, L_{ij}(4)}\right)\right\}_{j=1}^3$. Observe that for each $i = 1, 2, 3, 4$ and any of two distinct $j \in \{1, 2, 3\}$, the pairs of Latin squares Observe that for each $i = 1, 2, 3, 4$ and any two distinct $j \in \{1, 2, 3\}$, the pair of Latin squares $Y_0\left(R_{L_{ij}(1), L_{ij}(2), \cdots, L_{ij}(4)}\right)$ are mutually orthogonal. Hence, $\mathcal{M}_i$ is a set of MOLS for each $i = 1, 2, 3, 4$. □

Designs 1, 2, 3 and 4 are now presented using Theorem 3.1. Each design is a distinct set of MOLS represented as $MOLS_{(1)}, MOLS_{(2)}, MOLS_{(3)}$ and $MOLS_{(4)}$.

The Construction of $MOLS_{(1)}$ from $L_1 = [(\ ), (234), (243)]$ is presented.

$$I = (\ ) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = (1)$$

$$(234) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix} = [1342]$$

$$(234) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{pmatrix} = [1423]$$

$\mathcal{L}_1 = [[1234], [1342], [1423]]$. Then, $M_{11}, M_{12}$ and $M_{13}$ are obtained from $Y_0(\mathcal{L}_1)$ respectively as

$$\mathcal{M}_1 = \left\{ M_{11} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \quad M_{12} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \quad M_{13} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \end{pmatrix} \right\}.$$

Thus, $M_{11}, M_{12}$ and $M_{13}$ form a the set of $MOLS_{(1)}$.

The Construction of $MOLS_{(2)}$ from $L_2 = [(\ ), (234), (243)]$ is presented.

$$() = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = I$$

$$(123) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix} = (2314)$$

$$(132) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} = (3124)$$

$$\mathcal{L}_2 = [(1234), (2314), (3124)].$$

Then, $M_{21}, M_{22}$ and $M_{23}$ are obtained from $Y_0(\mathcal{L}_2)$ respectively as

$$\mathcal{M}_2 = \left\{ M_{21} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix} , M_{22} = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} , M_{23} = \begin{pmatrix} 3 & 4 & 1 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \end{pmatrix} \right\}.$$

Thus, $M_{21}, M_{22}$ and $M_{23}$ form a set of $MOLS_{(2)}$.

The Construction of $MOLS_{(3)}$ from $L_3 = [(\ ), (124), (243)]$ is presented.

$$(\ ) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = I$$

$$(123) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} = (2431)$$

$$(132) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix} = (4132)$$

$$\mathcal{L}_3 = [(1234), (2431), (4132)].$$

Then, $M_{31}, M_{32}$ and $M_{33}$ are obtained from $Y_0(\mathcal{L}_2)$ respectively as

$$\mathcal{M}_3 = \left\{ M_{31} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \quad M_{32} = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \\ 3 & 4 & 1 & 2 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad M_{33} = \begin{pmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \end{pmatrix} \right\}.$$

Thus, $M_{31}, M_{32}$ and $M_{33}$ form a set of $MOLS_{(3)}$.

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The Construction of $MOLS_{(4)}$ from $L_4 = [(\ ), (134), (143)]$ is presented.

$$(\ ) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = I$$

$$(134) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix} = (13241)$$

$$(143) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix} = (4213)$$

$$\mathcal{L}_4 = [(1234), (3241), (4213)].$$

Then, $M_{41}, M_{42}$ and $M_{43}$ are obtained from $Y_0\,(\mathcal{L}_4)$ respectively as

$$\mathcal{M}_4 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \quad M_{42} = \begin{pmatrix} 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad M_{43} = \begin{pmatrix} 4 & 3 & 2 & 1 \\ 2 & 1 & 4 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \right\}.$$

Thus, $M_{41}, M_{42}$ and $M_{43}$ form a set of $MOLS_{(4)}$.

## 4 Conclusion

The algebraic structure, Klein-4 Group was introduced in this research as a novel technique in the construction of MOLSs. The method resulted into generation of 4 distinct complete sets of mutually orthogonal Latin squares. The design could open ways for its application in cryptography, generation of one time password (OPT) and in cyber environments such as Internet of Things (IoT) needing lightweight cybersecurity measures.

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