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Derivation of a Stochastic Labour Market Model from a Semi – Markov Model

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Abstract

The labour Market which is a major component of any economy refers to the supply and demand for labour. The two possible labour market states are unemployment and employment, and the transitions between these states are described by Markov processes. This paper is aimed at deriving a two – state stochastic model from the interval transition probability of a Semi – Markov Model that can be used to study the transition rate between the two labour market states. A stochastic model of four equations is derived using the probable movement between the two labour market states. The solution of the model exist and it is unique. The derived model equations are solved analytically and matlab programmes are written to help in the computation of the probable transition probabilities. The results of applying this model to the Nigeria's labour market shows that the rate at which individuals are moving from unemployment state to employment state is very small compared to the rate at which individuals are remaining unemployed: about 559,948 persons (0.69%) are likely to enter the employment state from the unemployment state in 2035 while about 80,445,864 (99.13%) persons are likely to remain unemployed in that same year (2035). Also, the rate at which individuals are moving from employment state to unemployment state is very small compared to the rate at which individuals are remaining employed: about 4,604,040 (6.80%) persons are likely to enter the unemployment state from the employment state in 2035 while about 55,532,851 (82.02%) persons are likely to remain employed in that same year (2035) thereby increasing the number of unemployed. The result also indicates that this model can only measure effectively short term movements between the two labour market states.

Keywords: Labour Market, Stochastic Model, Markov Chain, Semi-Markov Model, Exponential Distribution.
MSC2010: 91B70

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1 Introduction

The major component of any economy which is the supply and demand for labour is called the labour market. In the labour market, employees provide the supply and employers provides the demand. There are two labour market states namely: unemployment and employment [1]. The transition from one state to another can be estimated using probability. The sum of Unemployment and employment persons gives rise to labour force. The labour force population covers all persons aged 15 – 64 years who are willing and able to work regardless of whether they have a job or not. The international definition of unemployment, employment is not a function of wages earned nor is it a function of job satisfaction. Rather employment and unemployment are treated as a function of a person's involvement or otherwise in economic activity even if that activity is performed solely to make end meet and not for satisfaction or enjoyment [2]. There is no universal standard definition of unemployment (and probably employment) as various countries adopt definitions that suit their local priorities. Virtually all countries however use the International Labour Organization (ILO) definition or a variant of it to compute unemployment. The ILO definition covers people aged 15 – 64 who during the reference period (which is usually the week preceding the time the survey is administered) were available for work, actively seeking for work but were unable to find work. Like most countries in the world, the Nigerian National Bureau of Statistics (NBS) uses a variant of the ILO definition such that the unemployment is the proportion of those in the labour forces who were actively looking for work but could not find it at least 20 hours during the reference period to the total labour force population. Also, a person is regarded as employment if he/she is engaged in the production of goods and services ([2],[3]).

Unemployment according to International Labour Organisation (ILO) is among the biggest threats to social stability in many countries including Nigeria ([4],[5],[6]). Unemployment has several causes and effects ([6],[7],[8]). Several studies have been conducted on modelling and forecasting of unemployment rate ([9],[10]) and employment generation ([4],[11],[12]). The forecasting of unemployment rate and employment generation in the above studies depends on the unemployment rate and employment generation of previous years. The limitation of these studies lies in the fact that, if there is no history of unemployment rate and employment generation, it will be impossible to predict unemployment rate and employment generation for the future: hence, the motivation for this research work. We derived a stochastic model from the interval transition probability of a Semi – Markov model that can help in the study of movement between the two labour market states, prediction of unemployment rate and employment generation depending on the unemployment rate and employment generation of current year and not on the history.

The remaining part of this research work shall be discussed under the following sub-headings: 2. Preliminaries (Stochastic Process, Markov and Semi-Markov Processess), 3. The Model derivation, 4. Analytical solution of the model, 5. Application, 6. Conclusion.

2 Preliminaries

Since a Semi – Markov process is a generalization of the Markov process and the Markov process is a stochastic process, we shall first introduce the concepts of stochastic process, Markov process and the Semi – Markov process.

2.1 Stochastic Process

A stochastic Process is a family of random variables $\{X_t, t \in T\}$, where T is some index set. The outcome of the process appears to be unpredictable. The possible values of the random variable

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X_t are called states and set of states is called a space state denoted by S which can be discrete or continuous [13]. The set of possible values of the indexing parameter t, is called the parameter space and it is denoted by T which can also be discrete or continues. According to [14], a stochastic process is a probability model that describes the evolution of a system evolving randomly in time. If the process is observed at a set of discrete times, say at the end of every day or every hour, we get a discrete – time stochastic process. On the other hand, if the process is observed continuously at all times, we get a continuous – time stochastic process.

As contained in [15], a stochastic process collects realizations of one or more random variables over time and the theory of stochastic processes tries to find models which describe such probabilistic systems. One can distinguish between discrete-time processes and continuous-time processes. While the system is observed at discrete points in time only in the first case, there is continuous observation given for the latter.

2.2 Markov Chain

A Markov Chain is a stochastic process in which the conditional probability of any future event given the past and present states is independent of the past states and depends only on the present state. ([13],[14],[16],[17]).
Mathematically, a stochastic process $\{X_t, t \in T\}$, is a Markov Chain if it has the following Markovian Property

$$P\{X_{t+1} = j \backslash X_0 = k_0, X_1 = k_1, ..., X_t = i\} = P\{X_{t+1} = j \backslash X_t = j\} = P_{ij} \tag{2.1}$$

P_{ij} must satisfy the transition probabilities conditions as follows,

$$P_{ij} \geq 0, \ and \ \sum_{j=1}^{n} P_{ij} = 1, \ \ i, j = 1, 2, ..., N. \tag{2.2}$$

N is the number of states in the system.

2.3 Semi–Markov Chain

We can think of the semi – Markov Chain as a process whose successive state occupancies are governed by the transition probabilities of a Markov Chain, but whose stay in any state is described by an integer – valued random variable that depends on the state presently occupied and on the state to which the next transition will be made. The parameters characterizing a semi – Markov process are the transition probabilities P_{ij} as well as the probability mass (Holding time) function. At transition instants the semi – Markov Chain behaves just like a Markov Chain hence the name semi – Markov Chain ([16],[18]). P_{ij} must satisfy the same equations (2) as the transition probabilities for a Markov process as in ([19],[20]).

The Semi – Markov Process is classified based on the relationship between its transition states. A Semi – Markov Process (SMP) is called ***regular*** if there is only a finite number of transitions possible in a finite time period. The SMP is ***irreducible*** if each state can be reached from any other state; the states are said to communicate with each other in this case. A state j is called ***recurrent*** if the process returns to this state j in a period less than infinity and it is called ***transient*** otherwise (if it never returns). A state is denoted as ***positive recurrent*** if it is recurrent and the expected returning time to state i, given the process started in i, is less than infinity. For a SMP, a recurrent state i is called ***aperiodic*** if it is possible to visit this state anytime. ***Periodicity*** with period d is given if a state i can only be visited at positive multiples integers of $d, d > 1$. Therefore, aperiodicity actually means $d = 1$ ([14],[21]).

2.3.1 The Semi – Markov Process with two States

The possible states of an individual in the labor market are ***unemployment or employment*** and the transitions between these states are described by Markov processes. Thus, four transition probabilities for the future: an unemployed or employed person can either be unemployed or employed at some future point after a period t. The state *unemployment* is accessible from the state *employment* and vice versa. Hence, the states communicate and the SMP is irreducible. The SMP is regular because the probability of very short durations is less than one. This means that finding a job or losing it normally needs some time. It is positive recurrent because the expected 'revisiting' duration for an unemployed or an employed is less than infinity. The SMP is aperiodic because obviously $d = 1$ in this two-state process [14].

2.3.2 The Holding Time and Waiting Time of a Semi – Markov Process ([16],[22])

The Holding Time
Whenever the process enters its current state, say i, it remains there for a time, say T_{ij}, (after choosing the next state) before making a transition to the next state, say j. T_{ij} is called the **Holding Time** in state i. The Holding Times are positive integer – valued random variables each governed by a probability distribution function $h_{ij}(.)$ called the holding time distribution function for a transition from state i to state j. Thus

$$P(T_{ij} = m) = h_{ij}(m), m = 1, 2, ...; i, j = 1, 2, ..., N \tag{2.3}$$

The Waiting Time
The waiting time Y_i is the time the process will spend in state i when we do not know the successor state. This is also described by a probability density function denoted by $w_i(.)$

2.3.3 Cumulative Probability Distribution for the Holding Times and Waiting Times of Semi – Markov Process ([16],[22])

Let $F_{ij}(.)$ be the cumulative probability distribution of the positive (continuous) integer-valued random variable (holding time) T_{ij},

$$F_{ij}(t) = P(T_{ij} \leq t) = \int_{m=0}^{t} h_{ij}(m)dm \tag{2.4}$$

and
let $\overline{F_{ij}}(.)$ be the complementary cumulative probability distribution of T_{ij},

$$\overline{F_{ij}}(t) = P(T_{ij} > t) = \int_{m=t+1}^{\infty} h_{ij}(m)dm = 1 - F_{ij}(t). \tag{2.5}$$

Let Y_i be the time the process spends in state i before moving out of the state i, then Y_i is called the waiting time in state i.
Let $w_i(m)$ be the probability that the system will spend m time units in state i called the probability distribution function of the waiting time Y_i, then

$$w_i(m) = P(Y_i = m) = \sum_{j=i}^{N} P_{ij} h_{ij}(m) \tag{2.6}$$

The cumulative probability distribution for waiting times is

$$w_i(n) = P(Y_i \leq m) = \int_{m=1}^{n} w_i(m)dm = \sum_{j=1}^{n} P_{ij}F_{ij}(n) \tag{2.7}$$

and the complementary cumulative probability distribution for waiting times is

$$\overline{w_i}(n) = P(Y_i > m) = 1 - w_i(n) = \int_{m=n+1}^{\infty} w_i(m)dm = \sum_{j=1}^{n} P_{ij}\overline{F_{ij}}(t) \tag{2.8}$$

3 The Model Derivation

3.1 The Interval Transition Probability from State i to State j in the Interval (0,t)

The probability that the process will occupy state j at time t if it entered state i at time zero (called the interval transition probability from state i to state j in the interval $(0,t)$) is given by [16] as

$$\Phi_{ij}(t) = \delta_{ij}\overline{w_i}(t) + \sum_{k=0}^{N} P_{ik} \int_{0}^{t} h_{ik}(m)\Phi_{kj}(t-m)dm \tag{3.1}$$

$$\delta_{ij} = \begin{cases} 1, & i = j \ ; \\ 0, & i \neq j, \end{cases} \qquad i,j = 1,2,3,...$$

3.2 The Semi – Markov Process in the Labour Market

The figure below is describing the possible movement between the two labour market states.

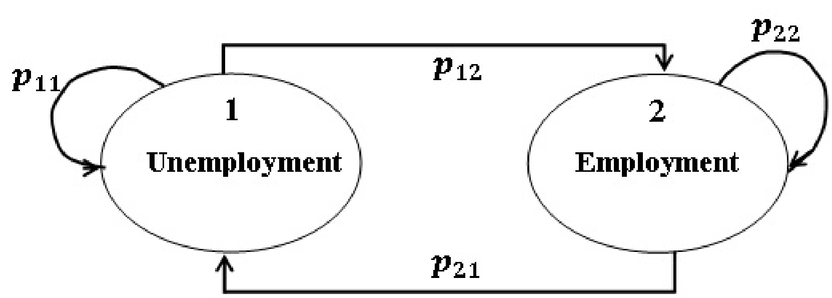

Figure 1: The flow diagram of unemployment and employment states

Typically as can be seen in Figure 1, the possible states of an individual in the labour market are ***unemployment or employment*** and the transitions between these states are described by Markov processes and the stay in a state before transition is governed by a probability distribution. Also, p_{12} is the transition rate from unemployment state to employment state, p_{11} is the rate of remaining unemployed, p_{21} is the transition rate from employment state to unemployment state and p_{22} is the rate of remaining employment. Thus, the four transition probabilities for the future are possible assuming that there is no movement into the labour market form an external source:

(i) an unemployed person can be employed after a period of time t,

(ii) an unemployed person can remain unemployed after a period of time t,

(iii) an employed person can be unemployed after a period of time t and,

(iv) an employed person can remain employed after a period of time t.

3.3 The Labour Market Equations

Since the two labour market states has no branching state, $k = j, m = t$, equation (9) now becomes

$$\Phi_{ij}(t) = \delta_{ij}\overline{w_i}(t) + \sum_{j=0}^{N} p_{ij} \int_0^t h_{ij}(m)dm \tag{3.2}$$

Therefore, the four (4) possible Stochastic Labour Market Model equations following from equation (10) and Figure 1 are thus

$$\Phi_{12}(t) = p_{12} \int_0^t h_{12}(n)dn \tag{3.3}$$

$$\Phi_{11}(t) = \overline{w_1}(t) + p_{11} \int_0^t h_{11}(n)dn \tag{3.4}$$

$$\Phi_{21}(t) = p_{21} \int_0^t h_{21}(n)dn \tag{3.5}$$

$$\Phi_{22}(t) = \overline{w_2}(t) + p_{22} \int_0^t h_{22}(n)dn \tag{3.6}$$

Equations (11)-(14) are the Stochastic Labour Market model derived from the interval transition probability of the Semi – Markov process described by equation (9). This Model equations can be used to study the probable movement from one state to the other within the labour market states if the stay in the present state is governed by a probability distribution. In this work, we used the exponential distribution [15].

3.4 The Exponential Distribution

The Exponential distribution is often used as a model for durations. It can be used to measure the time between successes intervals. Because the exponential represents time intervals, it is a continuous (not discrete) probability distribution. The exponential density function is given by [21] as

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases} = h_{ij}(\lambda) \tag{3.7}$$

The parameter λ is called rate parameter. It is the inverse of the expected duration, μ in a state before the next event. That is $\lambda = \frac{1}{\mu}$.
Substituting equation (15) into equations (11) – (14), we have

$$\Phi_{12}(t) = p_{12} \int_0^t \lambda e^{-\lambda m} dm \tag{3.8}$$

$$\Phi_{11}(t) = \overline{w_1}(t) + p_{11}\int_0^t \lambda e^{-\lambda m} dm \tag{3.9}$$

$$\Phi_{21}(t) = p_{21}\int_0^t \lambda e^{-\lambda m} dm \tag{3.10}$$

$$\Phi_{22}(t) = \overline{w_2}(t) + p_{22}\int_0^t \lambda e^{-\lambda m} dm \tag{3.11}$$

3.5 Existence and Uniqueness of Solution of the Model Equations

By the existence and uniqueness of solution in ([23],[24],[25]), equations (16) - (19) can be written as

$$f_1(t) = p_{12}\int_0^t \lambda e^{-\lambda m} dm \tag{3.12}$$

$$f_2(t) = p_{11}\left(\int_{m=t+1}^{\infty} \lambda e^{-\lambda m} dm\right) + p_{11}\int_0^t \lambda e^{-\lambda m} dm \tag{3.13}$$

$$f_3(t) = p_{21}\int_0^t \lambda e^{-\lambda m} dm \tag{3.14}$$

$$f_4(t) = p_{22}\left(\int_{m=t+1}^{\infty} \lambda e^{-\lambda m} dm\right) + p_{22}\int_0^t \lambda e^{-\lambda m} dm \tag{3.15}$$

Since $h_{ij}(m) = \lambda e^{-\lambda m}$ is a continuous probability distributions with initial condition $h_{ij}(0) = 0$, the functions f_1, f_2, f_3 and f_4 satisfies the continuity conditions of a function. So, the functions are continuous functions within the interval $(0, t)$.
Next,

$$\frac{\partial f_1}{\partial m} = p_{12}\frac{\partial}{\partial m}\int_0^t \lambda e^{-\lambda m} dm \tag{3.16}$$

$$\frac{\partial f_2}{\partial m} = p_{11}\frac{\partial}{\partial m}\left(\int_{m=t+1}^{\infty} \lambda e^{-\lambda m} dm\right) + p_{11}\frac{\partial}{\partial m}\int_0^t \lambda e^{-\lambda m} dm \tag{3.17}$$

$$\frac{\partial f_3}{\partial m} = p_{21}\frac{\partial}{\partial m}\int_0^t \lambda e^{-\lambda m} dm \tag{3.18}$$

$$\frac{\partial f_4}{\partial m} = p_{22}\frac{\partial}{\partial m}\left(\int_{m=t+1}^{\infty} \lambda e^{-\lambda m} dm\right) + p_{22}\frac{\partial}{\partial m}\int_0^t \lambda e^{-\lambda m} dm \tag{3.19}$$

Equations (24) – (27) also satisfies the conditions existence and uniqueness solution for a linear system and are continuous within the interval $(0, t)$. Hence, there exist a solution of the Model equations (16) – (19) and the solution is unique.

Table 1: Parameter Values of the Model

Parameter	Value	Source
p_{12}	0.0069	Table 3
p_{11}	0.9931	Table 3
p_{21}	0.0774	Amaefule et al.,2017 and Table 2
p_{22}	0.9226	Amaefule et al.,2017 and Table 2
μ_U	3	Table 4
λ_U	0.3333	Table 4
μ_E	9	Table 5
λ_E	0.1111	Table 5

4 Analytical Solution of the Model

Consider the integral part of the model equations (16) – (19)

$$\int_0^t \lambda e^{-\lambda m} dm = -\frac{\lambda}{\lambda} e^{-\lambda m}\Big|_0^t = 1 - e^{-\lambda t} \tag{4.1}$$

Equation (28) is the cumulative distribution function for the exponential distribution with rate parameter λ. Substituting equation (28) in the two state model equations (16) – (19), we have

$$\Phi_{12}(t) = p_{12}\left[1 - e^{-\lambda t}\right] \tag{4.2}$$

$$\Phi_{11}(t) = \overline{w}_1(t) + p_{11}\left[1 - e^{-\lambda t}\right] \tag{4.3}$$

$$\Phi_{21}(t) = p_{21}\left[1 - e^{-\lambda t}\right] \tag{4.4}$$

$$\Phi_{22}(t) = \overline{w}_2(t) + p_{22}\left[1 - e^{-\lambda t}\right] \tag{4.5}$$

Equations (29) – (32) measures the transition probabilities between the labour market states if the stay in any of the state is following Exponential Distribution.

5 Application of the Derived Model equations to the Nigeria's Labour Market

5.1 The Results and Discussions

Using the data of Table 2, 3, 4 and 5 in the appendix, the values of the parameters of the model equations (29) - (32) are presented in Table 1 below.

The table generated from fixing the values of Table 1 to the model equations (29) – (32) is presented in Table 6 and 7 (appendix) and the graphs obtained from the tables are presented in Figure 2 and Figure 3.

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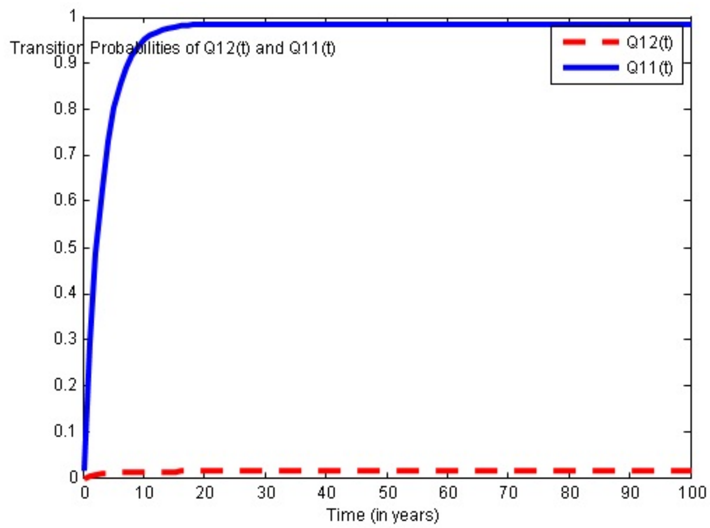

Figure 2: Graph showing the transition probabilities of $\Phi_{12}(t)$ and $\Phi_{11}(t)$ based on the exponential distribution.

Figure 2 and Figure 3 are representing the transition probabilities of moving from unemployment state to employment state after a spell of time and vice versa. The figures are also describing the transition probabilities of remaining in a state after a spell of time.

In particular, $\Phi_{12}(t)$ of Figure 2 is describing the transition probabilities of moving from unemployment state to employment state after a spell of time t. While $\Phi_{11}(t)$ is describing the transition probabilities of remaining in the unemployment state after a spell of time t.

It can be observed from the Figure 2 that there is a wide gap between the transition probabilities of moving from unemployment state and staying unemployed. This gap is simply indicating that the rate at which individuals are moving from unemployment state to employment state is very small compared to the rate at which individuals are remaining unemployed. From Table 2, the transition probability of entering the employment state from unemployment state in 2017 is 0.0020 representing 0.20% of the persons in the labour force. While the transition probability of remaining in the unemployment state is 0.2867 representing 28.67% of the persons in the labour force. This is simply indicating that in 2017, about 162,303 persons must have entered the employment state while about 23,241,900 persons must have remained unemployed. If this is allowed to continue without any intervention, then about 357,068 persons (0.44%), 413,875 persons (0.51%), 535602 persons (0.66%), 551,833 persons (0.68%) and 559,948 persons (0.69%) are likely to enter the employment state from the unemployment state in 2019, 2020, 2025, 2030 and 2035 respectively. Similarly about 23,241900 (28.64%) would have remained unemployed in 2017. Subsequently, about 51,150,033 (63.03%), 59,492,447 (73.31%), 76,607,379 (94.40%), 79,837,224 (98.39%) and 80,445,864 (99.13%) persons are likely to remain unemployed in the year 2019, 2020, 2025, 2030 and 2035 respectively.

Also, $\Phi_{21}(t)$ and $\Phi_{22}(t)$ of Figure 3 are describing the transition probabilities of moving from the employment state to the unemployment state after a period of time.

In particular, $\Phi_{21}(t)$ of Figure 3 is describing the transition probabilities of moving from employment state to unemployment state after a period of time t. While $\Phi_{22}(t)$ is describing the transition probabilities of remaining in the employment state after a period of time t.

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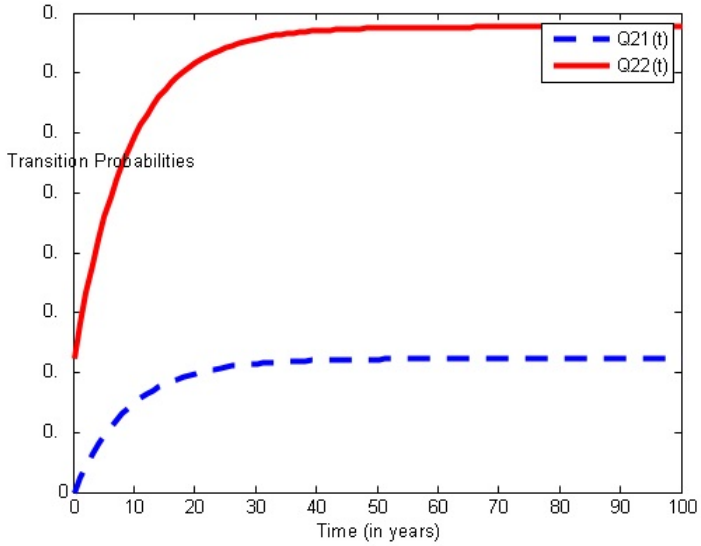

Figure 3: Graph showing the transition probabilities of $\Phi_{21}(t)$ and $\Phi_{22}(t)$ based on the exponential distribution

From Table 7, the transition probability of entering the unemployment state from employment state in 2017 is 0.0081 representing 0.81% of the persons in the labour force. While the transition probability of remaining in the employment state is 0.1663 representing 16.63% of the persons in the labour force. This is simply indicating that about 548,422 persons and 11,259,587 persons must have entered the unemployment state and employment respectively in 2017. If this continues about 1,482,772 (2.19%), 1,882,240 (2.78%), 3,310,847 (4.89%), 4,136,866 (6.11%) and 4,604,040 (6.8%) persons are likely to enter the unemployment state from the employment state in 2019, 2020, 2025, 2030 and 2035 respectively. In a similar manner, 21,462953 (31.70%), 25,775,855 (38.07%), 41,409,280 (61.16%), 50,387,159 (74.42%) and 55,532,851 (82.02%) persons are likely to remain in the employment state in 2017, 2019, 2020, 2025, 2030 and 2035 respectively.

Also, it can be observed from the Figure 3 that there is wide gap between the transition probabilities of moving from employment state and staying employed. This gap is simply indicating that the rate at which individuals are moving from employment state to unemployment state is very small compared to the rate at which individuals are remaining employed. This explains the reason why only few persons are leaving the unemployment state to the employment state.

Figure 2 and Figure 3 shows that both the transition probabilities of leaving a state and remaining in a state are almost stable from 15 years upward respectively. This shows that this Model based on Exponential distribution cannot measure the movements between the two labour market states from fifteen (15) years upward which is a limitation. So, this Model that measures the movement between the two labour market states whose stay in a state before transition followed the exponential distribution can only be used to study short term movement between the two labour market states.

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6 Conclusion

The Stochastic Model derived from the interval transition probabilities of a semi – markov model in this work can be a very useful tool in predicting unemployment rate and employment generation of a country. It can give the future position of individuals in the labour market if the stay in the present state is described by a probability distribution. It is important to note that the model presented here can only give a probable short term prediction of fifteen (15) years and below.

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APPENDIX

Table 2: Nigeria's Labour force, employment and Unemployment Statistics from 2006-2016

Year	Labour Force	Employed (E)	E%	Unemployed (U)	U%
2006	57455701	50388650	87.7	7067051	12.3
2007	59294283	51763909	87.3	7530374	12.7
2008	61191700	52074137	85.1	9117563	14.9
2009	63149835	50709318	80.3	12440517	19.7
2010	65170629	57089471	78.6	8081158	21.4
2011	67256090	51181884	76.1	16074206	23.9
2012	69105775	50170793	72.6	18934982	27.4
2013	71105800	53542667	75.3	17563133	24.7
2014	72931608	55209227	75.7	17722381	24.3
2015	76957923	54486209	70.8	22471714	29.2
2016	81151885	52586421	64.8	28565464	35.2
Total	744771229	579202686	854.3	165568543	245.7
Av.	67706475	52654790	77.7	15051686	22.3

Source: [3]

Table 3: New Job Created by Sectors

Quarter	New Jobs	Public Sector	Private Sector	Formal Sector	Informal Sector	Number of Employed
2012, Q_1	427,296	22,644	404,652	164,293	240,359	18907180
Q_2	385,913	24,975	360,938	152,018	208,920	
2013, Q_1	431,021	24,368	406,653	174,326	232,327	
Q_2	221,054	28,075	192,979	80,412	112,567	
Q_3	245,989	28,931	217,058	76,385	140,673	
Q_4	265,702	20,827	244,875	101,597	143,278	17597322
2014, Q_1	240,871	5,959	234,912	76,018	158,894	18171402
Q_2	259,358	4,812	254,541	78,755	175,786	18138673
Q_3	349,343	5,735	343,608	145,464	198,144	18232434
Q_4	369,485	4,387	365,098	138,026	227,072	17724669
2015, Q_1	469,070	5,726	463,344	130,941	332,403	
Q_2	141,368	6,395	134,973	51,070	83,903	19634580
Q_3	475,180	4,818	470,362	41,672	428,698	20723606
Q_4	499,521	-4,288	503,809	27,246	476,563	22451816
2016, Q_1	79,465	-3,038	82,503	21,477	61,026	24508611
Q_2	155,444	-5,223	160,667	55,124	105,543	26059701
Q_3	187,226	-7,012	194,238	49,587	144,651	27115086
Average	306,077	9,888	296,189	92,024	204,165	20772090
%	100	3.23	96.77	30.07	66.70	

Source: [3, 26]

Table 4: Duration as Unemployed

Unemployment Duration (x) in years	Number of Respondents (f)
0	19
1	26
2	79
3	6
4	2
5	44
5	4
7	0
8	20
	$\sum f = 200$

Source: Questionnaire, 2019.

Table 5: Duration as Employed

Employment Duration (x) in years	Number of Respondents (f)
0	1
1	27
2	3
3	14
4	12
5	15
6	0
7	13
8	22
9	22
10	13
11	0
12	0
13	11
14	13
15	19
16	1
17	0
18	1
19	1
20	0
21	0
22	9
23	1
24	2
	$\sum f = 200$

Source: Questionnaire, 2019.

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Table 6: The Transition Probabilities of $\Phi_{12}(t)$ and $\Phi_{11}(t)$

Time (t) in years	$\Phi_{12}(t)$	$\Phi_{11}(t)$
0	0.0000	0.0069
1	0.0020	0.2864
2	0.0034	0.4867
3	0.0044	0.6303
4	0.0051	0.7331
5	0.0056	0.8068
6	0.006	0.8596
7	0.0062	0.8974
8	0.0064	0.9246
9	0.0066	0.944
10	0.0067	0.9579
11	0.0067	0.9679
12	0.0068	0.975
13	0.0068	0.9802
14	0.0068	0.9838
15	0.0069	0.9865
16	0.0069	0.9883
17	0.0069	0.9897
18	0.0069	0.9907
19	0.0069	0.9913
20	0.0069	0.9918
21	0.0069	0.9922
22	0.0069	0.9925
23	0.0069	0.9926
24	0.0069	0.9928
25	0.0069	0.9929
26	0.0069	0.9929
27	0.0069	0.993
28	0.0069	0.993
29	0.0069	0.993
30	0.0069	0.9931
31	0.0069	0.9931
32	0.0069	0.9931
33	0.0069	0.9931
34	0.0069	0.9931
35	0.0069	0.9931
36	0.0069	0.9931
37	0.0069	0.9931
38	0.0069	0.9931
39	0.0069	0.9931
40	0.0069	0.9931

Table 7: contd.

41	0.0069	0.9931
42	0.0069	0.9931
43	0.0069	0.9931
44	0.0069	0.9931
45	0.0069	0.9931
46	0.0069	0.9931
47	0.0069	0.9931
48	0.0069	0.9931
49	0.0069	0.9931
50	0.0069	0.9931
51	0.0069	0.9931
52	0.0069	0.9931
53	0.0069	0.9931
54	0.0069	0.9931
55	0.0069	0.9931
56	0.0069	0.9931
57	0.0069	0.9931
58	0.0069	0.9931
59	0.0069	0.9931
60	0.0069	0.9931
61	0.0069	0.9931
62	0.0069	0.9931
63	0.0069	0.9931
64	0.0069	0.9931
65	0.0069	0.9931
66	0.0069	0.9931
67	0.0069	0.9931
68	0.0069	0.9931
69	0.0069	0.9931
70	0.0069	0.9931
71	0.0069	0.9931
72	0.0069	0.9931
73	0.0069	0.9931
74	0.0069	0.9931
75	0.0069	0.9931
76	0.0069	0.9931
77	0.0069	0.9931
78	0.0069	0.9931
79	0.0069	0.9931
80	0.0069	0.9931

Table 8: contd.

81	0.0069	0.9931
82	0.0069	0.9931
83	0.0069	0.9931
84	0.0069	0.9931
85	0.0069	0.9931
86	0.0069	0.9931
87	0.0069	0.9931
88	0.0069	0.9931
89	0.0069	0.9931
90	0.0069	0.9931
91	0.0069	0.9931
92	0.0069	0.9931
93	0.0069	0.9931
94	0.0069	0.9931
95	0.0069	0.9931
96	0.0069	0.9931
97	0.0069	0.9931
98	0.0069	0.9931
99	0.0069	0.9931
100	0.0069	0.9931

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Table 9: The Transition Probabilities of $\Phi_{21}(t)$ and $\Phi_{22}(t)$

Time (t) in years	$\Phi_{12}(t)$	$\Phi_{11}(t)$
0	0	0.0774
1	0.0081	0.1663
2	0.0154	0.2458
3	0.0219	0.317
4	0.0278	0.3807
5	0.033	0.4376
6	0.0377	0.4886
7	0.0418	0.5343
8	0.0456	0.5751
9	0.0489	0.6116
10	0.0519	0.6443
11	0.0546	0.6736
12	0.057	0.6998
13	0.0591	0.7232
14	0.0611	0.7442
15	0.0628	0.7629
16	0.0643	0.7797
17	0.0657	0.7947
18	0.0669	0.8082
19	0.068	0.8202
20	0.069	0.831
21	0.0699	0.8406
22	0.0707	0.8492
23	0.0714	0.857
24	0.072	0.8639
25	0.0726	0.87
26	0.0731	0.8756
27	0.0735	0.8805
28	0.074	0.8849
29	0.0743	0.8889
30	0.0746	0.8924
31	0.0749	0.8956
32	0.0752	0.8984
33	0.0754	0.901
34	0.0756	0.9033
35	0.0758	0.9053
36	0.076	0.9071
37	0.0761	0.9087
38	0.0763	0.9102
39	0.0764	0.9115
40	0.0765	0.9127

Table 10: contd.

41	0.0766	0.9137
42	0.0767	0.9146
43	0.0767	0.9155
44	0.0768	0.9162
45	0.0769	0.9169
46	0.0769	0.9175
47	0.077	0.918
48	0.077	0.9185
49	0.0771	0.9189
50	0.0771	0.9193
51	0.0771	0.9197
52	0.0772	0.92
53	0.0772	0.9203
54	0.0772	0.9205
55	0.0772	0.9207
56	0.0772	0.9209
57	0.0773	0.9211
58	0.0773	0.9213
59	0.0773	0.9214
60	0.0773	0.9215
61	0.0773	0.9216
62	0.0773	0.9217
63	0.0773	0.9218
64	0.0773	0.9219
65	0.0773	0.922
66	0.0773	0.922
67	0.0774	0.9221
68	0.0774	0.9222
69	0.0774	0.9222
70	0.0774	0.9222
71	0.0774	0.9223
72	0.0774	0.9223
73	0.0774	0.9223
74	0.0774	0.9224
75	0.0774	0.9224
76	0.0774	0.9224
77	0.0774	0.9224
78	0.0774	0.9225
79	0.0774	0.9225
80	0.0774	0.9225

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Table 11: contd.

81	0.0774	0.9225
82	0.0774	0.9225
83	0.0774	0.9225
84	0.0774	0.9225
85	0.0774	0.9225
86	0.0774	0.9225
87	0.0774	0.9225
88	0.0774	0.9226
89	0.0774	0.9226
90	0.0774	0.9226
91	0.0774	0.9226
92	0.0774	0.9226
93	0.0774	0.9226
94	0.0774	0.9226
95	0.0774	0.9226
96	0.0774	0.9226
97	0.0774	0.9226
98	0.0774	0.9226
99	0.0774	0.9226
100	0.0774	0.9226